This print-out should have 21 questions. Multiple-choice questions may continue on the next column or page - find all choices before making your selection. The due time is Central time.

Chapters 2 and 3 problems.

001 (part 1 of 1) 10 points

The height of a helicopter above the ground is given by $h = c t^3$, where $c = 1.1 \text{ m/s}^3$, $h$ is in meters, and $t$ is in seconds. The helicopter takes off and after 4 s it releases a small mailbag. The acceleration of gravity is 9.8 m/s$^2$.

How long after its release does the mailbag reach the ground?

Correct answer: 11.9753 s.

Explanation:

Given: $t = 4 \text{ s}$.
Under free fall,

$$h(t) = y(t) = y_0 + v_0 t + \frac{1}{2} a t^2.$$ The initial height of the mailbag is the height of the helicopter 4 s after takeoff:

$$h_0 = h(t) = (1.1 \text{ m/s}^3)(4 \text{ s})^3 = 70.4 \text{ m}$$ and it starts its free fall motion from this point. Its initial velocity is equal to the velocity of the helicopter at that time:

$$v_0 = \frac{dh}{dt} = 3 c t^2 = 3 (1.1 \text{ m/s}^3)(4 \text{ s})^2 = 52.8 \text{ m/s}.$$ Thus the equation of motion governing the mailbag is

$$y(t) = 0 = h_0 + v_0 t - \frac{1}{2} g t^2.$$ In quadratic form,

$$\frac{1}{2} g t^2 - v_0 t - h_0 = 0.$$ From the quadratic formula,

$$t = \frac{v_0 \pm \sqrt{v_0^2 + 2 g h_0}}{g}$$ Since

$$D = v_0^2 + 2 g h = (52.8 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(70.4 \text{ m})$$

$$= 4167.68 \text{ m}^2/\text{s}^2,$$

the time for the mailbag to reach the ground is

$$t = \frac{v_0 \pm \sqrt{D}}{g} = \frac{52.8 \text{ m/s} \pm \sqrt{4167.68 \text{ m}^2/\text{s}^2}}{9.8 \text{ m/s}^2} = 11.9753 \text{ s}.$$ The negative solution is rejected.

002 (part 1 of 1) 0 points

A freely falling body has a constant acceleration of 9.8 m/s$^2$.

This means that:

1. the acceleration of the body decreases by 9.8 m/s$^2$ during each second.
2. the acceleration of the body increases by 9.8 m/s$^2$ during each second.
3. the body falls 9.8 m during each second.
4. the body falls 9.8 m during the first second only.
5. the speed of the body increases by 9.8 m/s during each second. **correct**

Explanation:

The acceleration is defined as the ratio of the change in velocity to the time period. If the acceleration is $g = 9.8 \text{ m/s}^2$, the change in velocity during one second is

$$\Delta v = g \Delta t$$

$$\Delta v = (9.8 \text{ m/s}^2)(1 \text{ s}) = 9.8 \text{ m/s}$$

003 (part 1 of 2) 5 points

A stone is thrown upwards from the edge of a
cliff $y_0 = 11.6$ m high. It just misses the cliff on the way back down and hits the ground below, at $y = 0$, with a speed of $v = 21.7$ m/s.

The acceleration of gravity is $9.8$ m/s$^2$.

With what velocity was it released? Take up to be positive. Correct answer: $15.6054$ m/s.

**Explanation:**

**Solution:**

Since the acceleration is constant, 

$$v^2 = v_0^2 + 2a(y - y_0)$$

$v = -21.7$ m/s, $y_0 = 11.6$ m, and $a = -g$, so

$$v_0^2 = v^2 - 2a(y - y_0)$$
$$= v^2 - 2(-g)(0 - y_0)$$
$$= (-21.7$ m/s$)^2$$
$$- 2(-9.8$ m/s$^2)(-11.6$ m$)$
$$= 243.53$ m$^2$/s$^2$, 

so that

$$v_0 = 15.6054$$ m/s.

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**004 (part 2 of 2) 5 points**

During the flight, what is the maximum height of the stone measured with respect to the ground?

Correct answer: 24.025 m.

**Explanation:**

When the stone is at its maximum height $h$, its velocity is zero:

$$v^2 = 2gh \Rightarrow h = \frac{v^2}{2g}$$

so

$$h = \frac{(21.7$ m/s$)^2}{2(9.8$ m/s$^2)} = 24.025$$ m

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**005 (part 1 of 1) 0 points**

A stone is thrown straight upward and at the top of its trajectory its velocity is momentarily zero.

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What is its acceleration at this point?

