PH641, FINAL, June 9, 2009

Note: always indicate what you try to do, even if you are not able to finish the actual work.

Problem 1: An electron has three possible orbitals, given by a quantum number m that can be $-1, 0, +1$. The energy of the orbital is $\epsilon_m = -mB$. Give an expression that allows you to find $\mu(T, V, N)$ for this system. If the average number of particles in state $m = 0$ is n_0, find μ as a function of n_0. Find n_1 and n_{-1} as a function of B and n_0. Write an equation that allows you to find n_0 as a function of B and N.

Problem 2: Consider a wire of length L. Treat this as a one dimensional system. The frequency of the lattice vibrations (phonons) in this wire is given by $\omega = v_s|k|$, where v_s is the speed of sound in the wire. The wave vector k is quantized according to $k = n\frac{\pi}{L}$, with $n = 0, \pm 1, \pm 2, \cdots$. Find the one-phonon partition function for this system. Use the formulas $\mathcal{Z}(T, L, N) = \frac{1}{N!}\mathcal{Z}_1^N(T, L)$ and $\log(N!) \approx N\log(N) - N$ to find the Helmholtz free energy for the system. Calculate the stress τ, which is the equivalent of pressure for this one-dimensional system.

Problem 3: The multiplicity function for the states of a single particle as a function of energy is given by $g_1(\epsilon)$. Consider a system of two particles with combined energy E. The multiplicity function for the combined system is $g_2(E)$. Find an expression for g_2 in terms of g_1. Generalize this result for N particles. Construct an argument why we have $S(E, N) \approx Nk_B \log g_1(\frac{E}{N})$.