1. (a) \[ F_1 = \frac{m_1 F_1}{d-x} \quad \text{and} \quad F_2 = \frac{m_2 F_2}{d-x} \]

\[ F_1 = \frac{G m_1}{x^2} \quad \text{and} \quad F_2 = \frac{G m_2}{(d-x)^2} \]

\[ m_1 (d-x)^2 = m_2 x^2 \]

\[ \frac{m_1 (d-x)}{m_2} = \frac{x}{d-x} \quad \text{or} \quad x = \frac{d}{1 + \sqrt{m_1/m_2}} \]

(b) Moves back to equilibrium point, if displaced in y-n direction, moves away if displaced in x direction.

2. \[ F_1 = -\frac{G m_1 m_2}{d^2} \hat{e} \]

\[ F_3 = -\frac{G m_2 m_3}{d^2} \hat{j} \]

\[ F_4 = \frac{G m_2 m_4}{(\sqrt{2}d)^2} \left( -\frac{\sqrt{2}}{2} \hat{e} - \frac{\sqrt{2}}{2} \hat{j} \right) \]

\[ F_1 + F_3 + F_4 = \frac{G m_2}{d^2} \left[ -(m_1 + \frac{\sqrt{2}}{4} m_4) \hat{e} - (m_2 + \frac{\sqrt{2}}{4} m_4) \hat{j} \right] \]

3. (a) \[ F = m_2g = (1\text{kg})(9.8\text{N/kg}) = 9.8\text{N} \]

(b) \[ F = \frac{G m M_5}{(\Delta s - R)^2} = \frac{(6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2)(1\text{kg})(1.0\text{kg})(1.96 \times 10^{30} \text{kg})}{(1.50 \times 10^8 \text{m} - 6.27 \times 10^6 \text{m})^2} = 6.00 \times 10^{-9} \text{N} \]

(c) \[ F = \frac{G m M_m}{(\Delta m - R)^2} = \frac{(6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2)(1\text{kg})(2.36 \times 10^{22} \text{kg})}{(3.82 \times 10^8 \text{m} - 6.27 \times 10^6 \text{m})^2} = 0.000035 \text{N} \]
\[ \Delta F = \frac{G M M_s}{(d_0 - R_e)^2} - \frac{G M M_s}{d^2} = \frac{G M M_s}{d^2} \left[ \frac{1}{(1 - \frac{R_e}{d})^2} - \frac{1}{(1 + \frac{R_e}{d})^2} \right] \]

\[ \approx \frac{G M M_s}{d^2} \left( 4 \frac{R_e}{d} \right) = 1.0 \times 10^{-6} \text{ N} \]

\[ \Delta F = \frac{G M M_M}{d^2} \left( 4 \frac{R_e}{d} \right) = 2.2 \times 10^{-6} \text{ N} \]

Note that (from #3), the Sun's force is stronger but the force difference is larger for the Moon (which is why the moon has a greater effect on tides).

By the 2nd shell theorem, all the mass beyond radius \( r \) does not contribute to the gravitational force at \( r \).

By the first shell theorem, all the mass inside \( r \) can be replaced by a point mass at the center.

Assuming the Earth to be of uniform density, the fraction of the mass inside radius \( r \) is

\[ M_e = \frac{4}{3} \pi r^3 \]

\[ M_e = \frac{M_e}{R_e^3} \]

\[ F = \frac{G m}{r^2} \frac{M_e r^3 / R_e^3}{r^2} = \frac{G M e m}{R_e^2} r \]
The mass of the hollow sphere is $M = \frac{4}{3} \pi (\frac{3}{2} R)^3 M = \frac{7}{8} M$.

$$F = G \frac{m (\frac{7}{8} M)}{d^2}$$

Let body $A = \text{original solid sphere}$

$B = \text{sphere with hole}$

$C = \text{sphere of radius} \frac{R}{2} \text{ removed to make hole}$.

$A = B + C$

$$F_B = F_A - F_C = \frac{G m M}{d^2} - \frac{G m (\frac{7}{8} M)}{(d-R/2)^2}$$

$$F = -G \frac{M m e}{R_e^2} \Rightarrow -mg \hat{r} = \hat{g} = G \frac{M m e}{R_e^2}$$

$$g' = \frac{G m e}{(R e)^2} = \frac{6 m e}{R_e^3} = \frac{4}{3} g = \frac{4}{3} \times 9.8 \text{ m/s}^2$$

$$g' = 39.2 \text{ m/s}^2$$