1. Unable to determine
2. Zero
3. 9.8 m/s$^2$ down correct
4. 9.8 m/s$^2$ up

**Explanation:**

**Basic Concepts:** The gravitational acceleration near the surface of the earth is considered constant, for all practical purposes. This acceleration of $9.8$ m/s$^2$ is pointing downward.

**Solution:** To illustrate how it works, let’s take, for example, an upward initial velocity of $9.8$ m/s. One second later the velocity will be zero. One second after that it will be at $-9.8$ m/s. In other words, in each second the velocity is decreased by $9.8$ m/s.

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**006 (part 1 of 1) 0 points**

At a particular instant, a stationary observer on the ground sees a package falling with speed $v_1$ at an angle to the vertical. To a pilot flying horizontally at constant speed relative to the ground, the package appears to be falling vertically with a speed $v_2$ at that instant.

What is the speed of the pilot relative to the ground?

1. $v_2 - v_1$
2. $v_1 - v_2$
3. $\sqrt{v_1^2 - v_2^2}$ correct
4. \( v_1 + v_2 \)

5. \( \sqrt{v_1^2 + v_2^2} \)

**Explanation:**

**Basic Concepts**
Vector Addition

**Solution:**

The stationary observer sees the resultant velocity of the horizontal and vertical components. The pilot sees the vertical component.

Thus

\[
v = \sqrt{v_1^2 - v_2^2}.
\]

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**007** (part 1 of 1) 0 points

**Given:** The battleship and enemy ships A and B lie along a straight line. Neglect air friction.

A battleship simultaneously fires two shells (with the same muzzle velocity) at these two enemy ships.

If the shells follow the parabolic trajectories shown in the figure, which ship gets hit first?

1. A
2. B correct
3. both at the same time
4. need more information

**Explanation:**

The time interval for the entire projectile motion is given by

\[
t_{\text{trip}} = t_{\text{rise}} + t_{\text{fall}} = 2t_{\text{rise}},
\]

where \( t_{\text{rise}} \) is the rising time from 0 to the maximum height, and \( t_{\text{fall}} \) the falling time from \( h \) to 0. In the absence of air resistance

\[
t_{\text{rise}} = t_{\text{fall}}, \quad h = \frac{1}{2}gt_{\text{fall}}^2, \quad \text{or} \quad t_{\text{trip}} = 2\sqrt{\frac{2h}{g}}. \quad \text{So the smaller is } h, \text{ the smaller is } t_{\text{trip}}. \text{ In other words, B should get hit first.}
\]

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**008** (part 1 of 3) 0 points

A cannon sends a projectile towards a target a distance 1280 m away. The initial velocity makes an angle \( 29^\circ \) with the horizontal. The target is hit.

The acceleration of gravity is 9.8 m/s\(^2\).

What is the magnitude of the initial velocity?

Correct answer: 121.621 m/s.

**Explanation:**

For a projectile, we have

\[
d = \frac{v_0^2 \sin 2\theta}{g}
\]

\[
v_0 = \sqrt{\frac{dg}{\sin(2\theta)}}
\]

\[
= \sqrt{(1280 \text{ m})(9.8 \text{ m/s}^2)} \frac{\sin(2 \times 29^\circ)}{\sin(2 \times 29^\circ)}
\]

\[
= 121.621 \text{ m/s}.
\]

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**009** (part 2 of 3) 0 points

How high is the highest point of the trajectory?

Correct answer: 177.379 m.

**Explanation:**

\[
h = \frac{v_0^2 \sin^2 \theta}{2g}
\]

\[
= \frac{(121.621 \text{ m/s})^2 \sin^2 29^\circ}{2(9.8 \text{ m/s}^2)}
\]

\[
= 177.379 \text{ m}.
\]

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**010** (part 3 of 3) 0 points

How long does it take for the projectile to reach the target? (Assume no friction)

Correct answer: 12.0332 s.

**Explanation:**

\[
t = \frac{2v_0 \sin \theta}{g}
\]

\[
= \frac{2(121.621 \text{ m/s}) \sin 29^\circ}{9.8 \text{ m/s}^2}
\]

\[
= 12.0332 \text{ s}.
\]
011 (part 1 of 4) 3 points
A ball is thrown horizontally from the top of a building 22.2 m high. The ball strikes the ground at a point 77.4 m from the base of the building.

The acceleration of gravity is 9.8 m/s². Find the time the ball is in motion.

Correct answer: 2.12852 s.

