**EXAMPLE 36.1 Finding resistor voltages**

In the circuit of FIGURE 36.6, what are (a) the peak voltage across each resistor and (b) the instantaneous resistor voltages at $t = 20$ ms?

**VISUALIZE** Figure 36.6 shows the circuit diagram. The two resistors are in series.

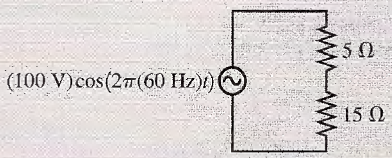
**SOLVE** a. The equivalent resistance of the two series resistors is $R_{eq} = 5\ \Omega + 15\ \Omega = 20\ \Omega$. The instantaneous current through the equivalent resistance is

$$i_R = \frac{v_R}{R_{eq}} = \frac{\mathcal{E}_0 \cos\omega t}{R_{eq}} = \frac{(100\text{ V})\cos(2\pi(60\text{ Hz})t)}{20\ \Omega}$$
$$= (5.0\text{ A})\cos(2\pi(60\text{ Hz})t)$$

The peak current is $I_R = 5.0$ A, and this is also the peak current through the two resistors that form the 20 Ω equivalent resistance. Hence the peak voltage across each resistor is

$$V_R = I_R R = \begin{cases} 25\text{ V} & 5\ \Omega\text{ resistor} \\ 75\text{ V} & 15\ \Omega\text{ resistor} \end{cases}$$

**FIGURE 36.6** An AC resistor circuit.




b. The instantaneous current at $t = 0.020$ s is

$$i_R = (5.0\text{ A})\cos\big(2\pi(60\text{ Hz})(0.020\text{ s})\big) = 1.55\text{ A}$$

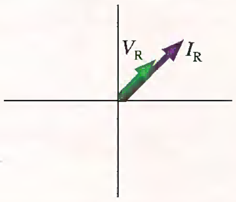
The resistor voltages at this time are

$$v_R = i_R R = \begin{cases} 7.7\text{ V} & 5\ \Omega\text{ resistor} \\ 23.2\text{ V} & 15\ \Omega\text{ resistor} \end{cases}$$

**ASSESS** The sum of the instantaneous voltages, 30.9 V, is what you would find by calculating $\mathcal{E}$ at $t = 20$ ms. This self-consistency gives us confidence in the answer.

**STOP TO THINK 36.2** The resistor whose voltage and current phasors are shown here has resistance $R$

a. $>1\ \Omega$
b. $<1\ \Omega$
c. It's not possible to tell.




## 36.2 Capacitor Circuits

FIGURE 36.7a shows current $i_C$ charging a capacitor with capacitance $C$. The instantaneous capacitor voltage is $v_C = q/C$, where $\pm q$ is the charge on the two capacitor plates at this instant of time. It is useful to compare Figure 36.7a to Figure 36.3 for a resistor.

FIGURE 36.7b, where capacitance $C$ is connected across an AC source of emf $\mathcal{E}$, is the most basic capacitor circuit. The capacitor is in parallel with the source, so the capacitor voltage equals the emf: $v_C = \mathcal{E} = \mathcal{E}_0 \cos\omega t$. It will be useful to write

$$v_C = V_C \cos\omega t \tag{36.6}$$

where $V_C$ is the peak or maximum voltage across the capacitor. You can see that $V_C = \mathcal{E}_0$ in this single-capacitor circuit.

To find the current to and from the capacitor, we first write the charge

$$q = Cv_C = CV_C \cos\omega t \tag{36.7}$$

The current is the *rate* at which charge flows through the wires, $i_C = dq/dt$, thus

$$i_C = \frac{dq}{dt} = \frac{d}{dt}(CV_C \cos\omega t) = -\omega C V_C \sin\omega t \tag{36.8}$$

**FIGURE 36.7** An AC capacitor circuit.

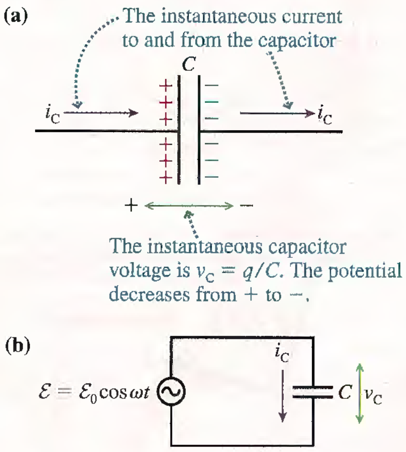
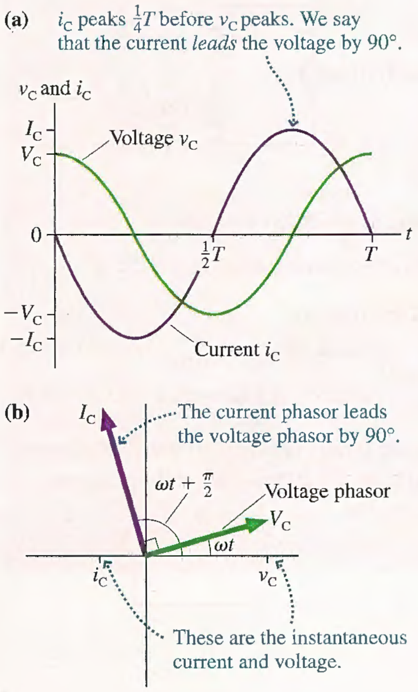


