This print-out should have 18 questions. Check that it is complete before leaving the printer. Also, multiple-choice questions may continue on the next column or page: find all choices before making your selection.

Some solutions may be found on the class home page.

## AP B 1998 MC 7

12:01, trigonometry, multiple choice, $<1 \mathrm{~min}$. 001
Three forces act on an object. IF the object is in translational equilibrium, which of the following must be true?
I. The vector sum of the three forces must equal zero.
II. The magnitude of the three forces must be equal.
III. All three forces must be parallel.

## 1. I only correct

2. III only
3. I and II only
4. II and III only
5. I, II and III

## Explanation:

If an object is in translational equilibrium, the vector sum of all forces acting on it must equal zero.

Thus, I only is the correct answer.

## Resting Beam

12:01, calculus, numeric, > 1 min .
003
A uniform beam of weight $w$ and length $l$ has weights $W_{1}$ and $W_{2}$ at two positions, as in the figure. The beam is resting at two points. For what value of x will the beam be balanced at $P$ such that the normal force at $O$ is zero?

$$
\begin{aligned}
& \text { 1. } x=\frac{\left(W_{1}+W\right) d+W_{1} \frac{l}{2}}{W_{2}} \text { correct } \\
& \text { 2. } x=\frac{\left(W_{2}+W\right) d+W_{1} \frac{l}{2}}{W_{2}}
\end{aligned}
$$

3. $x=\frac{\left(W_{1}+W\right) d+W_{2} \frac{l}{2}}{W_{2}}$
4. $x=\frac{\left(W_{1}+W_{2}\right) d+W_{1} \frac{l}{2}}{W_{2}}$
5. $x=\frac{\left(W_{1}+W\right) d+W_{2} \frac{l}{2}}{W_{1}}$

## Explanation:

Basic Concepts: In equilibrium,

$$
\begin{aligned}
& \sum \overrightarrow{\mathbf{F}}=0 \\
& \sum \vec{\tau}=0
\end{aligned}
$$

Take the torques about the point $P$.

$$
\begin{gathered}
\sum \tau_{P}=0: \\
-N_{0}\left(\frac{l}{2}+d\right)+W_{1}\left(\frac{l}{2}+d\right)+W d-W_{2} x=0
\end{gathered}
$$

We want to find $x$ for which $N_{0}=0$

$$
x=\frac{\left(W_{1}+W\right) d+W_{1} \frac{l}{2}}{W_{2}}
$$

## Balanced Rod

12:01, calculus, multiple choice, $>1 \mathrm{~min}$. 004
Two objects are to be dragged along a table by attaching them to the ends of $\mathrm{a}(\mathrm{n}) 2.3 \mathrm{~m}$ rod. One weighs 26 N and has a coefficient of friction of 0.22 . The other weighs 22 N and has a coefficient of friction of 0.54 .


How far from the 26 N attachment should the string be positioned to drag them evenly (all angles $=90^{\circ}$ ) and uniformly?
Correct answer: 1.5525 m .
Explanation:

## Basic Concepts

Rotational and Translational Equilibrium

$$
\Sigma \tau=\Sigma \tau_{c w}-\Sigma \tau_{c c w}=0
$$

## Solution

Let the fulcrum be at the point of attachment of the pulling string.


The weight $W_{1}$ exerts a frictional force of $\mu_{1} W_{1}$ on the bar at a distance $x$ from the point of attachment. The weight $W_{2}$ exerts a frictional force of $\mu_{2} W_{2}$ on the bar at a distance $\ell-x$ from the point of attachment. Thus by rotational equilibrium

$$
\begin{aligned}
& \mu_{1} W_{1} x-\mu_{2} W_{2}(\ell-x)=0 \\
& \left(\mu_{1} W_{1}+\mu_{2} W_{2}\right) x=\mu_{2} W_{2} \ell \\
& \quad \Longrightarrow x=\frac{\mu_{2} W_{2} \ell}{\mu_{1} W_{1}+\mu_{2} W_{2}}
\end{aligned}
$$

