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How energy efficient is your car?

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A detailed energy balance indicating how fuel energy is transferred from the engine to the wheels of a commercial car is obtained using non-specialized experiments that can be readily understood using elementary mechanics. These experiments allow us to determine the engine's thermal efficiency, its mechanical losses, and the rolling (friction) and aerodynamic (drag) coefficients. We find that approximately 28% of the fuel energy is transferred to the wheels.

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I. INTRODUCTION

The car is the main form of transportation used by people in developed countries. It offers a high degree of freedom that will probably never be surpassed by public transportation vehicles such as the train or bus. Unfortunately, current car engine technology is based on the internal combustion of petroleum-derived fuels, so the use of such engines results in high pollution levels in populated areas and contributes significantly to global warming. Thus, improvements in efficiency would reduce its negative impact on the environment.

We recently came across a detailed energy balance of a conventional gasoline vehicle in the USA (Ref. 1) and were surprised by the fact that only 12%–13% of the fuel energy is transferred to the wheels. Since many cars measure fuel consumption, we proposed to use this information to determine experimentally the energy balance for a particular car. We designed simple experiments based on basic physics and found results that were quite satisfactory. In view of this relative success, we believe our study will be useful for students at the undergraduate level interested in applied mechanics. Although the experiments are based on elementary physics, the analysis of the results requires knowledge of concepts that are developed in introductory courses on mechanics (air drag and rolling resistance) and thermodynamics (thermal cycles).

The paper is organized as follows: after a brief introduction to the particular car studied, we describe the experiment to measure the thermal efficiency of the engine. Next, we explain how air drag and rolling resistance are measured. We then compile the information from the experiments and draw up the car's energy balance for a typical trip. Lastly, some general conclusions are given.

II. THE CAR

All experiments are performed with a Volkswagen Lupo 3L, a car that is designed to be extremely efficient. It is powered by a three-cylinder 45-kW diesel engine and weighs 830 kg. At present, it probably still holds the world fuel-economy record for a four-seat car at 3.0 l/100 km or 78 miles per gallon.

Before performing any experiments, we calibrate the car's fuel consumption display by comparing the displayed average consumption (in l/100 km) with the actual consumption over a distance of 850 km. We find that the display underestimates the actual consumption by a factor of 0.93. All values reported below have been corrected accordingly.

III. EXPERIMENT 1: ENGINE THERMAL EFFICIENCY

Engine thermal efficiency is obtained by comparing the fuel consumption of the car between the upward and downward directions of several road sections while traveling in a fixed gear at a constant speed. The work done by the engine in the upward direction is

$$W_u = E_e + E_R + E_D + mgh, \tag{1}$$

where E_e is the mechanical losses of the engine itself (friction of the moving parts, fuel pumping, etc.) plus losses due to friction of the parts connecting the engine to the gear box, E_R includes the losses due to tire deformation (rolling resistance) plus the friction of the parts connecting the wheels to the gear box, E_D are the losses due to air resistance, and mgh is the change in potential energy of the car plus passenger. When calculating the work done in the downward direction W_d, the drag loss takes exactly the same value as in the upward direction because the car speed is identical. Because all parts move at the same speed, it is also reasonable to assume that $E_e + E_R$ will remain approximately constant, although slight differences might arise because the internal forces and torques are higher in the upward direction. Finally, the change in potential energy will reverse its sign. Consequently, the predicted difference of fuel consumption (the fuel consumed by the engine per unit distance) between the upward (c_u) and downward (c_d) directions is

$$\Delta c = c_u - c_d = \frac{1}{\eta}\frac{W_u - W_d}{Q_F d} = \frac{1}{\eta}\frac{2mgh}{Q_F d}, \tag{2}$$

where d is the length of the road section, Q_F the fuel energy density (3.56×10^7 J/l for diesel),[2,3] and η the thermal efficiency of the engine.

