This print-out should have 8 questions. Check that it is complete before leaving the printer. Also, multiple-choice questions may continue on the next column or page: find all choices before making your selection.

Note that only a few (usually 4) of the problems will have their scores kept for a grade. You may make multiple tries to get a problem right, although it's worth less each time. Worked solutions to a number of these problems (even some of the scored ones) may be found on the Ph211 home page.

Dragging a Block

07:01, calculus, numeric, > 1 min.

001

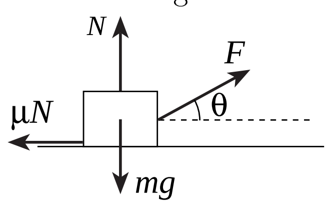
A 15.5 kg block is dragged over a rough, horizontal surface by a constant force of 106 N acting at an angle of angle 26.8° above the horizontal. The block is displaced 75.3 m, and the coefficient of kinetic friction is 0.2.

Find the work done by the force 106 N.

Correct answer: 7124.44 J.

Explanation:

Consider the force diagram



Work is $W = \vec{F} \cdot \vec{s}$, where $\vec{s}$ is the distance traveled. In this problem $\vec{s} = 5\,\hat{\imath}$ is only in the x direction.

$$\begin{aligned} \Rightarrow W_A &= F_x\, s_x = F(\cos\theta) s_x \\ &= (106\text{ N})\,\cos(26.8^\circ)\,(75.3\text{ m}) \\ &= 7124.44\text{ J}\,. \end{aligned}$$

004

A force F = $(4.0\,x\,\hat{\imath} + 4.74\,y\,\hat{\jmath})$ N acts on an object as the object moves in the $\hat{\imath}$ direction from the origin to $b = 7.2$ m.

Find the work done on the object by the force.

Correct answer: 103.68 J.

Explanation:

The work is found through the expression,

$$W = \int_i^f \vec{F} \cdot d\vec{s}$$

For the force given,

$$W = \int_0^{7.2\text{ m}} (4\,x\,\hat{\imath} + 4.74\,y\,\hat{\jmath}) \cdot dx\,\hat{\imath}$$

Evaluation of the integral yields,

$$W = (4\text{ N/m}) \;*\; \frac{x^2}{2}\Big|_0^{7.2\text{ m}}$$

Algorithm

$$\begin{aligned} a &= 4.74\ \left\{{1.2 \atop 4.8}\right\} && (1) \\ b &= 7.2\text{ m}\ \left\{{2 \atop 8}\right\} && (2) \\ c &= 4.0\text{ N/m} && (3) \\ W &= \frac{c\,b^{2.0}}{2.0} && (4) \\ &= \frac{\langle 4\rangle\,\langle 7.2\rangle^{2.0}}{2.0} \\ &= 103.68\text{ J} \\ \langle\text{J}\rangle &= \frac{\langle\text{N/m}\rangle\,\langle\text{m}\rangle^{2.0}}{\langle\rangle} && \text{units} \end{aligned}$$

005

When a spring is stretched near its elastic limit, the spring force satisfies the equation,

$$F = -k\,x + \beta\,x^3$$

If $k = 9.23$ N/m and $\beta = 149$ N/m^3, calculate the work done by this force when the spring is stretched 0.0514 m from its equilibrium position.

Correct answer: −0.0119326 J.

Explanation:

The work done is found by integrating the force of the spring over the distance it is compressed,

$$W = \int_{x_i}^{x_f} F\,dx$$

For this problem,

$$W = \int_{x_i}^{x_f} (-k\,x + \beta\,x^3)\;dx$$

Evaluating the integral (where $x_i = 0$) we find,

$$\begin{aligned} W &= \frac{-k\,x_f^2}{2} + \frac{\beta\,x_f^4}{4} \\ &= \frac{-(9.23\ \text{N/m})\,(0.0514\ \text{m})^2}{2} \\ &\quad + \frac{(149\ \text{N/m}^3)\,(0.0514\ \text{m})^4}{4} \\ &= -0.0119326\ \text{J} \end{aligned}$$

