Ph 365 Final Exam Wed 12 May 2002

2-4 PM   Closed Book   No Notes

1. Imagine a different universe in which Newton's second law has the form,

$$F(x,t) = m\frac{d^3x}{dt^3}. \tag{1}$$

   (a) Express this equation in the dynamical form appropriate for a Runge-Kutta algorithm:

$$\frac{d\mathbf{y}(t)}{dt} = \mathbf{f}(\mathbf{y},t) \tag{2}$$

   (b) What explicit equations would you solve if the force arouse from the quartic potential

$$V(x) = kx^4? \tag{3}$$

   (c) What would be the components of $\mathbf{f}(\mathbf{y},t)$ if there was a harmonic restoring force plus a periodic driving force:

$$F(x,t) = -kx + A\cos\omega t? \tag{4}$$

2. Consider a random walk starting at the origin in which there are N steps each of length 1 and each made in a random direction. Derive an expression for $< R(N)^2 >$, the average squared distance from the origin after a large number N steps.

3. Here is a series of (about) 40 numbers between 0 and 99:

$$53, 81, 98, 12, 60, 40, 41, 39, 43, 69, 97, 70, 58, 92, 26, 17, 83, 97, 93, 78, 66, 38, 75, 71, 85,$$
$$19, 99, 65, 50, 60, 00, 76, 03, 43, 53, 82, 15, 26, 25, 21, 30, 49, 92, 38, 48, 27, 84, 45, 69, 5.$$

   (a) Use an argument or graphical technique based on science to decide if these are distributed uniformly.

   (b) Use an argument or graphical technique based on science to decide if these are distributed randomly.

   (c) Assume these numbers are distributed randomly and uniformly. Use them to simulate the spontaneous decay of 15 hydrogen atoms from an excited state to the ground state.

4. Explain briefly the difference between a cubic spline fit to a table of data and a least square fit of a cubic polynomial to a set of data.

5. On your homework you solved
$$AX = B \tag{5}$$

   with A a three by three matrix. Write down the explicit simultaneous equations that were solved (you can use symbolic values for individual elements).