To determine η, we choose four sections of a road and measure the height along the road using an altimeter with a precision of ± 1 m. The mean slope varied from 5 m/km for the flattest section to 45 m/km for the steepest. We chose three velocities in several gears for the experiments and the values of c_i measured for several representative experiments are shown in Fig. 1, along with the four road sections. As expected from Eq. (2), Δc increases with road slope, but this equation systematically delivers higher values of η for the two largest slopes (unrealistic values of $\eta \approx 0.6$ were obtained). This apparent slope-dependence is fictitious, however, and related to the fact that the average experimental fuel consumption

$$\bar{c} \equiv \frac{c_u + c_d}{2} = \frac{1}{\eta}\frac{E_e + E_R + E_D}{Q_F d}, \tag{3}$$

increases with slope (see Fig. 1). We do not believe this slope dependence is related to any dependence of E_e or E_R on the upward or downward directions, but instead is a result of the highest downward slopes causing the experiment to deviate from the programmed conditions. We find that when the downward local slope is too high, the instantaneous fuel consumption falls to zero and the car speed is actually higher than the programmed speed (braking was never applied). Thus, the force of gravity is doing work on the engine and the fuel consumption should in fact be negative. In other words, c_d—and therefore $\bar{c}$—are overestimated for the highest slopes. Despite this experimental limitation, we can still make use of the experiments carried out on the steepest sections. First, we notice that the $\bar{c}$ values for the two road sections with the lowest slopes [Fig. 2(a)] are not affected by this problem. Therefore, because $\bar{c}$ should be independent of slope, we write $c_d = 2\bar{c} - c_u$ where we use the lowest-slope values of $\bar{c}$. Using this procedure to determine η via Eq. (2) does not exhibit any systematic dependence on the section slope.

The values of η obtained for each velocity/gear pair are averaged over the four road sections, and the results are collected in Fig. 2(b); the error bars indicate the maximum and minimum values of η. In view of this data, we conclude that the engine thermal efficiency is approximately 40% and that, within experimental uncertainties, we observe no dependence on velocity or gear except possibly for the lowest value at 3700 rpm (65 km/h, second gear).

Diesel engines follow a cycle of compression and expansion close to the ideal diesel cycle. Efficiencies above 40% are only expected in heavy-duty engines for trucks or electricity generation[3] because higher pressures are used in

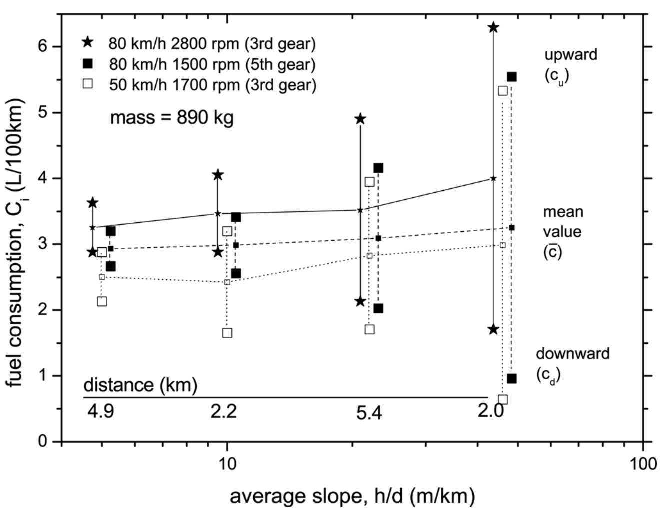

Fig. 1. Fuel consumption for upward and downward directions on four road sections with different mean slope. Only the results of several representative experiments are shown.