## Algorithm

$$
\begin{align*}
\ell & =2.3 \mathrm{~m}\left\{\begin{array}{l}
2 \\
5
\end{array}\right\}  \tag{1}\\
W_{1} & =26 \mathrm{~N}\left\{\begin{array}{l}
20 \\
40
\end{array}\right\}  \tag{2}\\
\mu_{1} & =0.22\left\{\begin{array}{l}
0.18 \\
0.24
\end{array}\right\}  \tag{3}\\
W_{2} & =22 \mathrm{~N}\left\{\begin{array}{l}
15 \\
25
\end{array}\right\}  \tag{4}\\
\mu_{2} & =0.54\left\{\begin{array}{l}
0.1 \\
0.7
\end{array}\right\}  \tag{5}\\
x & =\frac{\mu_{2} W_{2} \ell}{\mu_{1} W_{1}+\mu_{2} W_{2}}  \tag{6}\\
& =\frac{\langle 0.54\rangle\langle 22\rangle\langle 2.3\rangle}{\langle 0.22\rangle\langle 26\rangle+\langle 0.54\rangle\langle 22\rangle} \\
& =1.5525 \mathrm{~m} \\
\langle\mathrm{~m}\rangle & =\frac{\langle \rangle\langle\mathrm{N}\rangle\langle\mathrm{m}\rangle}{\langle \rangle\langle\mathrm{N}\rangle+\langle \rangle\langle\mathrm{N}\rangle}
\end{align*}
$$

## Bricks on the Brink

12:03, trigonometry, numeric, $>1 \mathrm{~min}$.

## 005

A uniform brick of length 19.9 cm is placed over the edge of a horizontal surface with the maximum overhang $x$ possible without falling.


Find $x$ for a single block.
Correct answer: 9.95 cm .

## Explanation:

Basic Concepts: The definition of the center of mass ( $n$ bricks):

$$
x_{c m} \equiv \frac{\sum_{i=1}^{n} x_{i} m_{i}}{\sum_{i=1}^{n} m_{i}}=\frac{1}{n} \sum_{i=1}^{n} x_{i}
$$

where $x_{i}$ is the center of mass position of the $i^{t h}$ brick and $m_{i}$ is the mass of the $i^{t h}$ brick.

Solution: The center of mass of a single brick is in its middle or $\frac{1}{2}$ of a brick's length from its maximum overhang. Since $\frac{x_{1}}{L}=\frac{1}{2}$, as measured from the maximum overhang,

$$
\left.\frac{x_{c m}}{L}\right|_{n=1}=\frac{\frac{1}{2}}{1}=\frac{1}{2}
$$

## Bricks on the Brink

12:03, calculus, numeric, $>1 \mathrm{~min}$.
007
Three identical uniform bricks of length 19.9 cm are stacked over the edge of a horizontal surface with the maximum overhang $x$ possible without falling.


Find $x$ for three blocks.
Correct answer: 18.2417 cm .

## Explanation:

The top two bricks can extend $\frac{3}{4}$ of a brick's length from the maximum overhang. When the top two bricks extended $\frac{3}{4}$ of their length past the third brick, the center of mass of the top three bricks is in their middle or $\frac{11}{12}$ of a brick's length from the maximum overhang. Since $\frac{x_{3}}{L}=\left.\frac{x_{c m}}{L}\right|_{n=2}+\frac{1}{2}=\frac{3}{4}+\frac{1}{2}=\frac{5}{4}$, as measured from the maximum overhang,

$$
\left.\frac{x_{c m}}{L}\right|_{n=3}=\frac{\frac{1}{2}+1+\frac{5}{4}}{3}=\frac{11}{12}
$$

## 009

When water freezes, it expands about $9 \%$. The bulk modulus of ice is $1.97 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}$.

