This print-out should have 13 questions. Check that it is complete before leaving the printer. Also, multiple-choice questions may continue on the next column or page: find all choices before making your selection.

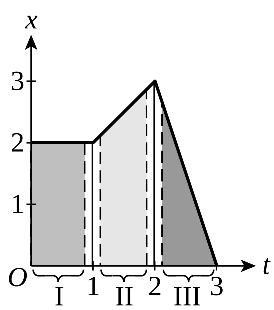
Note that only a few (usually 4) of the problems will have their scores kept for a grade. You may make multiple tries to get a problem right, although it's worth less each time. Worked solutions to a numner of these problems (even some of the scored ones) may be found on the Ph211 home page.

Displacement Curve

02:01, trigonometry, multiple choice, > 1 min.

001

Consider a moving object whose position x is plotted as a function of the time t on the following figure:



Clearly, the object moved in different ways during the time intervals denoted I, II and III on the figure.

During these three intervals, when was the object's *speed* highest? Caution: Do not confuse the speed with the velocity.

1. During interval I.

2. During interval II.

3. During interval III. **correct**

4. During intervals II and III (same speed during those two intervals).

5. Same speed during each of the three intervals.

Explanation:

The velocity $\mathbf{v}$ is the slope of the $x(t)$ curve; the magnitude $v = |\mathbf{v}|$ of this slope is the speed. Looking at the picture we see that the curve is steepest (in absolute magnitude) during the interval III and that is when the object had the highest speed.

002

During which interval(s) did the object have no change in displacement?

1. During each of the three intervals.

2. During interval II only.

3. During interval III only.

4. During interval I only. **correct**

5. During none of the three intervals.

6. During intervals II and III.

Explanation:

The figure shows no change in displacement during the interval I but there are changes in displacements during the other intervals II and III.

003

During which interval(s) did the object's velocity remain constant?

1. During each of the three intervals. **correct**

2. During interval II only.

3. During interval III only.

4. During interval I only.

5. During none of the three intervals.

Explanation:

For each of the three intervals I, II or III, the $x(t)$ curve is linear, so its slope — the velocity $\mathbf{v}$ — is constant during each interval. *Between* the intervals, the velocity did change in a rather abrupt manner — but it did remain constant **during** each interval.

004

During which interval(s) does the object have non-zero, positive acceleration?

1. During interval I only.

2. During interval II only.

3. During interval III only.

4. During none of the three intervals. **correct**

5. During each of the three intervals.

Explanation:

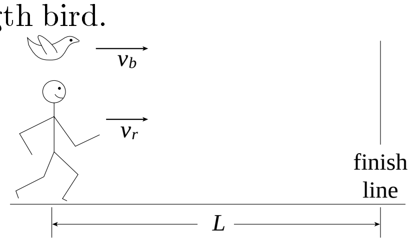
During each interval the velocity remained constant (see previous explanation), so the acceleration was zero. Between the intervals, the velocity changed abruptly, which means the accelerations were undefined — but the question is about the intervals, not the transition instants.

Bird and Runner

02:01, trigonometry, numeric, > 1 min.

005

A runner is jogging at a steady $v_r = 8.3$ km/hr. When the runner is $L = 2.7$ km from the finish line a bird begins flying from the runner to the finish line at $v_b = 33.2$ km/hr (4 times as fast as the runner). When the bird reaches the finish line, it turns around and flies back to the runner. Even though the bird is a dodo, we will assume that it occupies only one point in space, *i.e.*, a zero length bird.



How far does the bird travel?

Correct answer: 4.32 km.

Explanation:

Let, dodo birds fly, and

d_r be the distance the runner travels.

d_b be the distance the bird travels.

v_r be the speed of the runner.

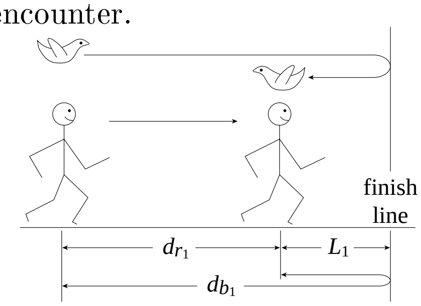
v_b be the speed of the bird.

$L = d_r$ be the original distance to the finish line.

L_1 be the distance to the finish line after the first encounter.

$\vdots$

L_i be the distance to the finish line after the i^{th} encounter.