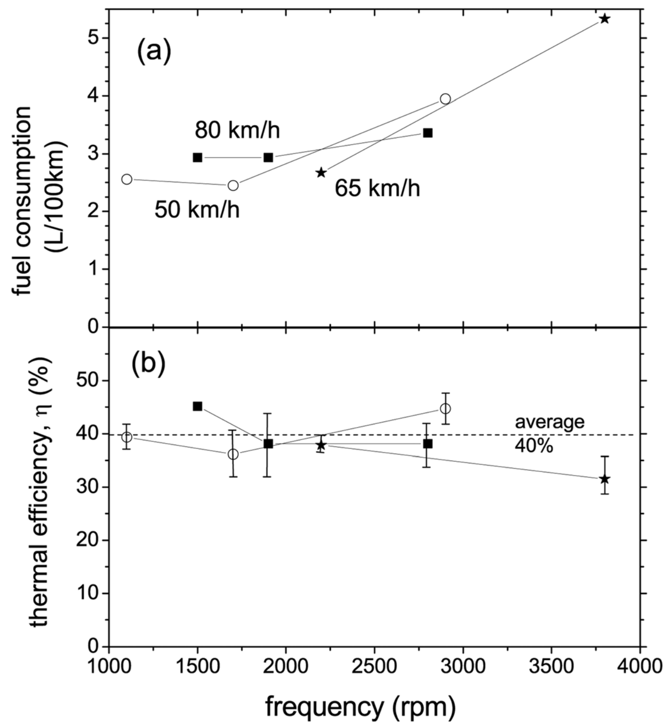

Fig. 2. (a) Average fuel consumption and (b) engine thermal efficiency as a function of frequency for several car velocities obtained from data like those in Fig. 1.

larger engines. Thus, our measured efficiency seems quite high for a light-duty engine. Unfortunately, we were unable to find any published data on engine efficiencies of commercial cars for comparison. Moreover, despite the fact that shorter cycles correspond to a higher deviation from thermodynamic reversibility, thermal efficiencies of combustion engines are known to remain almost constant within a large range of frequencies.[3] This fact suggests that the decrease in efficiency at 3700 rpm is due to higher deviations from the ideal diesel cycle rather than in terms of reversibility. In any event, the relative constancy of η should not lead the reader to the erroneous conclusion that the engine's overall efficiency is independent of velocity and gear. The engine's mechanical losses (E_e) must also to be taken into account to evaluate the engine's overall efficiency (see Sec. V).

IV. EXPERIMENT 2: AIR DRAG AND ROLLING RESISTANCE

If the car is in motion and the transmission is put in neutral, it will eventually come to rest due to air friction and rolling resistance. Rolling resistance usually refers to the torque that arises from the tire deformation that opposes the rolling motion.[4,5] To quantify this rolling force F_R, a rolling coefficient α_R is usually defined from the equation

$$F_R = \alpha_R N, \tag{4}$$

where N is the force acting on the wheels normal to the road surface ($N = mg$ for a horizontal road). In this experiment, the wheels are still connected to the gearbox so friction acting on these elements is included in the rolling resistance term. We expect no significant dependence of the rolling resistance on the car speed v.[6] In contrast, the drag force caused by the air is proportional to v^2 and takes the form[7]

$$F_D = \alpha_D \frac{1}{2} \rho A v^2, \quad (5)$$

where ρ is the air density (1.22 kg/m^3 at 500 m above sea level), A is the cross-sectional area of the car, and α_D is the so-called drag coefficient that depends on the vehicle shape.

Experiment 2 allows us to measure the drag (E_D) and rolling (E_R) loss terms from Eq. (1). To this aim, we choose a straight road section that is as flat as possible. Once the car achieves a set speed v_0, the transmission is shifted into neutral and we measure the time Δt needed to achieve a given speed reduction Δv. This speed reduction corresponds to a loss of kinetic energy given by

$$\Delta E_c = m\Delta v\left(v_0 - \frac{1}{2}\Delta v\right) = m\Delta v \cdot \bar{v}. \quad (6)$$

If the force opposing the motion is constant, then the car will undergo a uniform deceleration. We can expect such a motion if the velocity changes are small ($\Delta v \ll v_0$) or if the drag force is negligible compared with the rolling force ($F_R \gg F_D$); one of these conditions is always (approximately) met in our experiments. In such cases, the displacement after Δt is

$$\Delta x = \bar{v}\Delta t = \left(v_0 - \frac{1}{2}\Delta v\right)\Delta t. \quad (7)$$

Combining Eqs. (6) and (7) results in the energy lost per unit distance due to the rolling resistance and air drag

$$\frac{\Delta(E_R + E_D)}{\Delta x} = m\frac{\Delta v}{\Delta t}. \quad (8)$$

Because the loss of energy per unit distance has the dimensions of force, we rewrite Eq. (8) as

$$F_R + F_D \pm F_g = m\frac{\Delta v}{\Delta t_\pm}, \quad (9)$$

where we have added the gravitational force F_g that appears because the road section is not perfectly flat (+ and – signs apply for the upward and downward directions, respectively). The gravitational term disappears when the equations for the upward and downward directions are added, leaving us with the force that would act on an ideal flat surface

$$F \equiv F_R + F_D = \frac{m\Delta v}{2}\left(\frac{1}{\Delta t_+} + \frac{1}{\Delta t_-}\right). \quad (10)$$

Experiment 2 is performed on a straight road section with a slope of around 10 m/km and generalizes a similar experiment reported in Ref. 8 on a level surface. Figure 3 shows the times needed to reduce the car speed by 10 km/h for various starting speeds. The lack of a horizontal surface is reflected in the difference between Δt_+ and Δt_-.