**FIGURE 36.8** Graph and phasor diagrams of the capacitor current and voltage.




We can most easily see the relationship between the capacitor voltage and current if we use the trigonometric identity $-\sin(x) = \cos(x + \pi/2)$ to write

$$i_C = \omega C V_C \cos\left(\omega t + \frac{\pi}{2}\right) \tag{36.9}$$

In contrast to a resistor, a capacitor's current and voltage are *not* in phase. In **FIGURE 36.8a**, a graph of the instantaneous voltage $v_C$ and current $i_C$, you can see that the current peaks one-quarter of a period *before* the voltage peaks. The phase angle of the current phasor on the phasor diagram of **FIGURE 36.8b** is $\pi/2$ rad—a quarter of a circle—larger than the phase angle of the voltage phasor.

We can summarize this finding:

**The AC current to and from a capacitor *leads* the capacitor voltage by $\pi/2$ rad, or 90°.**

The current reaches its peak value $I_C$ at the instant the capacitor is fully discharged and $v_C = 0$. The current is zero at the instant the capacitor is fully charged. You saw a similar behavior in the oscillation of an $LC$ circuit in Chapter 34.

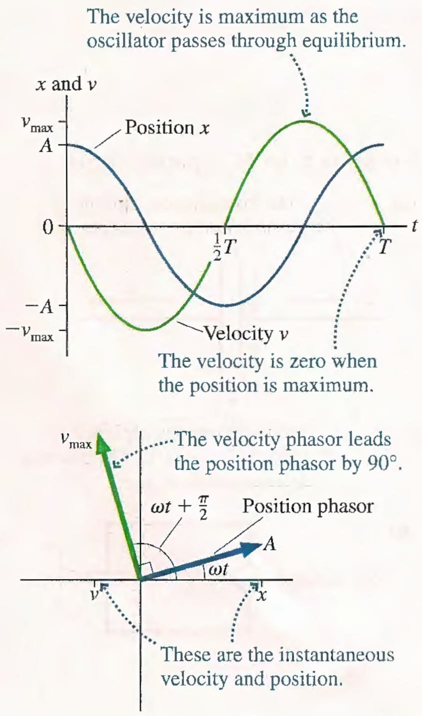
A simple harmonic oscillator provides a mechanical analogy of the 90° phase difference between current and voltage. You learned in Chapter 14 (refer to Section 14.1 and Figure 14.5) that the position and velocity of a simple harmonic oscillator are

$$x = A\cos\omega t$$

$$v = \frac{dx}{dt} = -\omega A \sin\omega t = -v_{max}\sin\omega t = v_{max}\cos\left(\omega t + \frac{\pi}{2}\right)$$

You can see in **FIGURE 36.9** that the velocity leads the position by 90° in the same way that the capacitor current (which is proportional to the charge velocity) leads the voltage.

**FIGURE 36.9** In a mechanical analogy, the velocity of a simple harmonic oscillator leads the position by 90°.




## Capacitive Reactance

We can use Equation 36.9 to see that the peak current to and from a capacitor is $I_C = \omega C V_C$. This relationship between the peak voltage and peak current looks much like Ohm's law for a resistor if we define the **capacitive reactance** $X_C$ to be

$$X_C \equiv \frac{1}{\omega C} \tag{36.10}$$

With this definition,

$$I_C = \frac{V_C}{X_C} \quad \text{or} \quad V_C = I_C X_C \tag{36.11}$$

The units of reactance, like those of resistance, are ohms.

