Chapter 1.

Laser: Theory and Applications

Reading: Siegman, Chapter 6 and 7
Laser Basics

Light Amplification by Stimulated Emission of Radiation

Stimulated emission

Population inversion \((N_1 > N_0) \Rightarrow \text{laser, maser}\)

Transition rate for stimulated emission

\[
\bar{W}_{01} = \frac{4\pi^2}{m^2 c} \left( \frac{e^2}{4\pi \varepsilon_0} \right) \frac{I(\omega_{01})}{\omega_{01}^2} \left| M_{01}(\omega_{01}) \right|^2 \propto I(\omega_{01})
\]

Incident light intensity

Pumping (optical, electrical, etc.) for population inversion

High reflector

Gain medium

Out coupler

Optical cavity
Ruby Laser

- This system is a **three level laser** with lasing transitions between $E_2$ and $E_1$.
- The excitation of the Chromium ions is done by **light pulses** from flash lamps (usually Xenon).
- The **Chromium ions** absorb light at wavelengths around 545 nm (500-600 nm). As a result the ions are transferred to the excited energy level $E_3$.
- From this level the ions are going down to the **metastable energy level** $E_2$ in a **non-radiative transition**. The energy released in this non-radiative transition is transferred to the **crystal vibrations** and changed into **heat** that must be removed away from the system.
- The lifetime of the metastable level ($E_2$) is about 5 msec.

http://web.phys.ksu.edu/vqm/laserweb/Ch-6/C6s2t1p2.htm
He-Ne Laser

Energy level diagram

Excimer Lasers

- Gain medium: inert gas (Ar, Kr, Xe etc.) + halide (Cl, F etc.)
- Excited state is induced by an electrical discharge or high-energy electron beams.
- Laser action in an excimer molecule occurs because it has a bound (associative) excited state, but a repulsive (disassociative) ground state.

<table>
<thead>
<tr>
<th>Excimer</th>
<th>Wavelength</th>
</tr>
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<tbody>
<tr>
<td>Fe</td>
<td>157 nm</td>
</tr>
<tr>
<td>ArF</td>
<td>193 nm</td>
</tr>
<tr>
<td>KrF</td>
<td>248 nm</td>
</tr>
<tr>
<td>XeCl</td>
<td>308 nm</td>
</tr>
</tbody>
</table>

Applications
- Marking
- Micromachining
- Laser Ablation
- Laser Annealing
- Surface Structuring
- Laser Vision Correction
- Optical Testing and Inspection
- Pulsed Laser Deposition
- Fiber Bragg Gratings

http://tftlcd.khu.ac.kr/research/poly-Si/chapter4.html
Semiconductor Lasers – laser diodes

Band structure near a semiconductor p-n junction

Diode laser structure

Vertical cavity surface emitting lasers (VCSEL) structure

Specific Laser Systems

Laser media:
gas, dye, chemical, excimer, solid-state, fiber, semiconductor, free-electron

Longitudinal Modes in an Optical Cavity

Boundary condition:

\[
\begin{align*}
L &= \frac{\lambda}{2} = \frac{c\tau}{2} = \frac{\pi c}{m \omega} \\
\Rightarrow \quad \lambda &= \frac{2L}{m}, \quad \nu = \frac{c}{2L} m, \quad \omega = \frac{\pi c}{L} m
\end{align*}
\]

Round-trip time of flight: \( T = \frac{2L}{c} = m\tau \)

Typical laser cavity: \( L = 1.5 \text{ m}, \lambda = 0.75 \text{ \mu m} \)

\[
T = \frac{2L}{c} = \frac{3 m}{3 \times 10^8 \text{ m/sec}} = 10^{-8} \text{ sec} = 10 \text{ nsec}
\]

\[
\Rightarrow \quad \nu_R = \frac{1}{T} = 10^8 \text{ Hz} = 100 \text{ MHz}
\]

\[
m = \frac{2L}{\lambda} = \frac{3 m}{0.75 \times 10^{-6} \text{ m}} = 4 \times 10^6 = 4 \text{ milion!!}
\]
Single mode

\[ I(t) = |\cos \omega_0 t|^2 = |\cos t|^2 \]
Cavity Quality Factors, $Q_c$

End mirror $R \approx 1$ \quad $L$ \quad Out coupler $T = 1 \sim 5\%$

Energy loss by reflection, transmission, etc.

