This print-out should have 10 questions, check that it is complete. Multiple-choice questions may continue on the next column or page: find all choices before making your selection. The due time is Central time. 
Chapter 18 problems.

001 (part 1 of 2) 5 points
A standing wave is formed on a string that is 27 m long, has a mass per unit length \( \mu = 0.00641 \, \text{kg/m} \), and is stretched to a tension of 25 N.
Find the fundamental frequency.
Correct answer: 1.1565 s\(^{-1}\).
Explanation:
Basic Concepts: standing wave
Solution:
\[
f_1 = \frac{v}{2L} = \frac{(62.4512 \, \text{m/s})}{(2)(27 \, \text{m})} = 1.1565 \, \text{s}^{-1},
\]
where
\[
v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{(25 \, \text{N})}{(0.00641 \, \text{kg/m})}} = 62.4512 \, \text{m/s}.
\]

002 (part 2 of 2) 5 points
Find the next frequency that could cause a standing wave pattern.
Correct answer: 2.31301 s\(^{-1}\).
Explanation:
\[
f_2 = 2f_1 = 2 \times 1.1565 = 2.31301 \, \text{s}^{-1}.
\]

003 (part 1 of 1) 0 points
If the air in an organ pipe with one end closed is in resonance with a tuning fork so that the air column (including the two ends) contains two nodes and two antinodes, the length of the pipe is approximately:

1. \( 2\lambda \)
2. \( \frac{3\lambda}{4} \) correct
3. \( \lambda \)
4. \( \frac{\lambda}{4} \)
5. \( \frac{\lambda}{2} \)

Explanation:
The neighboring node and antinode are \( \frac{\lambda}{4} \) apart. Since there are two nodes and two antinodes, the length of the pipe is given by
\[
\frac{\lambda}{4} \times 3 = \frac{3\lambda}{4}.
\]

004 (part 1 of 2) 0 points
Given: The speed of sound in air is 344 m/s. If an organ pipe is to resonate at 28.8 Hz, what is its required length if it is open at both ends?
Correct answer: 5.97222 m.
Explanation:
The wavelength of the standing sound wave is
\[
\lambda = \frac{v_s}{f} = \frac{344 \, \text{m/s}}{28.8 \, \text{Hz}} = 11.9444 \, \text{m}.
\]
If the pipe is open at both ends, then its length \( L_o \) must be half of the wavelength of the standing sound wave
\[
L_o = \frac{\lambda}{2} = \frac{(11.9444 \, \text{m})}{2} = 5.97222 \, \text{m}.
\]

005 (part 2 of 2) 0 points
What is its required length if it is closed at one end?
Correct answer: 2.98611 m.
**Explanation:**

When the pipe is closed at one end, its length \( L_c \) has to be one fourth of the wavelength \( \lambda \)

\[
L_c = \frac{\lambda}{4} = \frac{(11.9444 \text{ m})}{4} = 2.98611 \text{ m}.
\]

006 (part 1 of 1) 10 points
A tuning fork vibrating at frequency \( f = 868 \text{ Hz} \) is held above a long cylindrical pipe with an open top end. The pipe is partially filled with water up to a slowly rising height \( h \).

For some values of the water level \( h = h_1, h_2, h_3, \ldots \) the air column in the pipe (above the water) resonates to the tuning fork. A student performing this experiment notices equal height differences between successive resonances,

\[
h_2 - h_1 = h_3 - h_2 = h_4 - h_3 = \cdots = \Delta h.
\]

Given the speed of sound in air \( v_s = 344 \text{ m/s} \), calculate the height difference \( \Delta h \).
Correct answer: 19.8157 cm.

Explanation:

The resonance happens when the standing sound wave of wavelength

\[
\lambda = \frac{v_s}{f}
\]

has a pressure node at the open top of the pipe and a displacement node at the water level — which acts as a closed bottom of the air column. The pressure nodes of a standing wave are displacement antinodes and vice versa, hence the length of the resonant air column should be a half-integral multiple of the internode distance \( \lambda/2 \). Thus,

\[
L = \frac{1}{2} \times \frac{\lambda}{2}, \frac{3}{2} \times \frac{\lambda}{2}, \frac{5}{2} \times \frac{\lambda}{2}, \ldots.
\]

Suppose the first resonance happens when

\[
L = h_{\text{top}} - h_{\text{water}}^1 = (n + \frac{1}{2}) \frac{\lambda}{2},
\]

then the next resonance happens when the water rises by \( \lambda/2 \) and the air column shortens by the same amount,

\[
\begin{align*}
h_{\text{top}} - h_{\text{water}}^2 &= (n + \frac{1}{2}) \frac{\lambda}{2}, \\
h_{\text{top}} - h_{\text{water}}^3 &= (n - \frac{1}{2}) \frac{\lambda}{2}, \\
\ldots \\
h_{\text{top}} - h_{\text{water}}^n &= (\frac{1}{2}) \frac{\lambda}{2},
\end{align*}
\]

and therefore

\[
h_2 - h_1 = h_3 - h_2 = \cdots = h_n - h_{n-1} = \frac{\lambda}{2}.
\]

In other words,

\[
\Delta h = \frac{\lambda}{2} = \frac{v_s}{2f} = 19.8157 \text{ cm}.
\]

007 (part 1 of 1) 0 points
An open pipe 0.55 m in length is placed vertically in a cylindrical bucket with a cross-sectional area of 0.1 m\(^2\). Water is poured into the bucket until a tuning fork of frequency 774 Hz, placed over the pipe, produces resonance.

Note: If more water is poured into the bucket after reaching this resonance, no further resonance occurs.