Since the bird travels 4 times as fast as the runner at the first meeting between the bird and runner,

$$d_{b_1} = 4\, d_{r_1}\,. \tag{1}$$

The sum of the bird's and runner's distances is 4 times L.

$$d_{b_1} + d_{r_1} = 2\, L\,. \tag{2}$$

Therefore, substituting for d_{b_1} from Eq. (1)

$$d_{r_1} + 4\, d_{r_1} = 2\, L$$

$$d_{r_1} = \frac{2}{5}\, L = \frac{2}{5}\,(2.7 \text{ km}) = 1.08 \text{ km}\,. \tag{3}$$

Thus the distance the bird flies is

$$\begin{aligned} d_{b_1} &= 4\, d_{r_1} = \frac{8}{5}\, L \\ &= \frac{8}{5}\,(2.7 \text{ km}) = 4.32 \text{ km}\,, \end{aligned} \tag{4}$$

and the distance for the runner to travel after this first encounter is

$$L_1 = \frac{3}{5}\, L = \frac{3}{5}\,(2.7 \text{ km}) = 1.62 \text{ km}\,.$$

006

After this first encounter, the bird then turns around and flies from the runner back to the finish line, turns around again and flies back to the runner. The bird repeats the back and forth trips until the runner reaches the finish line.

How far does the bird travel from the beginning? (i.e. include the distance traveled to

the first encounter)
Correct answer: 10.8 km.
Explanation:

Repeating this scenario a second time the distance for the runner to travel after the second encounter is

$$L_2 = \frac{3}{5}\,L_1 = \left(\frac{3}{5}\right)^2 L\,,$$

and the third time

$$L_3 = \frac{3}{5}\,L_2 = \left(\frac{3}{5}\right)^3 L\,,$$

and the i^{th} time

$$L_i = \frac{3}{5}\,L_{i-1} = \left(\frac{3}{5}\right)^i L\,. \qquad (5)$$

Note: The distance the bird travels between the $(i-1)^{th}$ and i^{th} time is [see Eq. (4)]

$$d_{b_i} = \frac{8}{5}\,L\left(\frac{3}{5}\right)^i \qquad (6)$$

and summing over all terms d_{b_i}

$$d_b = \sum_{i=0}^{\infty} d_{b_i} = \frac{8}{5}\,L\left[\sum_{i=0}^{\infty}\left(\frac{3}{5}\right)^i\right] \qquad (7)$$

$$= \frac{8}{5}\,L\left[1 + \frac{3}{5} + \left(\frac{3}{5}\right)^2 + \left(\frac{3}{5}\right)^3 + \left(\frac{3}{5}\right)^4 + \left(\frac{3}{5}\right)^5 + \left(\frac{3}{5}\right)^6 + \cdots\right] \qquad (8)$$

Or, by factoring $\frac{1}{5}$ from the second term on

$$d_b = \frac{8}{5}\,L\left\{1 + \frac{3}{5}\left[1 + \frac{3}{5} + \left(\frac{3}{5}\right)^2 + \left(\frac{3}{5}\right)^3 + \left(\frac{3}{5}\right)^4 + \left(\frac{3}{5}\right)^5 + \cdots\right]\right\} \qquad (9)$$

By comparing Eq. (8) with (9), and generalizing ($\ell = 1$, and $k = 3$), the infinite series

$$\sum_{i=0}^{\infty}\left(\frac{\ell}{k}\right)^i = 1 + \frac{\ell}{k}\sum_{i=0}^{\infty}\left(\frac{\ell}{k}\right)^i \qquad (10)$$

then solving Eq. (10) for $\sum_{i=0}^{\infty}\left(\frac{\ell}{k}\right)^i$

$$\sum_{i=0}^{\infty}\left(\frac{\ell}{k}\right)^i = \left.\frac{k}{k-\ell}\right|_{k=5,\,\ell=3} \qquad (11)$$

$$\sum_{i=0}^{\infty}\left(\frac{3}{5}\right)^i = \frac{5}{5-3} = \frac{5}{2}\,.$$

Therefore [from Eq. (7)]

$$\begin{aligned} d_b &= \frac{8}{5}\,L\sum_{i=0}^{\infty}\left(\frac{3}{5}\right)^i \\ &= \frac{8}{5}\,L\,\frac{5}{2} = 4\,L \\ &= 4\,(2.7\text{ km}) = 10.8\text{ km}\,. \end{aligned}$$