$E(\omega) = \int_{0}^{\infty} E_0 e^{-\delta_c t/2T} e^{-i\omega_0 t} e^{i\omega t} dt$

$= \frac{E_0}{-i(\omega - \omega_0) + \delta_c / 2T}$

$|E(\omega)|^2 = \frac{E_0^2}{(\omega - \omega_0)^2 + \delta_c^2 / 4T^2}$

Emission spectrum

$\omega_0$  $\delta_c / T = \frac{\omega}{Q_c}$

Lorentzian
Energy of circulating EM wave, $I_{circ}(t)$

$$I_{circ}(t) = I_{circ}(0) \times \exp\left[-\delta_c \left(\frac{t}{T}\right)\right], \quad T = \frac{2L}{c}$$

: round-trip time of flight

Number of round trips in $t$

$$\Rightarrow I_{circ}(t) = I_{circ}(0) \times \exp\left[-\frac{\omega}{Q_c} t\right]$$

where

$$Q_c = \frac{\omega T}{\delta_c} = \frac{4\pi L}{\lambda} \frac{1}{\delta_c}$$

Q-factor of a RLC circuit

$$Q = \frac{\omega}{\Delta \omega} = \frac{\omega L}{R}$$

Typical laser cavity: $L = 1$ m, $\lambda = 0.8 \, \mu$m, $\delta_c = 0.01$ ($\sim 1\%$ loss/RT)

$$Q_c = \frac{4\pi}{0.8 \times 10^{-6}} \frac{1}{0.01} \approx 1.6 \times 10^9 \quad \Rightarrow \quad \Delta \omega \approx 10^6 \, Hz$$
Two Level Rate Equations and Saturation

Two Level Rate Equation

\[
\frac{dN_1(t)}{dt} = -\frac{dN_2(t)}{dt} = -W_{12}N_1(t) + (W_{21} + \gamma_{21})N_2(t)
\]

\[
= -W_{12}[N_1(t) - N_2(t)] + \gamma_{21}N_2(t)
\]

Total number of atoms: \( N = N_1(t) + N_2(t) = \text{constant} \)

Population difference: \( \Delta N(t) = N_1(t) - N_2(t) \)

\[
\frac{d}{dt} \Delta N(t) = -2W_{12}[N_1(t) - N_2(t)] + 2\gamma_{21}N_2(t)
\]

\[
= -2W_{12}[N_1(t) - N_2(t)] - \gamma_{21}[N_1(t) - N_2(t) - N_1(t) - N_2(t)]
\]

\[
\frac{d}{dt} \Delta N(t) = -2W_{12}\Delta N(t) - \frac{\Delta N(t) - N}{T_1}
\]

\[\gamma_{21} = \frac{1}{T_1} \]: non-radiative decay rate
Steady-State Atomic Response: Saturation

\[
\frac{d}{dt} \Delta N(t) = 0 = -2W_{12} \Delta N(t) - \frac{\Delta N(t) - N}{T_1}
\]

\[
\Delta N = \Delta N_{ss} = \frac{N}{1 + 2W_{12}T_1}
\]

\[
\frac{\Delta N_{ss}}{N} = \frac{1}{1 + W_{12}/W_{sat}}, \quad W_{sat} \equiv \frac{1}{2T_1}
\]

Gain coefficient in laser materials

\[
\alpha_m \propto \Delta N
\]

\[
\alpha_m(I) = \frac{\alpha_{m0}}{1 + I/I_{sat}}
\]
Transient Two-Level Solutions

\[ \frac{d}{dt} \Delta N(t) = -2W_{12} \Delta N(t) - \frac{\Delta N(t) - N}{T_1} = -\left(2W_{12} + \frac{1}{T_1}\right) \Delta N(t) + \frac{N}{T_1} \]