Applying Eq. (10) to the data in Fig. 3 yields the net rolling plus drag force, which is plotted as a function of car speed in Fig. 4. Because air drag is proportional to v^2, the experimental points can be fit with a parabolic function

$$F = A + Bv^2. \quad (11)$$

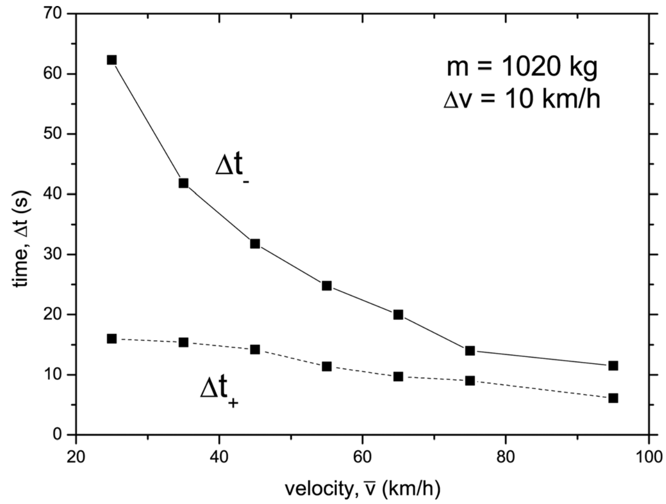

Fig. 3. Time needed to reduce the car velocity by 10 km/h on an almost flat road section with transmission in neutral (below 20 km/h—not shown—the velocity reduction was only 5 km/h). The ± sign refers to the downward/upward direction.

Our fit parameter of $A = 110$ N represents the (constant) rolling resistance F_R, and Eq. (4) then determines the rolling coefficient to be $\alpha_R = 0.011$. This value falls in the lower range of values for ordinary tires (0.010–0.015).[4,5] The other fit parameter of $B = 0.33$ Ns^2/m^2, in conjunction with Eq. (5), serves to deduce the drag coefficient $\alpha_D = 0.35$. This value falls in the upper range of most cars ($0.3 < \alpha_D < 0.35$).[9] It is worth mentioning that because the wheels are still connected to the gear box in this experiment, the values obtained here should be viewed as an upper bound to the actual values.

V. BRAKE EFFICIENCY AND ENERGY BALANCE

Our experimental results allow us to understand the fuel consumption measurements shown in Fig. 2(a). In particular, we can understand why at less than 2000 rpm the consumption at 80 km/h is higher than that at 50 km/h, whereas at 2700 rpm, this order is reversed.

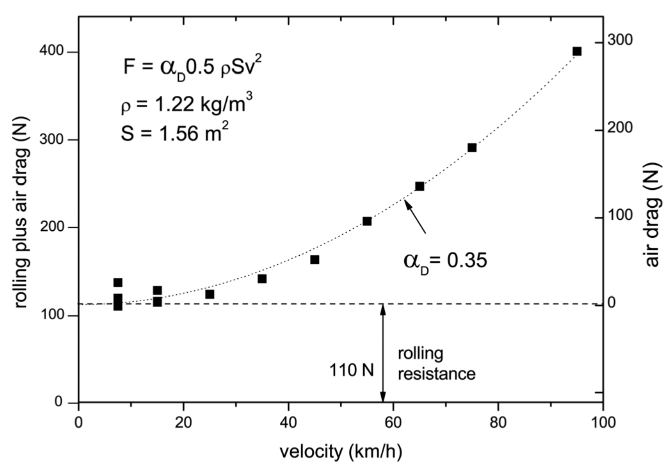

Fig. 4. Fuel consumption to oppose the air drag and rolling forces obtained from the data in Fig. 3. From the fitting parabola, the aerodynamic (α_D) and rolling (α_R) coefficients are obtained.

