This print-out should have 8 questions. Multiple-choice questions may continue on the next column or page – find all choices before making your selection. The due time is Central time.

Chapter 9 problems.

001 (part 1 of 2) 0 points
Two blocks of masses $M$ and $3M$ are placed on a horizontal, frictionless surface. A light spring is attached to one of them, and the blocks are pushed together with the spring between them.

**Explanation:**

Let $v = 2.83$ m/s.

From conservation of momentum $\Delta p = 0$, in our case we obtain

$$0 = M v_M - 3 M v$$

Therefore

$$v_M = 3 v = (3) (2.83 \text{ m/s}) = 8.49 \text{ m/s}.$$ 

002 (part 2 of 2) 0 points
If $M$ is 5 kg and the spring between the masses has a spring constant of 6300 N/m, how much was the spring originally compressed from its equilibrium length?

**Explanation:**

Let $m_1 = 1070$ kg, $v_1 = 278.08 \text{ km/h}$, $m_2 = 2107.9$ kg, and $v_2 = 176 \text{ km/h}$. 

003 (part 1 of 2) 0 points
A 1070 kg car skidding due north on a level frictionless icy road at 278.08 km/h collides with a 2107.9 kg car skidding due east at 176 km/h in such a way that the two cars stick together.

At what angle ($-180^\circ \leq \theta \leq +180^\circ$) East of North do the two coupled cars skid off at?

**Explanation:**
Basic Concepts: Momentum Conservation, \( K = \frac{1}{2} m v^2 \), \( \vec{p} = m \vec{v} \).

During the collision, the total momentum of the two car system will be conserved
\[
\vec{p}_f = \vec{p}_1 + \vec{p}_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2
\]

\[
p_{fx} \hat{i} + p_{fy} \hat{j} = m_1 v_1 \hat{i} + m_2 v_2 \hat{i}.
\]

Looking at the \( x \) and \( y \) components of momentum,
\[
p_{fx} = m_2 v_2
\]
\[
= (2107.9 \text{ kg}) (176 \text{ km/h}) \times \left( \frac{10^3 \text{ m}}{\text{km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right)
\]
\[
= 103053 \text{ kg m/s}, \ \text{and}
\]
\[
p_{fy} = m_1 v_1
\]
\[
= (1070 \text{ kg}) (278.08 \text{ km/h}) \times \left( \frac{10^4 \text{ m}}{\text{km}} \right) \left( \frac{1 \text{ h}}{3600 \text{ s}} \right)
\]
\[
= 82651.6 \text{ kg m/s}.
\]

Since we are asked to find the angle from the \( y \)-axis instead of the \( x \),
\[
\tan \theta = \left( \frac{p_{fx}}{p_{fy}} \right)
\]
\[
= \left( \frac{103053 \text{ kg m/s}}{82651.6 \text{ kg m/s}} \right)
\]
\[
= 1.24684
\]
\[
\theta = \arctan \left( \frac{103053 \text{ kg m/s}}{82651.6 \text{ kg m/s}} \right)
\]
\[
= 51.2693^\circ.
\]

How much kinetic energy is lost in the collision?
Correct answer: 2.96555 \times 10^6 \text{ J}.

Explanation:
To find the energy lost in the collision, we need to know the total kinetic energy of the system before and after the collision. (There is no change in potential energy since the cars remain on level ground.) The initial and final energies are given by
\[
E_i = K_i = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2
\]
\[
= \frac{1}{2} (1070 \text{ kg}) (77.2444 \text{ m/s})^2
\]
\[
+ \frac{1}{2} (2107.9 \text{ kg}) (48.8889 \text{ m/s})^2
\]
\[
= 5.71126 \times 10^6 \text{ J}, \ \text{and}
\]
\[
E_f = K_f = \frac{1}{2} (m_1 + m_2) v_f^2 .
\]

To find \( v_f \), we need to go back to the momentum equation;
\[
\vec{v}_f = \frac{\vec{p}_f}{m_1 + m_2} ,
\]
or in component form,
\[
v_{fx} = \frac{p_{fx}}{m_1 + m_2} , \ \text{and}
\]
\[
v_{fy} = \frac{p_{fy}}{m_1 + m_2}.
\]

Now that we know the components of \( \vec{v}_f \), we can find its magnitude
\[
v_f^2 = v_{fx}^2 + v_{fy}^2 .
\]

We can substitute this into the equation for the final kinetic energy to get
\[
E_f = \frac{1}{2} \frac{p_{fx}^2 + p_{fy}^2}{m_1 + m_2}
\]
\[
= 2.74571 \times 10^6 \text{ J}.
\]

The energy lost is then
\[
E_{\text{lost}} = -\Delta E
\]
\[
= E_i - E_f
\]
\[
= (5.71126 \times 10^6 \text{ J}) - (2.74571 \times 10^6 \text{ J})
\]
\[
= 2.96555 \times 10^6 \text{ J}.
\]
Two people are initially standing still on frictionless ice. They push on each other so that one person, of mass 120 kg, moves to the left at 2 m/s, while the other person, of mass 80 kg, moves to the right at 3 m/s.