Thus, we have

\[ \Delta N(t) = \Delta N_{ss} + A \exp\left[-\left(2W_{12} + \frac{1}{T_1}\right)t\right], \quad \Delta N_{ss} = \frac{N}{1 + 2W_{12}T} \]

Initial population difference at \( t = 0 \): \( \Delta N(0) = \Delta N_{ss} + A \)

\[ \Delta N(t) = \frac{N}{1 + 2W_{12}T} + \left[\Delta N(0) - \frac{N}{1 + 2W_{12}T}\right] \exp\left[-\left(1 + 2W_{12}T_1\right) \frac{t}{T_1}\right] \]

Transient saturation behavior following sudden turn-on of an applied signal
Two-Level Systems with Degeneracy

Stimulated transition rates: \( g_1 W_{12} = g_2 W_{21} \)

Rate equation:
\[
\frac{dN_1(t)}{dt} = - \frac{dN_2(t)}{dt} = -W_{12}N_1(t) + (W_{21} + \gamma_{21})N_2(t)
\]

Population difference:
\[
\Delta N(t) \equiv \left( \frac{g_2}{g_1} \right) N_1(t) - N_2(t)
\]

Effective signal-stimulated transition probability:
\[
W_{\text{eff}} \equiv \frac{1}{2}(W_{12} + W_{21})
\]

Rate equation:
\[
\frac{d}{dt} \Delta N(t) = -2W_{\text{eff}} \Delta N(t) - \frac{\Delta N(t) - N}{T_1}
\]

Atomic time constants: \( T_1 \) and \( T_2 \)

\( T_1 \): longitudinal (on-diagonal) relaxation time
- population recovery or energy decay time

\( T_2 \): dephasing time, transverse (off-diagonal) relaxation time
- time constant for dephasing of coherent macroscopic polarization
Steady state population:

\[ N_4 = \frac{W_p \tau_4}{1 + W_p \tau_4} N_1 \approx W_p \tau_4 N_1, \quad \text{if } W_p \tau_4 << 1 \]

Normalized pumping rate

Rate equation for level 4

\[ \frac{dN_4}{dt} = W_p (N_1 - N_4) - (\gamma_{43} + \gamma_{42} + \gamma_{41})N_4 \]

\[ = W_p (N_1 - N_4) - \frac{N_4}{\tau_4}, \]

where \( \frac{1}{\tau_4} \equiv \gamma_4 = \gamma_{43} + \gamma_{42} + \gamma_{41} \)

Four-level pumping analysis

Steady-State Laser Pumping and Population Inversion
Rate equations for level 2 and 3

\[
\frac{dN_3}{dt} = \gamma_{43} N_4 - (\gamma_{32} + \gamma_{31}) N_3 = \frac{N_4}{\tau_{43}} - \frac{N_3}{\tau_3}
\]

\[
N_3 = \frac{\tau_3}{\tau_{43}} N_4
\]

at steady state

In a good laser system, \( \tau_3 >> \tau_{43} \) so that \( N_3 >> N_4 \)

\[
\frac{dN_2}{dt} = \gamma_{42} N_4 + \gamma_{32} N_3 - \gamma_{21} N_2 = \frac{N_4}{\tau_{42}} + \frac{N_3}{\tau_{32}} - \frac{N_2}{\tau_{21}}
\]

\[
N_2 = \left( \frac{\tau_{21}}{\tau_{32}} + \frac{\tau_{43} \tau_{21}}{\tau_{42} \tau_3} \right) N_3 = \beta N_3
\]

where \( \beta \equiv \frac{\tau_{21}}{\tau_{32}} + \frac{\tau_{43} \tau_{21}}{\tau_{42} \tau_3} \)

If \( \beta < 1 \), \( N_2 < N_3 \) : population inversion on the \( 3 \rightarrow 2 \) transition

In a good laser system, \( \gamma_{42} \approx 0 \) so that the level 4 will relax primarily into the level 3.