In Fig. 5, we present a detailed balance of the fuel consumed at 50 km/h and 80 km/h for several gears. The points correspond to the average fuel consumption $\bar{c}$ calculated using Eq. (3) [see Fig. 2(a)]. The thick line corresponds to the mechanical work done by the engine, calculated as $\bar{c}\eta$, where the thermal efficiency η is determined in experiment 1 [see Fig. 2(b)]. At 50 km/h [see Fig. 5(a)], approximately 60% of the fuel is lost as heat inside the engine, and only 40% of the fuel energy has done mechanical work (thick line). For a flat section of road, this work is decomposed into three components: E_e, E_R, and E_D of Eq. (1). The air drag and rolling resistance forces have been measured and converted into fuel consumption by dividing by the fuel energy density

$$c_{D/R} = F_{D/R}/Q_F. \tag{12}$$

Once the rolling and air drag components of this work have been calculated with this equation, their difference from the total work must be interpreted as the engine friction losses E_e. For a given velocity, we see that these friction losses are larger for higher frequency and this behavior can be qualitatively understood as follows. For a fixed distance, the engine completes more cycles at a higher frequency, and consequently the work done by friction (per unit distance) will be higher.

To test the self-consistency of our analysis, we measured E_e directly by monitoring the instantaneous fuel consumption of the car at 50 km/h in neutral at several frequencies. After correcting for the thermal efficiency ($\eta = 0.40$), the values are plotted as vertical bars in Fig. 5. The agreement with the expected values is reasonably good and it confirms the dependence of the engine friction losses on frequency.

A look at the detailed balance at 80 km/h [see Fig. 5(b)] shows that there is an overall increase in fuel consumption due to air drag (the rolling resistance remains unchanged). This is why, generally speaking, fuel consumption is higher at 80 km/h than at 50 km/h. However, notice that this order reverses above about 2000 rpm [see Fig. 2(a)]. This surprising behavior is due to the engine friction losses. Although friction losses increase with frequency, they increase more slowly at 80 km/h compared with 50 km/h because of the lower number of engine cycles per unit distance at 80 km/h. At some point, the friction losses at 50 km/h overtake the friction and the increased drag losses at 80 km/h.

Figure 5 can be used to quantify the overall engine's efficiency—the so called "brake" efficiency—defined as the useful work done by the engine divided by the fuel energy

$$\eta_B \equiv \frac{W_{\text{use}}}{Q_F c}, \tag{13}$$

where W_{use} is the useful work per unit distance and c is the fuel consumed along a given road section and direction. At constant velocity on a flat surface, the useful work is just $E_R + E_D$, so η_B can be calculated using the data in Fig. 5. Results for the average consumption over the four road sections at 50 and 80 km/h are shown in Fig. 6. In contrast with the thermal efficiency, the brake efficiency is not constant but improves for higher gear. This result agrees with the well-known recommendation of driving in the highest gear compatible with a given speed in order to save fuel. It is also interesting to note that η_B increases when the work done by the engine is higher. This point is readily illustrated by the fact that η_B reaches a much higher value when the car travels up the steepest section of road (see Fig. 6). In this case, to evaluate η_B the consumption related to the gravitational energy has been added to the rolling and drag contributions, and the total engine's consumption—c in Eq. (13)—was 5.3 l/100 km (empty square in Fig. 1).

Lastly, we analyze the average consumption on a long road section that included the four sections studied above.