Algorithm

$$k = 9.23\ \text{N/m}\ \left\{{4 \atop 16}\right\} \quad (1)$$

$$\beta = 149\ \text{N/m}^3\ \left\{{40 \atop 160}\right\} \quad (2)$$

$$x_f = 0.0514\ \text{m}\ \left\{{0.04 \atop 0.16}\right\} \quad (3)$$

$$\begin{aligned} W1 &= \left(-\frac{k\,x_f^{2.0}}{2.0}\right) \quad (4) \\ &= \left(-\frac{\langle 9.23\rangle\,\langle 0.0514\rangle^{2.0}}{2.0}\right) \\ &= -0.0121926\ \text{J} \\ \langle \text{J}\rangle &= \left(-\frac{\langle \text{N/m}\rangle\,\langle \text{m}\rangle^{2.0}}{\langle\rangle}\right) \quad \text{units} \end{aligned}$$

$$\begin{aligned} W2 &= \frac{\beta\,x_f^{4.0}}{4.0} \quad (5) \\ &= \frac{\langle 149\rangle\,\langle 0.0514\rangle^{4.0}}{4.0} \\ &= 0.000260003\ \text{J} \\ \langle \text{J}\rangle &= \frac{\langle \text{N/m}^3\rangle\,\langle \text{m}\rangle^{4.0}}{\langle\rangle} \quad \text{units} \end{aligned}$$

$$\begin{aligned} W &= W1 + W2 \quad (6) \\ &= \langle -0.0121926\rangle + \langle 0.000260003\rangle \\ &= -0.0119326\ \text{J} \\ \langle \text{J}\rangle &= \langle \text{J}\rangle + \langle \text{J}\rangle \quad \text{units} \end{aligned}$$

Energies

07:03, calculus, numeric, > 1 min.

006

As an object moves from point A to point B only two forces act on it: one force is conservative and does −70 J of work, the other force is nonconservative and does +50 J of work.

Between A and B,

1. kinetic energy of the object increases, mechanical energy increases.

2. None of these.

3. kinetic energy of the object decreases, mechanical energy decreases.

4. kinetic energy of the object increases, mechanical energy decreases.

5. kinetic energy of the object decreases, mechanical energy increases. **correct**

Explanation:

$$W = -70 + 50 = -20J.$$

Therefore,

$$W = \Delta E_K < 0,$$

and E_K decreases. Since

$$W_{non-conserve} = 50 = \Delta E_{Mechanical} > 0,$$

the total mechanical energy increases.

Pushing a Crate 03

07:03, calculus, numeric, > 1 min.

008

You push a 36.9 kg crate at a constant speed of 2.08 m/s across a horizontal floor with coefficient of kinetic friction 0.13.

At what rate is work being done on the crate by you?

Correct answer: 97.782 W.

Explanation:

In order to keep the crate moving at a constant speed, the force you apply needs to exactly cancel the frictional force. Therefore the force that you need to apply is

$$F = \mu_k\,m\,g$$

The work you need to apply per unit time (power) is given by

$$P = F\,v$$

Therefore, the rate at which you do work on the crate is

$$\begin{aligned} P &= \mu_k\,m\,g\,v \\ &= (0.13)\,(36.9\ \text{kg})\,(9.8\ \text{m/s}^2)\,(2.08\ \text{m/s}) \\ &= 97.782\ \text{W} \end{aligned}$$

Pushing a Crate 03
07:04, trigonometry, multiple choice, > 1 min.
009

At what rate is energy being dissipated by the frictional force?

Correct answer: -97.782 W.

Explanation:

To keep the crate moving at a constant speed, the work per unit time that you apply must exactly cancel the energy per unit time that is dissipated by the frictional force. Therefore, the rate at which energy is being dissipated by friction is

$$\begin{aligned} P &= -\mu_k \, m \, g \, v \\ &= -97.782 \text{ W} \end{aligned}$$

Algorithm

$$\begin{aligned}
m &= 36.9 \text{ kg } \left\{ {16 \atop 64} \right\} && (1) \\
v &= 2.08 \text{ m/s } \left\{ {0.56 \atop 2.24} \right\} && (2) \\
\mu_k &= 0.13 \ \left\{ {0.1 \atop 0.4} \right\} && (3) \\
g &= 9.8 \text{ m/s}^2 && (4) \\
f &= \mu_k \ m \ g && (5) \\
&= \langle 0.13 \rangle \, \langle 36.9 \rangle \, \langle 9.8 \rangle \\
&= 47.0106 \text{ N} \\
\langle \text{N} \rangle &= \langle \rangle \, \langle \text{kg} \rangle \, \langle \text{m/s}^2 \rangle && \text{units} \\
P &= f \ v && (6) \\
&= \langle 47.0106 \rangle \, \langle 2.08 \rangle \\
&= 97.782 \text{ W} \\
\langle \text{W} \rangle &= \langle \text{N} \rangle \, \langle \text{m/s} \rangle && \text{units} \\
P_f &= (-P) && (7) \\
&= (-\langle 97.782 \rangle) \\
&= -97.782 \text{ W} \\
\langle \text{W} \rangle &= (-\langle \text{W} \rangle) && \text{units}
\end{aligned}$$

Tree Stops a Bullet
07:04, calculus, numeric, > 1 min.
010

A 4.62 g bullet moving at 891 m/s penetrates a tree to a depth of 2.06 cm.