What would be the pressure increase inside your automobile engine block if the water in it froze?
Correct answer: 1750.25 atm .

## Explanation:

## Basic Concept:

$$
\Delta P=B \frac{\Delta V}{V}
$$

Solution:

$$
\begin{aligned}
\Delta P & =B \frac{\Delta V}{V} \\
& =\left(1.97 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}\right)(0.09) \\
& =1.773 \times 10^{8} \mathrm{~Pa} \\
& =1750.25 \mathrm{~atm}
\end{aligned}
$$

## Algorithm

$$
\begin{align*}
\left\langle\begin{array}{l}
\text { Pa } \\
\mathrm{atm}
\end{array}\right. & =101300 \mathrm{~Pa} / \mathrm{atm}  \tag{1}\\
\delta & =9.0 \%  \tag{2}\\
\delta_{100} & =\frac{\delta}{100.0}  \tag{3}\\
& =\frac{\langle 9\rangle}{100.0} \\
& =0.09 \\
\rangle & =\frac{\langle \%\rangle}{\langle \rangle}  \tag{units}\\
B & =1.97 \times 10^{9} \mathrm{~N} / \mathrm{m}^{2}\left\{\begin{array}{c}
8 \times 10^{8} \\
3.2 \times 10^{9}
\end{array}\right\}  \tag{4}\\
d P_{p a} & =\frac{B \delta}{100.0} \tag{5}
\end{align*}
$$

$$
\begin{align*}
& =\frac{\left\langle 1.97 \times 10^{9}\right\rangle\langle 9\rangle}{100.0} \\
& =1.773 \times 10^{8} \mathrm{~Pa} \\
\langle\mathrm{~Pa}\rangle & =\frac{\left\langle\mathrm{N} / \mathrm{m}^{2}\right\rangle\langle \%\rangle}{\langle \rangle} \\
d P_{\text {atm }} & =\frac{d P_{p a}}{\left\langle\mathrm{P}_{\mathrm{atm}}\right\rangle}  \tag{6}\\
& =\frac{\left\langle 1.773 \times 10^{8}\right\rangle}{\langle 101300\rangle} \\
& =1750.25 \mathrm{~atm} \\
\langle\mathrm{~atm}\rangle & =\frac{\langle\mathrm{Pa}\rangle}{\langle\mathrm{Pa} / \mathrm{atm}\rangle}
\end{align*}
$$

units

## 010

A 450 kg load is hung on a wire of length 4.5 m , cross sectional area $2.8 \times 10^{-5} \mathrm{~m}^{2}$, and Young's modulus $6.8 \times 10^{10} \mathrm{~N} / \mathrm{m}^{2}$. What is its increase in length?
Correct answer: 10.4228 mm .

## Explanation:

## Basic Concepts:

$$
\frac{F}{A}=Y \frac{\Delta l}{l}
$$

$\frac{F}{A}=Y \frac{\Delta l}{l}$, and for $\Delta l$ we have

$$
\begin{aligned}
\Delta l & =\frac{m g}{A} \frac{l}{Y} \\
& =\frac{(450 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)}{\left(2.8 \times 10^{-5} \mathrm{~m}^{2}\right)} \frac{(4.5 \mathrm{~m})}{\left(6.8 \times 10^{10} \mathrm{~N} / \mathrm{m}^{2}\right)} \\
& =0.0104228 \mathrm{~m} \\
& =10.4228 \mathrm{~mm}
\end{aligned}
$$