Explanation:
In this solution, we take the origin of the x and y axes at the base of the building. The positive y direction is taken to point upward, so that the vertical acceleration is \(-g\). The time the ball is in flight is found from

\[
y = y_0 + v_{y0} t + \frac{1}{2} a_y t^2
\]

\[
0 = y_0 + 0 - \frac{1}{2} g t^2
\]

\[
t = \sqrt{\frac{2 y_0}{g}}
\]

\[
= \sqrt{\frac{2 (22.2 \text{ m})}{9.8 \text{ m/s}^2}}
\]

\[
= 2.12852 \text{ s} .
\]

012 (part 2 of 4) 3 points
Find the initial velocity of the ball.

Correct answer: 36.3632 m/s.

Explanation:
There is no acceleration in the horizontal direction. Therefore, we have

\[
x = x_0 + v_{x0} t
\]

\[
x = 0 + v_{x0} t
\]

\[
v_{x0} = \frac{x}{t}
\]

\[
= \frac{77.4 \text{ m}}{2.12852 \text{ s}}
\]

\[
= 36.3632 \text{ m/s} .
\]

013 (part 3 of 4) 2 points
Find the x component of its velocity just before it strikes the ground.

Correct answer: 36.3632 m/s.

Explanation:
Since there is no acceleration in the horizontal direction, the x component of the velocity is constant. So

\[
v_x = v_{x0} = 36.3632 \text{ m/s} .
\]

014 (part 4 of 4) 2 points
Find the y component of its velocity just before it strikes the ground.

Correct answer: -20.8595 m/s.

Explanation:
The y component of the velocity just before the ball strikes the ground is

\[
v_y = v_{y0} + a_y t
\]

\[
= 0 - g t
\]

\[
= -(9.8 \text{ m/s}^2) (2.12852 \text{ s})
\]

\[
= -20.8595 \text{ m/s} .
\]

015 (part 1 of 4) 3 points
Consider the setup of a gun aimed at a target (such as a monkey) as shown in the figure below. The target is to be dropped from the point A at \(t = 0\), the same moment as the gun is fired. The bullet hits the target at a point P. Let the initial speed of the bullet be \(v_0 = 108 \text{ m/s}\), let the angle between the vector \(\vec{v}_0\) and the horizontal (x-) direction be \(\theta = 54.3^\circ\) and let \(AB = 89.8 \text{ m}\). The distance \(d = OB\) is the x-coordinate of the target. Denote the time taken to hit the target by \(t\).

\[
\text{The acceleration of gravity is } 9.8 \text{ m/s}^2 .
\]

This time is given by

1. \(t = \frac{d}{v_0 \sin \theta} .\)

2. \(t = \frac{d}{v_0} .\)

3. \(t = \frac{d}{v_0 \tan \theta} .\)
4. \( t = \frac{d}{v_0 \cos \theta} \). correct

5. \( t = \frac{d}{v_0 \cot \theta} \).

**Explanation:**

**Basic Concepts:** Constant acceleration:

\[ x - x_0 = v_0 t + \frac{1}{2} a t^2 \quad (1) \]

\[ v = v_0 + at \quad (2) \]

**Solution:**

Think of this two-dimensional motion as two one-dimensional trajectories, one in the \( x \)- and one in the \( y \)-direction. The horizontal (\( x \)) initial velocity is

\[ v_{0x} = v_0 \cos \theta . \]

The \( x \)-motion is unaccelerated (gravity only affects the \( y \)-motion) so equation (1) gives

\[ d = v_{0x} t + 0 . \]

Thus

\[ t = \frac{d}{v_{0x}} = \frac{d}{v_0 \cos \theta} . \]

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**016 (part 2 of 4) 3 points**

**Assume:** Up is the positive direction.

With the time taken to hit the target given by \( t \), the vertical (\( y \)) component of the velocity of the bullet is

1. \( v_y = -v_0 \cos \theta + r t \)
2. \( v_y = v_0 \cos \theta + r t \)
3. \( v_y = v_0 \sin \theta - r t \) correct
4. \( v_y = r t \)
5. \( v_y = v_0 \cos \theta - r t \)
6. \( v_y = -v_0 \cos \theta - r t \)
7. \( v_y = -v_0 \sin \theta - r t \)
8. \( v_y = -v_0 \sin \theta + r t \)
9. \( v_y = v_0 \sin \theta + r t \)
10. \( v_y = -r t \)

**Explanation:**

The vertical (\( y \)) initial velocity is given by

\[ v_{0y} = v_0 \sin \theta \]

We take positive direction upward, so \( a = -g \) and equation (2) gives us for the vertical component of the velocity at \( P \)

\[ v_{Py} = v_{0y} - g t = v_0 \sin \theta - g t . \]

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**017 (part 3 of 4) 2 points**

The height \( BP \) where the collision takes place is

Correct answer: 84.6631 m.