Elegant Alternative Solution: The bird will travel 4 times as far as the runner in the same time. Since the bird and jogger travel for the same length of time, the bird will travel

$$d_b = 4 \times L = 4\,(2.7\text{ km}) = 10.8\text{ km}$$

Algorithm

$$\begin{aligned} i &= 4\ \left\{{2 \atop 6}\right\} && (1)\\ k &= i + 1.0 && (2)\\ &= \langle 4\rangle + 1.0 \\ &= 5 \\ \langle\rangle &= \langle\rangle + \langle\rangle.\langle\rangle && \text{units}\\ j &= 2.0\,i && (3)\\ &= 2.0\,\langle 4\rangle \\ &= 8 \\ \langle\rangle &= \langle\rangle\,\langle\rangle && \text{units}\\ \ell &= k - 2.0 && (4)\\ &= \langle 5\rangle - 2.0 \\ &= 3 \\ \langle\rangle &= \langle\rangle - \langle\rangle.\langle\rangle && \text{units}\\ v_r &= 8.3\text{ km/hr}\ \left\{{2 \atop 9}\right\} && (5)\\ v_b &= i\ v_r && (6)\\ &= \langle 4\rangle\,\langle 8.3\rangle \\ &= 33.2\text{ km/hr} \\ \langle\text{km/hr}\rangle &= \langle\rangle\,\langle\text{km/hr}\rangle && \text{units}\\ d_r &= 2.7\text{ km}\ \left\{{2 \atop 9}\right\} && (7) \end{aligned}$$

$$\begin{aligned} L_1 &= \frac{\ell\, d_r}{k} & (8)\\ &= \frac{\langle 3\rangle\,\langle 2.7\rangle}{\langle 5\rangle}\\ &= 1.62\ \text{km}\\ \langle \text{km}\rangle &= \frac{\langle\rangle\,\langle \text{km}\rangle}{\langle\rangle} & \text{units}\\ d_{r_1} &= \frac{2.0\, d_r}{k} & (9)\\ &= \frac{2.0\,\langle 2.7\rangle}{\langle 5\rangle}\\ &= 1.08\ \text{km}\\ \langle \text{km}\rangle &= \frac{\langle\rangle\,\langle \text{km}\rangle}{\langle\rangle} & \text{units}\\ d_{b_1} &= \frac{j\, d_r}{k} & (10)\\ &= \frac{\langle 8\rangle\,\langle 2.7\rangle}{\langle 5\rangle}\\ &= 4.32\ \text{km}\\ \langle \text{km}\rangle &= \frac{\langle\rangle\,\langle \text{km}\rangle}{\langle\rangle} & \text{units}\\ d_b &= i\, d_r & (11)\\ &= \langle 4\rangle\,\langle 2.7\rangle\\ &= 10.8\ \text{km}\\ \langle \text{km}\rangle &= \langle\rangle\,\langle \text{km}\rangle & \text{units} \end{aligned}$$

Acceleration of a Sled

02:02, trigonometry, numeric, > 1 min.

009

A rocket-driven sled running on a straight, level track has been used to study the physiological effects of large accelerations on astronauts. One such sled can attain a speed of 448 m/s in 1.9 s starting from rest.

What is the acceleration of the sled, assuming it is constant?

Correct answer: 235.789 m/s^2.

Explanation:

$v = v_0 + a\,t$, so

$$\begin{aligned} a &= \frac{v - v_0}{t}\\ &= \frac{(448\ \text{m/s}) - (0\ \text{m/s})}{1.9\ \text{s}}\\ &= 235.789\ \text{m/s}^2 \end{aligned}$$

010

How many g's would you pull? (What factor times $g = 9.80$ m/s^2 is this?)

Correct answer: 24.0602 .

Explanation:

$a = n\,g$, so

$$\begin{aligned} n &= \frac{a}{g}\\ &= \frac{235.789\ \text{m/s}^2}{9.8\ \text{m/s}^2}\\ &= 24.0602 \end{aligned}$$

Acceleration of a Sled

02:03, trigonometry, numeric, > 1 min.

011

How far does the sled travel in 1.9 s, starting from rest?

Correct answer: 425.6 m.