## Algorithm

$$
\begin{align*}
\left\langle\begin{array}{l}
\langle\mathrm{mm} \\
\mathrm{m}
\end{array}\right\rangle & =1000 \mathrm{~mm} / \mathrm{m}  \tag{1}\\
g & =9.8 \mathrm{~m} / \mathrm{s}^{2}  \tag{2}\\
m & =450 \mathrm{~kg}\left\{\begin{array}{l}
2500 \\
450
\end{array}\right\}  \tag{3}\\
l & =4.5 \mathrm{~m}\left\{\begin{array}{l}
3.5 \\
5.5
\end{array}\right\}  \tag{4}\\
A & =2.8 \times 10^{-5} \mathrm{~m}^{2}\left\{\begin{array}{l}
1.5 \times 10^{-5} \\
4.5 \times 10^{-5}
\end{array}\right\}  \tag{5}\\
Y & =6.8 \times 10^{10} \mathrm{~N} / \mathrm{m}^{2}\left\{\begin{array}{l}
6.5 \times 10^{10} \\
9.5 \times 10^{10}
\end{array}\right\}  \tag{6}\\
F & =m g  \tag{7}\\
& =\langle 450\rangle\langle 9.8\rangle \\
& =4410 \mathrm{~N}
\end{align*}
$$

$$
\begin{align*}
\langle\mathrm{N}\rangle & =\langle\mathrm{kg}\rangle\left\langle\mathrm{m} / \mathrm{s}^{2}\right\rangle \\
d l_{m} & =\frac{F l}{A Y}  \tag{8}\\
& =\frac{\langle 4410\rangle\langle 4.5\rangle}{\left\langle 2.8 \times 10^{-5}\right\rangle\left\langle 6.8 \times 10^{10}\right\rangle} \\
& =0.0104228 \mathrm{~m} \\
\langle\mathrm{~m}\rangle & =\frac{\langle\mathrm{N}\rangle\langle\mathrm{m}\rangle}{\left\langle\mathrm{m}^{2}\right\rangle\left\langle\mathrm{N} / \mathrm{m}^{2}\right\rangle} \\
d l_{m m} & =d l_{m}\left\langle\mathrm{~mm}_{\mathrm{m}}\right\rangle  \tag{9}\\
& =\langle 0.0104228\rangle\langle 1000\rangle \\
& =10.4228 \mathrm{~mm} \\
\langle\mathrm{~mm}\rangle & =\langle\mathrm{m}\rangle\langle\mathrm{mm} / \mathrm{m}\rangle
\end{align*}
$$

units units

## 011

The displacement in simple harmonic motion is maximum when the

1. acceleration is zero
2. kinetic energy is a maximum
3. velocity is a maximum
4. potential energy function has a maximum
5. velocity is zero correct
6. linear momentum is a maximum

## Explanation:

The maximum displacement occurs at the turning points, which are the points where the velocity is zero.

Piston in Harmonic Motion
12:04, calculus, numeric, $>1$ min.
012
A piston in an automobile engine is in simple harmonic motion. If its amplitude of oscillation from centerline is $\pm 4.62 \mathrm{~cm}$ and its mass is 2.97 kg , find the maximum velocity of the piston when the auto engine is running at the rate of $1850 \mathrm{rev} / \mathrm{min}$.
Correct answer: $8.95041 \mathrm{~m} / \mathrm{s}$.

## Explanation:

The simple harmonic motion is described
by

$$
x=A \cos \omega t
$$

where $\omega$ is the frequency in $\mathrm{rad} / \mathrm{s}$ if $t$ is in seconds.

To find $\omega$ from the given value in rev/min, note: $60 \mathrm{rev} / \mathrm{min}=1 \mathrm{rev} / \mathrm{s}=1 \mathrm{~Hz}$, so

$$
\begin{aligned}
\omega & =2 \pi f \\
& =2 \pi(30.8334 \mathrm{rev} / \mathrm{s}) \\
& =193.732 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

Differentiating $x$ above we find the velocity:

$$
v=\frac{d x}{d t}=-A \omega \sin (\omega t)
$$

meaning the maximum velocity is

$$
\begin{aligned}
v_{\max } & =A \omega \\
& =(0.0462 \mathrm{~m})(193.732 \mathrm{rad} / \mathrm{s}) \\
& =8.95041 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

since sine has a maximum value of 1 .