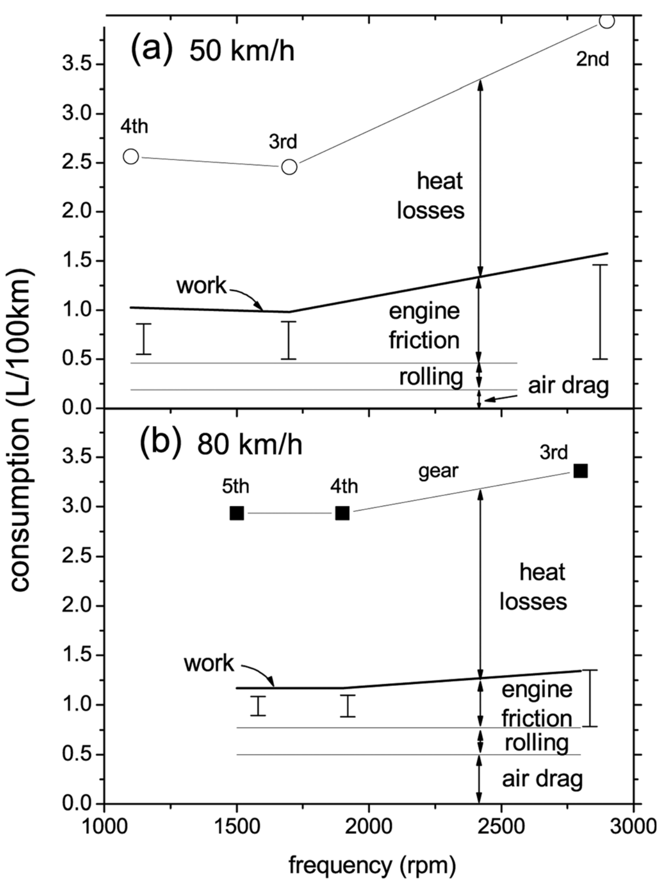

Fig. 5. Energy balance as a function of the frequency for two selected velocities deduced from experiments 1 and 2. The vertical bars indicate the engine friction losses obtained by an independent experiment as described in the text.

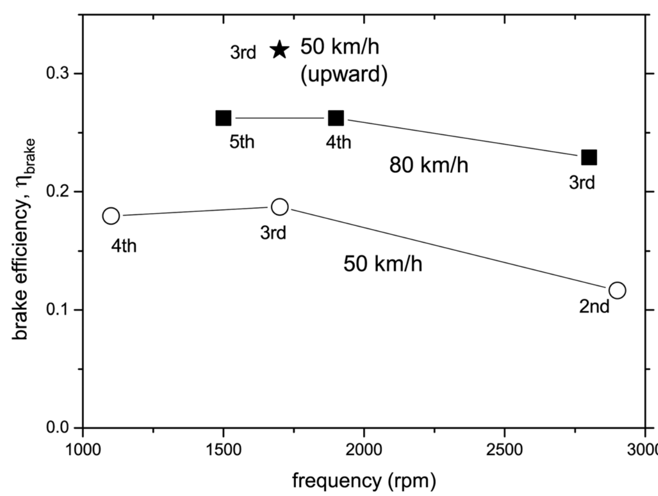

Fig. 6. Brake efficiencies deduced from the values of Fig. 5 (averaged over the four road sections) and Fig. 1 (star: steepest section, upward direction).

The section length and height are 38 km and 373 m, respectively. Several villages are encountered where the car must reduce its velocity or even stop. The consumption averaged over several round trips was 3.2 l/100 km. To determine the contributions of E_e and E_D, the average velocity and frequency must be estimated. For this particular problem, the velocity must be averaged over distance and instead of time, giving

$$\bar{v} \equiv \frac{\int v\, dx}{\int dx}. \tag{14}$$

Because the overall distance travelled inside the villages is very short (less than 4 km) compared to the overall trip, $\bar{v}$ is mainly determined by the inter-village sections. We estimate that $\bar{v} = 70$ km/h and that the average frequency is 1300 rpm. Given these conditions, all components of the fuel consumption defined so far can be determined and summed to give

$$\frac{1}{\eta}(E_e + E_R + E_D) = 2.65\,\mathrm{l}/100\,\mathrm{km}. \tag{15}$$

This value is lower than the measured fuel consumption (3.2 l/100 km) because, in contrast to the experiments done at constant velocity, extra work must to be done to restore the energy lost by braking E_B. In other words, E_B must be the fuel consumption difference (0.55 l/100 km) times the thermal efficiency ($\eta = 0.4$), i.e., 0.22 l/100 km.

To summarize, the energy balance of this representative trip has been plotted in Fig. 7. For every 100 l of fuel consumed, only 40 l are converted into work. A fraction of this work is lost as friction in the engine itself, so that 28 l are transferred to the wheels. This value is much higher than the 12–13 l reported in Sec. I for a conventional gasoline car.

VI. DISCUSSION: TECHNOLOGICAL INNOVATION AND CAR SIZE

Here, we want to analyze why the fuel economy is so high for the car in this study and to show that, in addition to technological innovation, car size and engine power play a crucial role.