Explanation:

Assuming an initial position and velocity of zero,

$$\begin{aligned} \Delta x &= v_0\, t + \frac{1}{2} a\, t^2\\ &= 0 + \frac{1}{2}(235.789\ \text{m/s}^2)(1.9\ \text{s})^2\\ &= 425.6\ \text{m} \end{aligned}$$

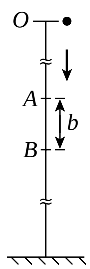
Algorithm

$$\begin{aligned} g &= 9.8\ \text{m/s}^2 & (1)\\ v_0 &= 0.0\ \text{m/s} & (2)\\ v &= 448\ \text{m/s}\ \left\{{}^{350}_{450}\right\} & (3)\\ t &= 1.9\ \text{s}\ \left\{{}^{1.5}_{2}\right\} & (4)\\ a &= \frac{v}{t} & (5)\\ &= \frac{\langle 448\rangle}{\langle 1.9\rangle}\\ &= 235.789\ \text{m/s}^2\\ \langle \text{m/s}^2\rangle &= \frac{\langle \text{m/s}\rangle}{\langle \text{s}\rangle} & \text{units}\\ n &= \frac{a}{g} & (6)\\ &= \frac{\langle 235.789\rangle}{\langle 9.8\rangle}\\ &= 24.0602\\ \langle\rangle &= \frac{\langle \text{m/s}^2\rangle}{\langle \text{m/s}^2\rangle} & \text{units} \end{aligned}$$

$$\begin{aligned}\Delta x &= \frac{a\,t^{2.0}}{2.0} \qquad (7)\\ &= \frac{\langle 235.789\rangle\,\langle 1.9\rangle^{2.0}}{2.0}\\ &= 425.6 \text{ m}\\ \langle \text{m}\rangle &= \frac{\langle \text{m/s}^2\rangle\,\langle \text{s}\rangle^{2.0}}{\langle\rangle} \qquad \text{units}\end{aligned}$$

013

A ball is dropped from rest at point O. After falling for some time, it passes by a window of height $b = 4$ m and it does so during time $t_{AB} = 0.46$ s.



The ball accelerates all the way down ($g = 9.8$ m/s^2); let v_A be its speed as it passes the window's top A and v_B its speed as it passes the window's bottom B.

How much did the ball speed up as it passed the window? Calculate $\Delta v_{\text{down}} = v_B - v_A$.

Correct answer: 4.508 m/s.

Explanation:

Assume down is positive. The ball falls under a constant acceleration g, so

$$g = \frac{\Delta v}{t} = \frac{v_B - v_A}{t}$$

and the change of its velocity during time t_{AB} is simply $\Delta \mathbf{v} = \mathbf{v}_B - \mathbf{v}_A = g t_{AB}$, assuming the downward direction to be positive.

$$\Delta v_{\text{down}} = (9.8 \text{ m/s}^2)(0.46 \text{ s}) = 4.508 \text{ m/s}.$$

014

Calculate the speed v_A at which the ball passes the window's top.

Correct answer: 6.44165 m/s.

Explanation:

Down is positive. Given the uniform downward acceleration g, we have

$$y(t) = y(t_A) + \mathbf{v}_A(t - t_A) + \tfrac{1}{2}g(t - t_A)^2$$

and hence

$$\begin{aligned}b &= y(t_B) - y(t_A)\\ &= \mathbf{v}_A(t_B - t_A) + \frac{1}{2}g(t_B - t_A)^2\\ &= \mathbf{v}_A t_{AB} + \tfrac{1}{2}g t_{AB}^2.\end{aligned}$$

Solving this equation for the velocity $\mathbf{v}_A$, we obtain

$$\begin{aligned}\mathbf{v}_A &= \frac{b}{t_{AB}} - \frac{g t_{AB}}{2}\\ &= \frac{4 \text{ m}}{0.46 \text{ s}} - \frac{(9.8 \text{ m/s}^2)(0.46 \text{ s})}{2} = 6.44165 \text{ m/s}.\end{aligned}$$

Thus the speed $v_A = |\mathbf{v}_A| = 6.44165$ m/s.

Ball Dropped From Rest 2

02:05, trigonometry, numeric, > 1 min.