## Piston in Harmonic Motion

12:04, calculus, numeric, > 1 min.
013
Find the maximum acceleration of the piston when the auto engine is running at this rate.
Correct answer: $1.73398 \mathrm{~km} / \mathrm{s}^{2}$.

## Explanation:

Similarly, the acceleration is

$$
a=\frac{d^{2} x}{d t^{2}}=-A \omega^{2} \cos (\omega t)
$$

so the maximum acceleration is

$$
\begin{aligned}
a_{\max } & =A \omega^{2}=(0.0462 \mathrm{~m})(193.732 \mathrm{rad} / \mathrm{s})^{2} \\
& =1.73398 \mathrm{~km} / \mathrm{s}^{2}
\end{aligned}
$$

since cosine has a maximum value of 1 .

## Algorithm

$$
\begin{align*}
&\left\langle\begin{array}{c}
\mathrm{m} \\
\mathrm{~cm} \\
\left\langle\begin{array}{c}
\mathrm{m} \\
\mathrm{~s}
\end{array}\right\rangle
\end{array}\right\rangle=0.01 \mathrm{~m} / \mathrm{cm}  \tag{1}\\
&\left\langle\begin{array}{c}
\mathrm{km} \\
\mathrm{~m}\rangle
\end{array}\right\rangle=0.001 \mathrm{~km} / \mathrm{m}  \tag{2}\\
& A=4.62 \mathrm{~cm}\left\{\begin{array}{l}
2 \\
8
\end{array}\right\}  \tag{3}\\
& m=2.97 \mathrm{~kg}\left\{\begin{array}{l}
0.8 \\
3.2
\end{array}\right\}  \tag{4}\\
& f=1850 \mathrm{rev} / \min \left\{\begin{array}{l}
1440 \\
5760
\end{array}\right\}  \tag{5}\\
& A_{u}=A\left\langle\begin{array}{l}
\mathrm{m} \\
\mathrm{~cm}
\end{array}\right\rangle \tag{6}
\end{align*}
$$

$$
\begin{array}{rlr} 
& =\langle 4.62\rangle\langle 0.01\rangle \\
& =0.0462 \mathrm{~m} \\
\langle\mathrm{~m}\rangle & =\langle\mathrm{cm}\rangle\langle\mathrm{m} / \mathrm{cm}\rangle & \text { units } \\
f_{u} & =f\left\langle\begin{array}{l}
\text { min } \\
\mathrm{s}
\end{array}\right\rangle \\
& =\langle 1850\rangle\langle 0.0166667\rangle \\
& =30.8334 \mathrm{rev} / \mathrm{s} \\
\langle\mathrm{rev} / \mathrm{s}\rangle & =\langle\mathrm{rev} / \mathrm{min}\rangle\langle\mathrm{min} / \mathrm{s}\rangle & \text { units } \\
\omega & =2.0 \pi f_{u} \\
& =2.0\langle 3.1415926\rangle\langle 30.8334\rangle \\
& =193.732 \mathrm{rad} / \mathrm{s} \\
\langle\mathrm{rad} / \mathrm{s}\rangle & =\langle \rangle\langle \rangle\langle\mathrm{rev} / \mathrm{s}\rangle \\
v_{m a x} & =A_{u} \omega \\
& =\langle 0.0462\rangle\langle 193.732\rangle \\
& =8.95041 \mathrm{~m} / \mathrm{s} \\
\langle\mathrm{~m} / \mathrm{s}\rangle & =\langle\mathrm{m}\rangle\langle\mathrm{rad} / \mathrm{s}\rangle \\
a_{m a x} & =A_{u} \omega^{2.0} \\
& =\langle 0.0462\rangle\langle 193.732\rangle^{2.0} \\
& =1733.98 \mathrm{~m} / \mathrm{s}^{2} \\
\left\langle\mathrm{~m} / \mathrm{s}^{2}\right\rangle & =\langle\mathrm{m}\rangle\langle\mathrm{rad} / \mathrm{s}\rangle^{2.0} \\
a_{u} & =a_{m a x}\langle\mathrm{~km}\rangle \\
& =\langle 1733.98\rangle\langle 0.001\rangle \\
& =1.73398 \mathrm{~km} / \mathrm{s}^{2} \\
\text { units } \\
\left\langle\mathrm{km} / \mathrm{s}^{2}\right\rangle & =\left\langle\mathrm{m} / \mathrm{s}^{2}\right\rangle\langle\mathrm{km} / \mathrm{m}\rangle & \text { units } \\
(12)
\end{array}
$$