Find the mass of water in the bucket at this moment. The speed of sound in air is 344 m/s.
Correct answer: 43.8889 kg.

Explanation:
The wavelength of the standing sound wave is
\[
\lambda = \frac{v_s}{f} = \frac{344 \text{ m/s}}{774 \text{ Hz}} = 0.444444 \text{ m}.
\]
The length of the resonant air column is
\[
d = \frac{\lambda}{4} = \frac{0.444444 \text{ m}}{4} = 0.111111 \text{ m},
\]
so the height of the water column is
\[
h = L - d = (0.55 \text{ m}) - (0.111111 \text{ m}) = 0.438889 \text{ m}.
\]
The mass of the water in the bucket is equal to the density \(\rho_{\text{water}}\) times the volume \(V\) of the water
\[
m = \rho_{\text{water}} V = \rho_{\text{water}} A h = (1000 \text{ kg/m}^3)(0.1 \text{ m}^2)(0.438889 \text{ m}) = 43.8889 \text{ kg}.
\]

008 (part 1 of 2) 5 points
An organ pipe of length \(L\) is open at one end and closed at the other. A tuning fork is brought near the pipe, exciting the air in the pipe with three nodes as represented in the figure.

With the speed of the sound denoted by \(v\), what is the frequency of the tuning fork?
1. \(\frac{2}{\ell} v\)
2. \(\frac{5}{4L} v\) correct
3. \(\frac{1}{4L} v\)
4. \(\frac{3}{L} v\)
5. \(\frac{7}{4L} v\)
6. \(\frac{1}{2L} v\)
7. \(\frac{1}{L} v\)
8. \(\frac{3}{4L} v\)
9. \(\frac{5}{2L} v\)
10. \(\frac{3}{2L} v\)

Explanation:
We must have a node (stationary point) at the closed end of our air column, and an anti-node (maximum oscillation point) at the open end of our air column. The distance between nodes is a half-wavelength, and the distance between a node and an anti-node is a quarter-wavelength. Therefore the length of the air column must correspond to an integer number of half-wavelengths plus one quarter-wavelength. This is equivalent to an odd integer number of quarter wavelengths. Therefore
\[
L = (2n - 1)\frac{\lambda}{4} \quad (n = 1, 2, 3, \ldots).
\]
This is not a harmonic series since the higher frequencies are not integral multiples of the fundamental frequency. We can express this as the odd members of a harmonic series as
\[
L = m\frac{\lambda}{4} \quad (m = 1, 3, 5, \ldots).
\]
The fifth harmonic is the third lowest frequency of the organ pipe, which corresponds to \(n = 3\) or \(m = 5\). The frequency is just \(f = v/\lambda\). Combining these equations and solving for the frequency gives:
\[
f = \frac{5}{4L} v.
\]

009 (part 2 of 2) 5 points
A string is stretched to a length $l$ and both ends are fixed. The density of the string is $\mu$ and its tension is $F$. A standing wave mode of the string with six nodes, including both end points (as pictured), is generated.

What is the frequency of this oscillation?

1. $\frac{5}{2l} \sqrt{\frac{F}{\mu}}$ correct
2. $\frac{3}{2l} \sqrt{\frac{F}{\mu}}$
3. $\frac{1}{2l} \sqrt{\frac{F}{\mu}}$
4. $\frac{5}{4l} \sqrt{\frac{F}{\mu}}$
5. $\frac{7}{2l} \sqrt{\frac{F}{\mu}}$
6. $\frac{2}{7} \sqrt{\frac{F}{\mu}}$
7. $\frac{9}{4l} \sqrt{\frac{F}{\mu}}$
8. $\frac{1}{7} \sqrt{\frac{F}{\mu}}$
9. $\frac{3}{7} \sqrt{\frac{F}{\mu}}$

Explanation:
Both ends of the string are fixed (immobile), so any standing waves on the string must have nodes (stationary points) at both ends. This requires the wavelength be such that an integer number of half-wavelengths (the distance between nodes) completely fills the string, i.e.

$$\frac{n \lambda}{2} = l \quad (n = 1, 2, 3, \ldots).$$

The resulting possible frequencies are:

$$f_n = \frac{v}{\lambda} = \frac{n}{2l} v = \frac{n}{2l} \sqrt{\frac{F}{\mu}} \quad (n = 1, 2, 3, \ldots).$$

In this case, we can clearly see that five half-wavelengths fill the string, therefore $n = 5$.

In certain ranges of a piano keyboard, more than one string is tuned to the same note to provide greater intensity. For example, the note at 134 Hz has two strings at this pitch.

If one string slips from its normal tension of 628 N to 546 N, what beat frequency will be heard when the two strings are struck simultaneously?

Correct answer: 9.0543 beats/s.

Explanation:
The beat frequency $\Delta f$ is equal to the difference of the frequencies $f_1$ and $f_2$

$$\Delta f = f_1 - f_2.$$

To calculate the frequency $f_2$, we use the fact that the fundamental frequency of vibrations in a string is proportional to the velocity of the waves along the string, which in turn is proportional to the square root of the tension $T$ in the string

$$f \propto v \propto \sqrt{T},$$

therefore

$$f_2 = \sqrt{\frac{T_2}{T_1}} f_1 = \sqrt{\frac{546 \text{ N}}{628 \text{ N}}} 134 \text{ Hz} = 124.946 \text{ Hz}.$$

So for the beat frequency we obtain

$$\Delta f = |f_1 - f_2| = |(134 \text{ Hz}) - (124.946 \text{ Hz})| = 9.0543 \text{ beats/s}.$$