What is the velocity of their center of mass?

1. $v_{cm} = 0$ correct
2. $v_{cm} = 2.4$ m/s to the left
3. $v_{cm} = 0.5$ m/s to the left
4. $v_{cm} = 1$ m/s to the right
5. $v_{cm} = 2.5$ m/s to the right

Explanation:
By definition, the velocity of the center of mass is

$$v_{cm} = \frac{v_1 m_1 + v_2 m_2}{m_1 + m_2}.$$  

Taking the left as the positive-$x$ direction, we know

$$v_{cm} = \frac{(2 \text{ m/s})(120 \text{ kg}) - (3 \text{ m/s})(80 \text{ kg})}{(120 \text{ kg}) + (80 \text{ kg})} = 0 \text{ m/kg}.$$ 

A uniform flat plate of metal is situated in the reference frame shown in the figure below.

Calculate the $x$ coordinate of the center of mass of the metal plate.
Correct answer: 8.

Explanation:
Basic Concept: The center of mass coordinate is

$$x = \frac{\int x \, dm}{m},$$

where $m = \int dm$, and $dm = \sigma y \, dx$, where $\sigma$ is the areal density ($\frac{\text{mass}}{\text{area}}$) of the plate.

Solution: Let

$$(x_1, y_1) = (6, 0)$$
$$(x_2, y_2) = (9, 0)$$
$$(x_3, y_3) = (9, 4).$$

The equation for the hypotenuse is

$$\frac{y - y_1}{x - x_1} = \frac{y_3 - y_1}{x_3 - x_1}.$$ 

The slope of the hypotenuse is

$$s = \frac{y_3 - y_1}{x_3 - x_1} = \frac{4 - 0}{9 - 6} = \frac{4}{3}.$$ 

Rewriting the equation, we have

$$y = s(x - x_1) + y_1 = \left(\frac{4}{3}\right)(x - 6) + 0.$$ 

The $x$-coordinate of the center of mass is

$$x = \frac{\sigma \int_{x_1}^{x_2} x y \, dx}{\sigma \int_{x_1}^{x_2} y \, dx} = \frac{\int_{x_1}^{x_2} x s(x - x_1) \, dx}{\int_{x_1}^{x_2} s(x - x_1) \, dx} = \frac{\int_{x_1}^{x_2} x(x - x_1) \, dx}{\int_{x_1}^{x_2} (x - x_1) \, dx} = \frac{1}{3} x^3 - \frac{1}{2} (x_1) x^2 \bigg|_x^{x_2} = \frac{1}{3} x^3 - \frac{1}{2} (x_1) x^2 \bigg|_x^{x_2}.$$
\[
= \frac{1}{3} (x_2^3 - x_1^3) - \frac{1}{2} (x_1) (x_2^2 - x_1^2)
\]
\[
= \frac{1}{2} (x_2^2 - x_1^2) - (x_1) (x_2 - x_1)
\]
\[
= \frac{3}{2} x_1 (x_2^2 - 2 x_1 x_2 + x_1^2)
\]
\[
= \frac{3}{2} (x_2^2 - 2 x_1 x_2 + x_1^2)
+ \frac{2}{3} (x_2 - x_1) (x_2^2 - 2 x_1 x_2 + x_1^2)
\]
\[
= x_1 + \frac{2}{3} (x_2 - x_1)
\]
\[
= 6 + \frac{2}{3} (9 - 6)
\]
\[
= 8.
\]

**Alternate solution:** The center of mass of a right triangle is \( \frac{1}{3} \) of the height or base of the triangle measured from its right angle. Therefore Eq. 1 is the \( x \)-coordinate of the center of mass of the metal plate.