AP B 1998 MC 38
13:01, noArithmetic, multiple choice, $<1$ min. 014
A block of mass 3.0 kg is hung from a string, causing it to stretch 12 cm at equilibrium, as shown.


The 3.0 kg block is then replaced by a 4.0 kg block, and the new block is released from the position shown above, at which the spring is unstretched. How far will the 4.0 kg block fall before its direction is reversed?
2. 18 cm
3. 24 cm

## 4. 9 cm

## 5. 48 cm

## Explanation:

When the 3.0 kg block is replaced by 4.0 kg block, the equilibrium point is $\frac{12 \mathrm{~cm}}{3.0 \mathrm{~kg}} \times$ $4.0 \mathrm{~kg}=16 \mathrm{~cm}$ below the unstretched point.

However, the point when the block reverses its direction is twice as far as the equilibrium point. Thus the block will go 32 cm before its direction is reversed.

## AP B 1998 MC 8

13:01, calculus, numeric, $>1 \mathrm{~min}$. 018
The graph below represents the potential energy $U$ as a function of displacement $x$ for an object on the end of a spring oscillating in simple harmonic motion with amplitude $x_{0}$.


Which of the following graphs represents the kinetic energy $K$ of the object as a function of displacement $x$ ?

## 1. 32 cm correct

1. 


2.

3.

4.

correct
5.


## Explanation:

In simple harmonic motion of an object on the end of a spring, the total energy is conserved. At the maximum displacement $x_{0}$, the kinetic energy is 0 , thus, $E=U\left(x_{0}\right)=U_{0}$.

$$
\begin{aligned}
& \\
& U+K
\end{aligned}=E=U_{0}, ~=K(x)=U_{0}-U(x) . ~ \$
$$

Thus, $K(x)$ looks like an upsidedown $U(x)$.

## 019

Assume a car's suspension behaves as a large spring with a spring constant $k$. The mass of the car without any passengers is 1400 kg . When three people with a combined mass of 274 kg sit in the car, the car drops 1.1 cm lower on its springs. When they leave the car, the car bounces up and down.

What is the frequency of the car's oscillation?
Correct answer: 2.10159 Hz .

## Explanation:

We find the spring constant from

$$
m g=k x,
$$

so that

$$
\begin{aligned}
k & =\frac{m g}{x} \\
& =\frac{(274 \mathrm{~kg})\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)}{(1.1 \mathrm{~cm})} \\
& =244109 \mathrm{~N} / \mathrm{m} .
\end{aligned}
$$

Then

$$
\begin{aligned}
f & =\frac{1}{2 \pi} \sqrt{\frac{k}{M}} \\
& =\frac{1}{2 \pi} \sqrt{\frac{244109 \mathrm{~N} / \mathrm{m}}{1400 \mathrm{~kg}}} \\
& =2.10159 \mathrm{~Hz} .
\end{aligned}
$$