**NOTE ►** Reactance relates the *peak* voltage $V_C$ and current $I_C$. But reactance differs from resistance in that it does *not* relate the instantaneous capacitor voltage and current because they are out of phase. That is, $v_C \neq i_C X_C$. ◄

A resistor's resistance $R$ is independent of the emf frequency. In contrast, as **FIGURE 36.10** shows, a capacitor's reactance $X_C$ depends inversely on the frequency. The reactance becomes very large at low frequencies (i.e., the capacitor is a large impediment to current). This makes sense because $\omega = 0$ would be a nonoscillating DC cir-

**EXAMPLE 36.4 Designing a filter**

For a science project, you've built a radio to listen to AM radio broadcasts at frequencies near 1 MHz. The basic circuit is an antenna, which produces a very small oscillating voltage when it absorbs the energy of an electromagnetic wave, and an amplifier. Unfortunately, your neighbor's short-wave radio broadcast at 10 MHz interferes with your reception. Having just finished physics, you decide to solve this problem by placing a filter between the antenna and the amplifier. You happen to have a 500 pF capacitor. What frequency should you select as the filter's crossover frequency? What value of resistance will you need to build this filter?

**MODEL** You need a low-pass filter to block signals at 10 MHz while passing the lower-frequency AM signal at 1 MHz.

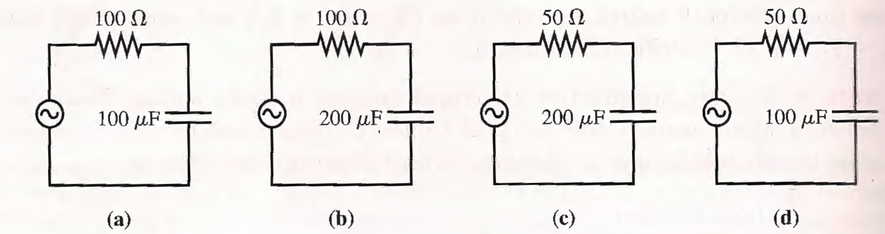
**VISUALIZE** The circuit will look like the low-pass filter in Figure 36.13a. The oscillating voltage generated by the antenna will be the emf, and $v_{out}$ will be sent to the amplifier.

**SOLVE** You might think that a crossover frequency near 5 MHz, about halfway between 1 MHz and 10 MHz, would work best. But 5 MHz is a factor of 5 higher than 1 MHz while only a factor of 2 less than 10 MHz. A crossover frequency the same factor above 1 MHz as it is below 10 MHz will give the best results. In practice, choosing $f_c = 3$ MHz would be sufficient. You can then use Equation 36.15 to select the proper resistor value:

$$R = \frac{1}{\omega_c C} = \frac{1}{2\pi(3 \times 10^6\ \text{Hz})(500 \times 10^{-12}\ \text{F})}$$
$$= 106\ \Omega \approx 100\ \Omega$$

**ASSESS** Rounding to 100 Ω is appropriate because the crossover frequency was determined to only one significant figure. Such "sloppy design" is adequate when the two frequencies you need to distinguish are well separated.

**STOP TO THINK 36.4** Rank in order, from largest to smallest, the crossover frequencies $(\omega_c)_a$ to $(\omega_c)_d$ of these four circuits.




## 36.4 Inductor Circuits

**FIGURE 36.14** Using an inductor in an AC circuit.

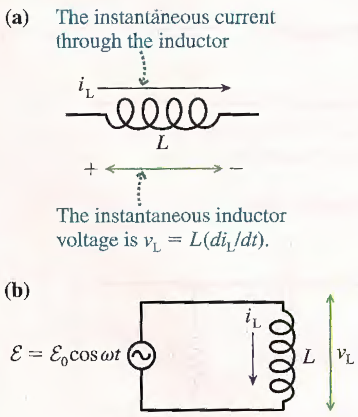



**FIGURE 36.14a** shows the instantaneous current $i_L$ through an inductor. If the current is changing, the instantaneous inductor voltage is

$$v_L = L\frac{di_L}{dt} \tag{36.16}$$

You learned in Chapter 34 that the potential decreases in the direction of the current if the current is increasing ($di_L/dt > 0$) and increases if the current is decreasing ($di_L/dt < 0$).

**FIGURE 36.14b** is the simplest inductor circuit. The inductor $L$ is connected across the AC source, so the inductor voltage equals the emf: $v_L = \mathcal{E} = \mathcal{E}_0 \cos\omega t$. We can write

$$v_L = V_L \cos\omega t \tag{36.17}$$

where $V_L$ is the peak or maximum voltage across the inductor. You can see that $V_L = \mathcal{E}_0$ in this single-inductor circuit.