\[
\beta \approx \frac{\tau_{21}}{\tau_{32}} \quad \text{condition for population inversion}
\]

\[
\beta \equiv \frac{N_2}{N_3} \approx \frac{\tau_{21}}{\tau_{32}} << 1
\]
**Fluorescent quantum efficiency**

The number of fluorescent photons spontaneously emitted on the laser transition divided by the number of pump photons absorbed on the pump transitions when the laser material is below threshold

\[
\eta = \frac{\gamma_{43}}{\gamma_4} \times \frac{\gamma_{rad}}{\gamma_3} = \frac{\tau_4}{\tau_{43}} \times \frac{\tau_3}{\tau_{rad}}
\]

Fraction of the total atoms excited to level 4 relax directly into the level 3

Fraction of the total decay out of level 3 is purely radiative decay to level 2

**Four level population inversion**

\[N = N_1 + N_2 + N_3 + N_4\]

\[
\frac{N_3 - N_2}{N} \approx \frac{(1-\beta)\eta W_p \tau_{rad}}{1 + (1 + \beta) \eta W_p \tau_{rad}} \approx \frac{\eta W_p \tau_{rad}}{1 + \eta W_p \tau_{rad}}
\]

In a good laser system, \(\tau_{rad} \gg \tau_{43}, \beta \rightarrow 0.\)

\[
\frac{N_3 - N_2}{N} \approx \frac{(1-\beta)\eta W_p \tau_{rad}}{1 + (1 + \beta) \eta W_p \tau_{rad}} \approx \frac{\eta W_p \tau_{rad}}{1 + \eta W_p \tau_{rad}}
\]
Laser gain saturation analysis

Pumping transition
\[ \frac{dN_3}{dt} \bigg|_{pump} = W_p (N_0 - N_3) \approx W_p N_0 \approx W_p N \]

Effective pumping rate: \( R_p = \eta_p W_p N_0 \)

quantum efficiency \( E_3 \rightarrow E_2 \)

Rate equations for laser levels 1 and 2: \( \gamma_2 = \gamma_{21} + \gamma_{20} \)

\[ \frac{dN_2}{dt} = R_p - W_{\text{sig}} (N_2 - N_1) - \gamma_2 N_2 \]

\[ \frac{dN_1}{dt} = W_{\text{sig}} (N_2 - N_1) + \gamma_{21} N_2 - \gamma_1 N_1 \]

\[ N_1 = \frac{W_{\text{sig}} + \gamma_{21}}{W_{\text{sig}} (\gamma_1 + \gamma_{20}) + \gamma_1 \gamma_2} R_p \]

\[ N_2 = \frac{W_{\text{sig}} + \gamma_1}{W_{\text{sig}} (\gamma_1 + \gamma_{20}) + \gamma_1 \gamma_2} R_p \]
Gain saturation behavior

\[ \Delta N_{21} = N_2 - N_1 = \left( \frac{\gamma_1 - \gamma_{21}}{\gamma_1 \gamma_2} \right) R_p \times \frac{1}{1 + \left[ (\gamma_1 + \gamma_{20}) / \gamma_1 \gamma_2 \right] W_{\text{sig}}} \]

\[ = \Delta N_0 \frac{1}{1 + W_{\text{sig}} \tau_{\text{eff}}} \]

Small signal or unsaturated population inversion

\[ \Delta N_0 = \left( \frac{\gamma_1 - \gamma_{21}}{\gamma_1 \gamma_2} \right) R_p = \left( 1 - \frac{\tau_1}{\tau_{21}} \right) \times R_p \tau_2 \]

Effective recovery time

\[ \frac{1}{\tau_{\text{eff}}} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_{20}} \quad \text{or} \quad \tau_{\text{eff}} = \tau_2 \left( 1 + \frac{\tau_1}{\tau_{20}} \right) \]

If \( \gamma_{20} \approx 0, \gamma_2 \approx \gamma_{21} \)

\[ \Delta N_{21} = R_p (\tau_2 - \tau_1) \times \frac{1}{1 + W_{\text{sig}} \tau_2} \]