## Algorithm

$$
\begin{aligned}
\left\langle\begin{array}{c}
\mathrm{m} \\
\mathrm{~cm}
\end{array}\right\rangle & =0.01 \mathrm{~m} / \mathrm{cm} \\
g & =9.8 \mathrm{~m} / \mathrm{s}^{2} \\
m & =274 \mathrm{~kg}\left\{\begin{array}{l}
100 \\
500
\end{array}\right\} \\
x & =1.1 \mathrm{~cm}\left\{\begin{array}{l}
0.2 \\
1.2
\end{array}\right\} \\
x_{u} & =x\left\langle{ }_{\mathrm{cm}}^{\mathrm{m}}\right\rangle \\
& =\langle 1.1\rangle\langle 0.01\rangle \\
& =0.011 \mathrm{~m} \\
\langle\mathrm{~m}\rangle & =\langle\mathrm{cm}\rangle\langle\mathrm{m} / \mathrm{cm}\rangle \\
M & =1400 \mathrm{~kg}\left\{\begin{array}{l}
1000 \\
5000
\end{array}\right\} \\
k & =\frac{m g}{x_{u}} \\
& =\frac{\langle 274\rangle\langle 9.8\rangle}{\langle 0.011\rangle} \\
& =244109 \mathrm{~N} / \mathrm{m} \\
\langle\mathrm{~N} / \mathrm{m}\rangle & =\frac{\langle\mathrm{kg}\rangle\left\langle\mathrm{m} / \mathrm{s}^{2}\right\rangle}{\langle\mathrm{m}\rangle} \\
f & =\frac{1.0 \quad \sqrt{\frac{k}{M}}}{2.0 \pi} \\
& =\frac{1.0 \quad \sqrt{\frac{\langle 244109\rangle}{\langle 1400\rangle}}}{2.0\langle 3.1415926\rangle} \\
& =2.10159 \mathrm{~Hz} \\
\langle\mathrm{~Hz}\rangle & =\frac{\langle \rangle \quad \sqrt{\frac{\langle\mathrm{N} / \mathrm{m}\rangle}{\langle\mathrm{kg}\rangle}}}{\langle \rangle\langle \rangle}
\end{aligned}
$$

units
units
units

AP M 1993 MC 33
13:02, trigonometry, multiple choice,$<1 \mathrm{~min}$. 020
A simple pendulum consists of a 1.0-kilogram brass bob on a string about 1.0 meter long. It has a period of 2.0 seconds. The pendulum would have a period of 1.0 second if the

1. string were replaced by one about 2.0 meters long
2. string were replaced by one about 0.25 meter long correct
3. bob were replaced by a $0.25-\mathrm{kg}$ brass sphere
4. bob were replaced by a $4.0-\mathrm{kg}$ brass sphere
5. amplitude of the motion were increased Explanation:

The period of the motion is

$$
T=2 \pi \sqrt{\frac{L}{g}}
$$

Only the change of $L$ will result in the change of the period. So if we want to reduce the period to half of its original value, we should replace the string with one about 0.25 meter long.

## AP M 1998 MC 10

13:02, calculus, numeric, $>1 \mathrm{~min}$.
021
A pendulum with a period of 1 s on Earth, where the acceleration due to gravity is $g$, is taken to another planet, where its period is 2 S . The acceleration due to gravity on the other planet is most nearly

1. $4 g$
2. $\frac{g}{2}$
3. $g$
4. $2 g$
5. $\frac{g}{4}$ correct

## Explanation:

For a pendulum, the relationship between the period and the acceleration is $T=2 \pi \sqrt{\frac{\ell}{g}}$. Now $T_{1}: T_{2}=1: 2$, so $g_{1}: g_{2}=4: 1$, which gives the acceleration on the other planet is $\frac{g}{4}$.

022
A uniform disk of radius 2.7 m and mass 9.1 kg is suspended from a pivot 0.432 m above its center of mass.


Find the angular frequency $\omega$ for small oscillations.
Correct answer: $1.05115 \mathrm{rad} / \mathrm{s}$.