The \( y \)-coordinate of the center of mass of the metal plate is

\[
y = y_2 + \frac{1}{3} (y_3 - y_2)
\]
\[
= 0 + \frac{1}{3} (4 - 0)
\]
\[
= 1.33333.
\]

**Note:** This problem has a different triangle for each student.

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**007 (part 1 of 1) 0 points**

A triangular wedge of height \( H = 1.368 \) m, base length \( L = 2.28 \) m and mass \( M = 8.95 \) kg is placed on a frictionless table. A small block of mass \( m = 4.42 \) kg is placed on top of the wedge as shown on the picture below:

All surfaces are frictionless, so the block slides down the wedge while the wedge slides sidewise on the table. By the time the block slides all the way down to the bottom of the wedge, how far does the wedge slide to the right?

Correct answer: 0.753747 m.

**Explanation:**

Consider the wedge and the block as a two-body system. The *external forces* acting on this system — the weight of the wedge, the weight of the block and the normal force from the table — are all vertical, hence the net horizontal momentum of the system is conserved,

\[
P^\text{wedge}_x + P^\text{block}_x = \text{const}.
\]

Furthermore, we start from rest \( \implies \) center of mass is not moving, and therefore the \( X \) coordinate of the center of mass will remain constant while the wedge slides to the right and the block slides down and to the left,

\[
X_{\text{cm}} = \frac{mX_{\text{block}} + MX_{\text{wedge}}}{m + M} = \text{const}.
\]

Note that only the \( X \) coordinate of the center of mass is a constant of motion: The \( Y_{\text{cm}} \) component of the net momentum is not conserved. Constant \( X_{\text{cm}} \) means \( \Delta X_{\text{cm}} = 0 \) and therefore

\[
m\Delta X_{\text{block}} + M\Delta X_{\text{wedge}} = 0.
\]

Note that this formula does not depend on where the wedge has its own center of mass; as long as the wedge is rigid, its overall displacement \( \Delta X_{\text{wedge}} \) is all we need to know.

Finally, consider the geometry of the problem: By the time the block slides all the way down, its displacement *relative to the wedge* is equal to the wedge length \( L \), or rather \(-L\) because the block moves to the left of the
wedge. In terms of displacements relative to the inertial frame of the table, this means

$$\Delta X_{\text{block}} - \Delta X_{\text{wedge}} = -L.$$  

Consequently,

$$0 = m\Delta X_{\text{block}} + M\Delta X_{\text{wedge}} = m(-L + \Delta X_{\text{wedge}}) + M\Delta X_{\text{wedge}}$$

and therefore

$$\Delta X_{\text{wedge}} = \frac{mL}{m + M} = 0.753747 \text{ m}.$$  

Finally, the center of mass of the boat-child system is

$$x_{\text{cm}} = \frac{(D + \frac{L}{2} - \chi) M + (D + L - \chi) m}{M + m},$$

where $\chi$ is the change in position of the center of mass of the boat. Since the center of mass of the system does not move, we can equate the above two expressions for $x_{\text{cm}}$

$$\left(D + \frac{L}{2}\right) M + D m$$

$$\frac{M + m}{M + m}$$

and, solving for $\chi$, we have

$$\left(D + \frac{L}{2} - \chi\right) M + (D + L - \chi) m$$

$$0 = -\chi M + (L - \chi) m$$

$$\chi (M + m) = L m$$

$$\chi = \frac{m}{m + M} L$$

$$= \frac{(53 \text{ kg})}{(53 \text{ kg}) + (72 \text{ kg})} \times (5.9 \text{ m})$$

$$= 2.5016 \text{ m}.$$  

The final distance $\ell$ of the child from the pier is

$$\ell = D + L - \chi$$

$$= (6 \text{ m}) + (5.9 \text{ m}) - (2.5016 \text{ m})$$

$$= 9.3984 \text{ m}.$$  

A(n) 72 kg boat that is 5.9 m in length is initially 6 m from the pier. A 53 kg child stands at the end of the boat closest to the pier. The child then notices a turtle on a rock at the far end of the boat and proceeds to walk to the far end of the boat to observe the turtle.

Assume: There is no friction between boat and water.

How far is the child from the pier when she reaches the far end of the boat?

Correct answer: 9.3984 m.

Explanation:

Let

$D$ = distance of the boat from the pier,

$L$ = length of the boat,

$M$ = mass of the boat,

$m$ = mass of the child, and

$\chi$ = change in position of the boat.

Initially, the center of mass of the boat-child system is

$$x_{\text{cm}} = \frac{(D + \frac{L}{2}) M + D m}{M + m}$$