Use energy considerations to find the average frictional force that stops the bullet.

Correct answer: 89022.6 N.

Explanation:

We can use conservation of energy to relate the initial kinetic energy of the bullet to the work done by the frictional force.

$$\frac{1}{2} \, m \, v^2 = f \cdot s$$

Solving for the frictional force, f,

$$\begin{aligned} f &= \frac{m \, v^2}{2 \, s} \\ &= \frac{(0.00462 \text{ kg}) \, (891 \text{ m/s})^2}{2 \, (0.0206 \text{ m})} \\ &= 89022.6 \text{ N} \, . \end{aligned}$$

Tree Stops a Bullet
07:04, calculus, numeric, > 1 min.
011

Assuming that the frictional force is constant, determine how much time elapsed between the moment the bullet entered the tree and the moment it stopped.

Correct answer: 4.62402×10^{-5} s.

Explanation:

If we assume the frictional force is constant, by Newton's second law we can assume constant deceleration, which implies

$$v_{avg} = \frac{v_{initial}}{2} \, .$$

The time of deceleration is then

$$\begin{aligned} t &= \frac{s}{v_{avg}} \\ &= \frac{0.0206 \text{ m}}{\frac{891 \text{ m/s}}{2}} \\ &= 4.62402 \times 10^{-5} \text{ s} \, . \end{aligned}$$

Algorithm

$$\begin{aligned}
\left\langle \tfrac{\text{kg}}{\text{g}} \right\rangle &= 0.001 \text{ kg/g} && (1) \\
\left\langle \tfrac{\text{m}}{\text{cm}} \right\rangle &= 0.01 \text{ m/cm} && (2) \\
m &= 4.62 \text{ g } \left\{ {2 \atop 8} \right\} && (3) \\
v &= 891 \text{ m/s } \left\{ {240 \atop 960} \right\} && (4) \\
d &= 2.06 \text{ cm } \left\{ {1.6 \atop 6.4} \right\} && (5) \\
m_u &= m \, \left\langle \tfrac{\text{kg}}{\text{g}} \right\rangle && (6) \\
&= \langle 4.62 \rangle \, \langle 0.001 \rangle \\
&= 0.00462 \text{ kg}
\end{aligned}$$

$$\langle \mathrm{kg} \rangle = \langle \mathrm{g} \rangle \, \langle \mathrm{kg/g} \rangle \qquad \text{units}$$

$$d_u = d \, \langle {}^{\mathrm{m}}_{\mathrm{cm}} \rangle \tag{7}$$

$$= \langle 2.06 \rangle \, \langle 0.01 \rangle$$

$$= 0.0206 \text{ m}$$

$$\langle \mathrm{m} \rangle = \langle \mathrm{cm} \rangle \, \langle \mathrm{m/cm} \rangle \qquad \text{units}$$

$$f = \frac{m_u \, v^{2.0}}{2.0 \, d_u} \tag{8}$$

$$= \frac{\langle 0.00462 \rangle \, \langle 891 \rangle^{2.0}}{2.0 \, \langle 0.0206 \rangle}$$

$$= 89022.6 \text{ N}$$

$$\langle \mathrm{N} \rangle = \frac{\langle \mathrm{kg} \rangle \, \langle \mathrm{m/s} \rangle^{2.0}}{\langle \rangle \, \langle \mathrm{m} \rangle} \qquad \text{units}$$

$$t = \frac{2.0 \, d_u}{v} \tag{9}$$

$$= \frac{2.0 \, \langle 0.0206 \rangle}{\langle 891 \rangle}$$

$$= 4.62402 \times 10^{-5} \text{ s}$$

$$\langle \mathrm{s} \rangle = \frac{\langle \rangle \, \langle \mathrm{m} \rangle}{\langle \mathrm{m/s} \rangle} \qquad \text{units}$$