- Population inversion requires \( \tau_{21} > \tau_1 \).
- \( \Delta N_0 \propto R_p \times \tau_2 (1 - \tau_1 / \tau_2) \)
- If \( \tau_1 \to 0, \tau_{\text{eff}} \approx \tau_2 \).
- The saturation intensity of the inverted population is independent of \( R_p \).
Wave Propagation in an Atomic Medium

Wave equation in a laser medium

\[
\left[ \nabla^2 + \omega^2 \mu \varepsilon \left( 1 + \chi_{at} - i \sigma / \omega \varepsilon \right) \right] E(x, y, z) = 0
\]

atomic susceptibility  

Ohmic loss

Plane wave approximation

\[
\left[ \frac{d^2}{dz^2} + \beta^2 \left( 1 + \chi_{at} - i \sigma / \omega \varepsilon \right) \right] E(z) = 0
\]

“Free-space” propagation constant:  \( \beta = \omega \sqrt{\mu \varepsilon} = \frac{\omega}{c} n = \frac{2\pi n}{\lambda} \)

Propagation factor:  \( E(z) = E_0 e^{-\Gamma z} \)

\[
\Gamma^2 = -\beta^2 \left( 1 + \chi_{at} - i \sigma / \omega \varepsilon \right)
\]

\[
\Gamma = i \beta \sqrt{1 + \chi_{at} - i \sigma / \omega \varepsilon} = i \beta \sqrt{1 + \chi'(\omega) + i \chi''(\omega) - i \sigma / \omega \varepsilon}
\]
Usually, $\chi_{at}, -i\sigma / \omega \varepsilon \ll 1$

$$\Gamma \approx i\beta \left[ 1 + \frac{1}{2} \chi'(\omega) + i \frac{1}{2} \chi''(\omega) - i \frac{\sigma}{2\omega \varepsilon} \right]$$

$$= i\beta + i \frac{1}{2} \beta \chi'(\omega) - \frac{1}{2} \beta \chi''(\omega) + \frac{\sigma}{2\varepsilon \nu}$$

$$= i\beta + i \frac{1}{2} \Delta \beta_m (\omega) - \alpha_m (\omega) + \alpha_0$$

Propagation of a $+z$ traveling wave

$$E(z, t) = \text{Re} \ E_0 \exp \left\{ i \omega t - i \left[ \beta + \Delta \beta_m (\omega) \right] z \right\} + \left\{ \alpha_m (\omega) - \alpha_0 \right\} z$$

Phase shift by atomic transition

Gain by atomic transiton + ohmic loss

The effects of ohmic losses and atomic transition are included.
Propagation factors

\[ \phi_{tot}(z, \omega) = \left[ \beta + \Delta \beta_m(\omega) \right] z = n(\omega) \frac{\omega}{c} z \]

“free-space” Contribution

\[ \beta(\omega) z \]

atomic phase-shift contribution \( \Delta \beta_m(\omega) z \)

atomic gain \( \alpha_m(\omega) z \)

\[ \Delta \beta_m \propto \chi'(\omega) = \frac{a(\omega^2 - \omega_a^2)}{(\omega^2 - \omega_a^2)^2 + \Gamma^2 \omega^2} \]

\[ \approx \frac{\chi'_0 \left[ 2(\omega - \omega_a)/\Gamma \right]}{1 + \left[ 2(\omega - \omega_a)/\Gamma \right]^2} \]
**Single-Pass Laser Amplification**

**Laser gain formulas**

**Complex amplitude gain:**
\[
g(\omega) \equiv \frac{E(L)}{E(0)} = \exp\left\{ -i[\beta + \Delta \beta_m(\omega)]L + [\alpha_m(\omega) - \alpha_0]L \right\}
\]
- **Total phase shift**
- **Amplitude gain or loss**

**Power or intensity gain:**
\[
G(\omega) \equiv \frac{I(L)}{I(0)} = |g(\omega)|^2 = \exp\left\{ 2[\alpha_m(\omega) - \alpha_0]L \right\}
\]
- \[= \frac{1}{2} \beta \chi''(\omega)\]