## Explanation:

Basic Concepts The physical pendulum:

$$
\begin{gathered}
\tau=I \alpha=-m g d \sin \theta \\
\alpha=\frac{d^{2} \theta}{d t^{2}}
\end{gathered}
$$

so that the angular frequency for small oscillations ( $\sin \theta \approx \theta$ ) is

$$
\omega=\sqrt{\frac{m g d}{I}} .
$$

Parallel axis theorem

$$
I=I_{0}+m a^{2}
$$

Solution: We need the moment of inertia of the disk about the pivot point, which we call P. The moment of inertia of a uniform disk about its center is

$$
I_{d i s k}=\frac{1}{2} m R^{2},
$$

but here the disk is rotating about P , a distance $d$ from the center of mass. The parallel
axis theorem lets us move the axis of rotation a distance $d$ :

$$
I_{P}=\frac{1}{2} m R^{2}+m d^{2}=m\left(\frac{R^{2}}{2}+d^{2}\right) .
$$

Then using the formula for the small angle oscillation frequency of a physical pendulum (see Basic Concepts above), we obtain

$$
\omega=\sqrt{\frac{m g d}{I_{P}}}=\sqrt{\frac{m g d}{m\left(\frac{R^{2}}{2}+d^{2}\right)}}
$$

or

$$
\begin{aligned}
\omega & =\sqrt{\frac{g d}{\frac{R^{2}}{2}+d^{2}}} \\
& =\sqrt{\frac{\left(9.8 \mathrm{~m} / \mathrm{s}^{2}\right)(0.432 \mathrm{~m})}{\frac{(2.7 \mathrm{~m})^{2}}{2}+(0.432 \mathrm{~m})^{2}}} \\
& =1.05115 \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

## Algorithm

$$
\begin{align*}
R & =2.7 \mathrm{~m}\left\{\begin{array}{l}
1 \\
5
\end{array}\right\}  \tag{1}\\
m & =9.1 \mathrm{~kg}\left\{\begin{array}{l}
1 \\
10
\end{array}\right\}  \tag{2}\\
\mathcal{R}_{1} & =0.16\left\{\begin{array}{l}
0.1 \\
0.7
\end{array}\right\}  \tag{3}\\
d & =\mathcal{R}_{1} R  \tag{4}\\
& =\langle 0.16\rangle\langle 2.7\rangle \\
& =0.432 \mathrm{~m} \\
\langle\mathrm{~m}\rangle & =\langle \rangle\langle\mathrm{m}\rangle \\
g & =9.8 \mathrm{~m} / \mathrm{s}^{2}  \tag{5}\\
\omega & =\sqrt{\frac{g d}{\frac{R^{2} .0}{2.0}+d^{2.0}}}  \tag{6}\\
& =\sqrt{\frac{\langle 9.8\rangle\langle 0.432\rangle}{\frac{\langle 2.7\rangle^{2.0}}{2.0}+\langle 0.432\rangle^{2.0}}} \\
& =1.05115 \mathrm{rad} / \mathrm{s} \\
\langle\mathrm{rad} / \mathrm{s}\rangle & =\sqrt{\frac{\left\langle\mathrm{m} / \mathrm{s}^{2}\right\rangle\langle\mathrm{m}\rangle}{\frac{\langle\mathrm{m}\rangle^{2.0}}{\langle \rangle}+\langle\mathrm{m}\rangle^{2.0}}}
\end{align*}
$$

025
The period of a simple pendulum swinging through a small angle is independent of

1. the acceleration due to gravity
2. the height above sea level
3. the location on the earth's surface
4. the length of the pendulum
5. the mass of the pendulum bob correct

## Explanation:

The period of a pendulum swinging in a small arc is

$$
T=2 \pi \sqrt{\frac{l}{g}}
$$

## Oscillations

13:03, trigonometry, multiple choice, $<1$ min. 026
An object with a potential energy $U(x)$ can oscillate around

1. Any point provided that the restoring force exerted on the object is given by Hooke's law
2. Any stable equilibrium point correct
3. Any point
4. Any unstable equilibrium point
5. Certain stable equilibrium points
6. Any equilibrium point

## Explanation:

Oscillations can only be done around points of stable equilibrium.