We can find the inductor current $i_L$ by integrating Equation 36.17. First, use Equation 36.17 to write Equation 36.16 as

$$di_L = \frac{v_L}{L}dt = \frac{V_L}{L}\cos\omega t\, dt \tag{36.18}$$

Integrating gives

$$i_L = \frac{V_L}{L}\int \cos\omega t\,dt = \frac{V_L}{\omega L}\sin\omega t = \frac{V_L}{\omega L}\cos\left(\omega t - \frac{\pi}{2}\right)$$
$$= I_L\cos\left(\omega t - \frac{\pi}{2}\right) \qquad (36.19)$$

where $I_L = V_L/\omega L$ is the peak or maximum inductor current.

**NOTE ▶** Mathematically, Equation 36.19 could have an integration constant $i_0$. An integration constant would represent a constant DC current through the inductor, but there is no DC source of potential in an AC circuit. Hence, on physical grounds, we set $i_0 = 0$ for an AC circuit. ◀

We define the **inductive reactance,** analogous to the capacitive reactance, to be

$$X_L \equiv \omega L \qquad (36.20)$$

Then the peak current $I_L = V_L/\omega L$ and the peak voltage are related by

$$I_L = \frac{V_L}{X_L} \quad \text{or} \quad V_L = I_L X_L \qquad (36.21)$$

**FIGURE 36.15** shows that the inductive reactance increases as the frequency increases. This makes sense. Faraday's law tells us that the induced voltage across a coil increases as the time rate of change of $\vec{B}$ increases, and $\vec{B}$ is directly proportional to the inductor current. For a given peak current $I_L$, $\vec{B}$ changes more rapidly at higher frequencies than at lower frequencies, and thus $V_L$ is larger at higher frequencies than at lower frequencies.

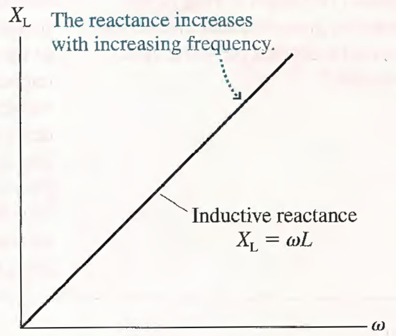
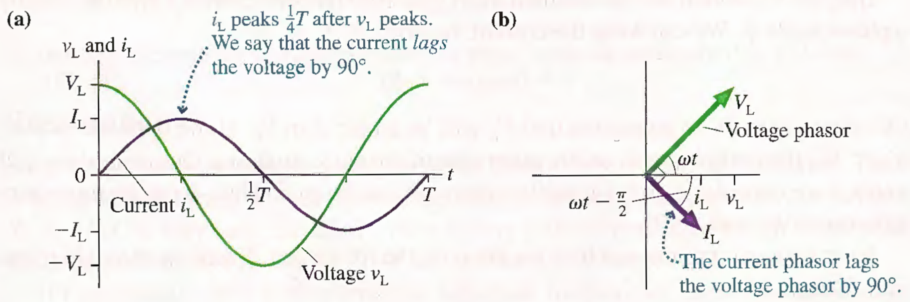



**FIGURE 36.15** The inductive reactance as a function of frequency.

**FIGURE 36.16a** is a graph of the inductor voltage and current. You can see that the current peaks one-quarter of a period *after* the voltage peaks. The angle of the current phasor on the phasor diagram of **FIGURE 36.16b** is $\pi/2$ rad less than the angle of the voltage phasor. We can summarize this finding:

**The AC current through an inductor *lags* the inductor voltage by $\pi/2$ rad, or 90°.**

**FIGURE 36.16** Graphs and phasor diagrams of the inductor current and voltage.




**EXAMPLE 36.5 Current and voltage of an inductor**

A 25 $\mu$H inductor is used in a circuit that oscillates at 100 kHz. The current through the inductor reaches a peak value of 20 mA at $t = 5.0\ \mu$s. What is the peak inductor voltage, and when, closest to $t = 5.0\ \mu$s, does it occur?

**MODEL** The inductor current lags the voltage by 90°, or, equivalently, the voltage reaches its peak value one-quarter period *before* the current.

**VISUALIZE** The circuit looks like Figure 36.14b.

**SOLVE** The inductive reactance at $f = 100$ kHz is

$$X_L = \omega L = 2\pi(1.0 \times 10^5\ \text{Hz})(25 \times 10^{-6}\ \text{H}) = 16\ \Omega$$

Thus the peak voltage is $V_L = I_L X_L = (20\ \text{mA})(16\ \Omega) = 320$ mV. The voltage peak occurs one-quarter period before the current peaks, and we know that the current peaks at $t = 5.0\ \mu$s. The period of a 100 kHz oscillation is 10.0 $\mu$s, so the voltage peaks at

$$t = 5.0\ \mu\text{s} - \frac{10.0\ \mu\text{s}}{4} = 2.5\ \mu\text{s}$$