**Explanation:**

The distance \( d \) is

\[ d = OB = \frac{AB}{\tan \theta} = \frac{89.8 \text{ m}}{\tan 54.3^\circ} = 64.5278 \text{ m} . \]

The time taken to go from O to P is, by Part 1,

\[ t = \frac{d}{v_0 \cos \theta} = \frac{(64.5278 \text{ m})}{(108 \text{ m/s}) (\cos 54.3^\circ)} = 1.02389 \text{ s} . \]

The initial \( y \)-velocity is

\[ v_{0y} = v_0 \sin \theta \]

\[ = (108 \text{ m/s}) \sin(54.3^\circ) \]

\[ = 87.705 \text{ m/s} . \]

To find the height, we utilize Eq. 1

\[ h - 0 = v_{0y} t + \frac{1}{2} (-g) t^2 , \]

so

\[ h = v_{0y} t - \frac{1}{2} g t^2 \]

\[ = (87.705 \text{ m/s}) (1.02389 \text{ s}) \]

\[ - \frac{1}{2} (9.8 \text{ m/s}^2) (1.02389 \text{ s})^2 \]

\[ = 84.6631 \text{ m} . \]
(part 4 of 4) 2 points

Reminder: Speed is the magnitude of the velocity vector, which has both \( x \)- and \( y \)-components.

Determine the speed of the bullet at the point of collision.
Correct answer: 100.023 m/s.

**Explanation:**

The reminder hints that for two perpendicular vectors we must use Pythagoras’ theorem

\[ v = \sqrt{v_{0x}^2 + v_{Py}^2}. \]

The numerical value of \( v_{Py} \) is

\[ v_{Py} = v_{0y} - gt = 87.705 \text{ m/s} - (9.8 \text{ m/s}^2)(1.02389 \text{ s}) = 77.671 \text{ m/s}, \]

so

\[ v = \sqrt{v_{0x}^2 + v_{Py}^2} = \sqrt{(63.0224 \text{ m/s})^2 + (77.671 \text{ m/s})^2} = 100.023 \text{ m/s}. \]

(part 1 of 3) 0 points

A ski jumper travels down a slope and leaves the ski track moving in the horizontal direction with a speed of 22 m/s as in the figure. The landing incline below her falls off with a slope of \( \theta = 38.9^\circ \).

The acceleration of gravity is 9.8 m/s\(^2\).

Calculate the distance \( d \) she travels along the incline before landing.
Correct answer: 102.413 m.

**Explanation:**

Excluding \( t \) rather than \( d \) from the system above, we obtain

\[ d \cos \theta = v_x t \]
\[ d \sin \theta = \frac{1}{2} g t^2. \]

Substituting these expressions for \( x \) and \( y \) into the two equations above, we obtain

\[ d \cos \theta = v_x t \]
\[ d \sin \theta = \frac{1}{2} g t^2. \]

Excluding \( t \) from these equations gives

\[ d = \frac{2 v_0^2 \sin \theta}{g \cos^2 \theta} = \frac{(2) (22 \text{ m/s})^2 \sin 38.9^\circ}{(9.8 \text{ m/s}^2) \cos^2 38.9^\circ} = 102.413 \text{ m}. \]

(part 2 of 3) 0 points

Determine how long the ski jumper is airborne.
Correct answer: 3.62281 s.

**Explanation:**

Excluding \( d \) rather than \( t \) from the system above, we obtain

\[ t = \frac{2 v_0 \tan \theta}{g} = \frac{(2) (22 \text{ m/s}) \tan(38.9^\circ)}{9.8 \text{ m/s}^2} = 3.62281 \text{ s}. \]

(part 3 of 3) 0 points

What is the relative angle \( \phi \) with which the ski jumper hits the slope?
Correct answer: 19.3153 \(^\circ\).

**Explanation:**
The direction \( \phi_t \) of the velocity vector (relative to the positive \( x \) axis) at impact is

\[
\phi_t = \arctan \left( \frac{v_y}{v_x} \right) \\
= \arctan \left( \frac{-35.5035 \text{ m/s}}{22 \text{ m/s}} \right) \\
= -58.2153^\circ,
\]

where \( v_y = -g t \) and \( v_x = v_0 \). Therefore the relative angle of impact \( \phi \) on the slope is

\[
\phi = |\phi_t| - \theta \\
= 58.2153^\circ - 38.9^\circ \\
= 19.3153^\circ.
\]