015

Now consider a new situation: The ball is thrown upward from the ground with an initial velocity that takes exactly the same time $t_{BA} = t_{AB} = 0.46$ s to pass by the window, with the ball moving up rather than down. Consider the ball's slowdown during this time: Let v'_B be the ball's speed (do not confuse the speed with the velocity) as it passes the window's bottom on the way up and let v'_A be its speed as it passes the window's top, also in its way up.

How does the ball's slowdown

$$\Delta v_{\text{up}} = v'_B - v'_A$$

compare to its speedup Δv_{down} on the way down?

1. $\Delta v_{up} > \Delta v_{down}$.

2. $\Delta v_{up} = \Delta v_{down}$. **correct**

3. $\Delta v_{up} < \Delta v_{down}$.

4. $\Delta v_{up} > \Delta v_{down}$ if the mass of the ball is less than 0.1 kg and $\Delta v_{up} < \Delta v_{down}$ if the mass of the ball is greater than 0.1 kg

5. $\Delta v_{up} < \Delta v_{down}$ if the mass of the ball is less than 0.1 kg and $\Delta v_{up} > \Delta v_{down}$ if the mass of the ball is greater than 0.1 kg

Explanation:

$$\Delta v_{up} = v'_A - v'_B$$

The free-fall acceleration always points downward, regardless of whether the velocity is downward or upward, so the velocity change during the same time period is always the same,

$$\mathbf{v}'_A - \mathbf{v}'_B = g t_{BA} = g t_{AB} = \Delta v_{down}.$$

Algorithm

$$\begin{aligned}
g &= 9.8 \text{ m/s}^2 & (1)\\
b &= 4 \text{ m} \left\{ {2 \atop 4} \right\} & (2)\\
t_{AB} &= 0.46 \text{ s} \left\{ {0.2 \atop 0.5} \right\} & (3)\\
dv &= g\ t_{AB} & (4)\\
&= \langle 9.8 \rangle \langle 0.46 \rangle \\
&= 4.508 \text{ m/s} \\
\langle \text{m/s} \rangle &= \langle \text{m/s}^2 \rangle \langle \text{s} \rangle & \text{units}\\
v_A &= \frac{b - 0.5\ g\ t_{AB}^{2.0}}{t_{AB}} & (5)\\
&= \frac{\langle 4 \rangle - 0.5 \langle 9.8 \rangle \langle 0.46 \rangle^{2.0}}{\langle 0.46 \rangle}\\
&= 6.44165 \text{ m/s}\\
\langle \text{m/s} \rangle &= \frac{\langle \text{m} \rangle - \langle \rangle \langle \text{m/s}^2 \rangle \langle \text{s} \rangle^{2.0}}{\langle \text{s} \rangle} & \text{units}
\end{aligned}$$

Stone Thrown Up

02:05, arithmetic, numeric, > 1 min.

017

What is the maximum distance from the ground below (not from the cliff) during its flight?

Correct answer: 29.8796 m.

Explanation:

When the stone is at its maximum height h, its velocity is zero:

$$v^2 = 2gh \Rightarrow h = \frac{v^2}{2g}$$

so

$$h = \frac{(24.2 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 29.8796 \text{ m}$$

Algorithm

$$\begin{aligned}
x_0 &= 17.9 \text{ m} \left\{ {10 \atop 18} \right\} & (1)\\
v &= 24.2 \text{ m/s} \left\{ {18.8 \atop 25} \right\} & (2)\\
g &= 9.8 \text{ m/s}^2 & (3)\\
v_0 &= \sqrt{v^{2.0} - 2.0\ g\ x_0} & (4)\\
&= \sqrt{\langle 24.2 \rangle^{2.0} - 2.0 \langle 9.8 \rangle \langle 17.9 \rangle}\\
&= 15.3232 \text{ m/s}\\
\langle \text{m/s} \rangle &= \sqrt{\langle \text{m/s} \rangle^{2.0} - \langle \rangle \langle \text{m/s}^2 \rangle \langle \text{m} \rangle} & \text{units}\\
h &= \frac{v^{2.0}}{2.0\ g} & (5)\\
&= \frac{\langle 24.2 \rangle^{2.0}}{2.0 \langle 9.8 \rangle}\\
&= 29.8796 \text{ m}\\
\langle \text{m} \rangle &= \frac{\langle \text{m/s} \rangle^{2.0}}{\langle \rangle \langle \text{m/s}^2 \rangle} & \text{units}
\end{aligned}$$