Innovation strategies can reduce the work needed to keep the car moving in the following ways:[1] (a) the air drag can be reduced by optimal aerodynamic design, (b) the rolling resistance can be minimized with adequate tires of lower mechanical hysteresis, and (c) the energy to accelerate the car (braking losses) can be lowered by using low-density metal alloys or fiber-reinforced plastic composites in several components. This work is done by the car engine, whose energy efficiency depends on the heat and mechanical losses. In our car, we have been able to verify that the heat losses are low (high thermal efficiency η), which probably constitutes one of the clues for its high fuel efficiency.

The size of the car and its engine power are also important. It has been shown that, on average, the fuel consumption of commercial cars increases by 32% when the car's mass increases by 50%, keeping its power per unit mass constant.[10] This dependence is not at all surprising in view of the proportionality of the rolling resistance and braking losses on mass, and that of the air drag force on the vehicle cross-section. On the other hand, for a given mass, the fuel consumption increases by around 17% for an engine power increase of 50%.[10] Although we do not know exactly the origin of this relationship, we believe it is mainly due to the engine's mechanical losses.[11] Without a doubt, the small size of the car in this study has also contributed to its extraordinarily high fuel efficiency.

We conclude this section with two significant facts. First, despite continuing technological innovation, the fuel economy of the average American car increased by 15% from 1980 to 2005.[12] This means that most of the benefits provided by technology were directed at the consumer by increasing car size and power (power doubled during this period), whereas they had a minimal effect on reducing the car's environmental impact. Second, in 2010, the average fuel consumption of cars sold in Europe decreased by 3.7%. However, it is estimated that the reduction due to technological innovation (i.e., if the car size and power were kept constant) was 5%, the difference being due to an increase in car size.[13]

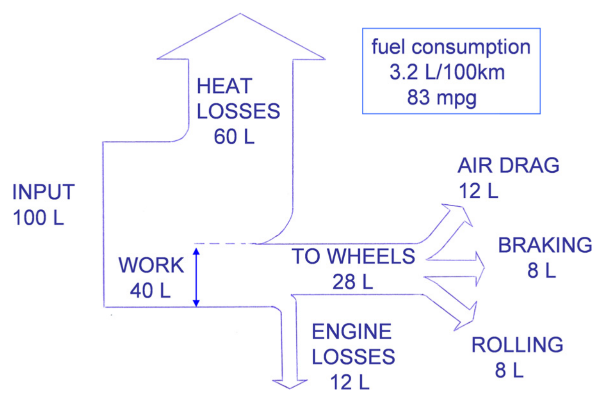

Fig. 7. Detailed energy balance for a significant round trip. Around 28% of the fuel energy is transferred to the wheels.

VII. CONCLUSIONS

The energy efficiency of a particular commercial car has been measured by suitable experiments based on elementary mechanics. From the difference in fuel consumption between the upward and downward directions on a given road section, the engine's thermal efficiency has been determined to be around 40%, a value independent of frequency. From the time needed to reduce the car velocity by a given amount, the force due to air drag plus rolling resistance has been measured as a function of velocity. Using a parabola to fit this data, the rolling and aerodynamic coefficients have been determined to be $\alpha_R = 0.011$ and $\alpha = 0.35$, respectively. Application of this information to a typical trip shows that around 28% of the fuel energy is transferred to the wheels of the car. Finally, we have concluded that technology alone is not able to reduce the environmental impact of cars. It is also necessary to limit car size and engine power.

ACKNOWLEDGMENTS

The authors are grateful for the assistance of F. Roura during the experiments, to Dr. J. Renart for fruitful discussions about car mechanics, and to Dr. A. González for his critical reading of the manuscript.

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Hilger Spectrometer. This is the largest spectrometer of the classic divided circle design that I have ever seen. It was made by the firm of Adam Hilger of London and in 1999 was on display at the museum of St. Patricks College in Maynooth, County Kildare, Ireland. The instrument stands on its own trolley, and the divided circle has a diameter of 45.7 cm. It was used by Walter Hartley (1846-1913) in his research on the spectra of elements. In 1883 he established that relationships exist between the wavelengths of the spectra of an element and its position in the periodic table. (Notes by Thomas B. Greenslade, Jr., Kenyon College; photograph by Sonia Greenslade)