**Lorenzian transition line shape:**
\[
\chi''(\omega) = \frac{\chi''_0}{1 + \left[2(\omega - \omega_a)/\Delta \omega_a\right]^2}
\]
Power gain: \[ G(\omega) = \exp \left\{ \frac{\omega L \chi''}{v} \times \frac{1}{1 + \left[2(\omega - \omega_a) / \Delta \omega_a\right]^2} \right\} \]

Power gain in decibels (dB):

\[ G_{dB}(\omega) \equiv 10 \log_{10} G(\omega) = 4.34 \ln G(\omega) = \frac{4.34 \omega_a L}{v} \chi''(\omega) \]

Amplification bandwidth and gain narrowing

3-dB amplifier bandwidth: \[ \Delta \omega_{3dB} = \Delta \omega_a \sqrt{\frac{3}{G_{dB}(\omega_a) - 3}} \]

Amplifier phase shift

Total phase shift: \[ \phi_{tot}(z, \omega) = \left[ \beta + \Delta \beta_m(\omega) \right] L = \frac{\omega L}{v} + \frac{\beta L}{2} \chi'(\omega) \]

Atomic transition phase shift:

\[ \Delta \beta_m(\omega) L = \left(2 \frac{\omega - \omega_a}{\Delta \omega_a}\right) \times \alpha_m(\omega) L = \frac{G_{dB}(\omega_a)}{20 \log_{10} e} \times \frac{2(\omega - \omega_a) / \Delta \omega_a}{1 + \left[2(\omega - \omega_a) / \Delta \omega_a\right]^2} \]
Saturation of the population difference

Traveling wave: \[ \frac{dI}{dz} = 2\alpha_m I = \Delta N_{\text{sat}} I \]

Stimulated transition cross-section

Population difference: \[ \Delta N = \Delta N_0 \times \frac{1}{1 + W\tau_{\text{eff}}} = \Delta N_0 \times \frac{1}{1 + I / I_{\text{sat}}} \]

where \[ 2\alpha_m = \Delta N_{0}\sigma \]

\[ \frac{1}{I(z)} \frac{dI(z)}{dz} = 2\alpha_m(z) = \frac{2\alpha_{m0}}{1 + I(z) / I_{\text{sat}}} \]

\[ \int_{I_{\text{in}}}^{I_{\text{out}}} \left[ \frac{1}{I} + \frac{1}{I_{\text{sat}}} \right] dI = 2\alpha_{m0} \int_0^L dz \]

unsaturated power gain

\[ \ln\left( \frac{I_{\text{out}}}{I_{\text{in}}} \right) + \frac{I_{\text{out}} - I_{\text{in}}}{I_{\text{sat}}} = 2\alpha_{m0} L = \ln G_0 \]

where \[ G_0 = \exp(2\alpha_{m0} L) \]
Overall power gain:

\[ G \equiv \frac{I_{out}}{I_{in}} = G_0 \times \exp \left[ - \frac{I_{out} - I_{in}}{I_{sat}} \right] \]

\[ = G_0 \times \exp \left[ - \frac{(G-1)I_{in}}{I_{sat}} \right] = G_0 \times \exp \left[ - \frac{(G-1)I_{out}}{GI_{sat}} \right] \]

\[ \rightarrow \frac{I_{in}}{I_{sat}} = \frac{1}{G-1} \ln \left( \frac{G_0}{G} \right) \quad \text{and} \quad \frac{I_{out}}{I_{sat}} = \frac{G}{G-1} \ln \left( \frac{G_0}{G} \right) \]

Power extraction and available power

\[ I_{extr} \equiv I_{out} - I_{in} = I_{sat} \times \ln \left( \frac{G_0}{G} \right) \]

\[ I_{avail} \equiv \lim_{G \to 1} \left[ I_{sat} \times \ln \left( \frac{G_0}{G} \right) \right] = I_{sat} \times \ln(G_0) = 2\alpha_m L \times I_{sat} \]