6.4 SUN PATH DEVELOPMENT USING MATHCAD

A number of web-based aids discuss all or some aspects of sun path computations. The website www.uni.edu/darrow/frames/geosol.html, contains a compilation of solar and geographical website addresses that provide information for many aspects of sun paths. The United States Naval Observatory (USNO) website, www.usno.navy.mil, provides a wide range of data and computation capabilities. USNO capabilities include not only site-specific solar and lunar daily/yearly characteristics, but also sun path generation in the form of tabular listings for United States locations or for specified latitudes and longitudes worldwide. This is a particularly useful website, but since all calculations are hidden, it is not a good instructional website for studying sun paths. The National Oceanic and Atmospheric Administration (NOAA) website, www.srrb.noaa.gov/highlights/sunrise/azel.html, has an online sun position calculator as well as some explanation of the approach. Additionally, www.susdesign.com/sunangle/ offers an online sun position calculator, but with no explanation as to the method(s) used.

Commercial software to compute solar positions and sun paths is also available. For example, the Florida Solar Energy Center (Cocoa, FL) markets SunPath™, a software element for sun path calculation. SUNPATH™, available from Filmtools (Burbank, CA), is designed for determining lighting issues associated with filming and photography but can also be used to generate conventional sun paths. An out-of-print book by Petherbridge (1966) presents sun paths and overlays for heat gain calculations.

With the Julian day and the latitude known, Eqs. (6-23)–(6-29) are sufficient to construct the sun path line for the corresponding day and location. Figure 6.13, the June 21 sun path line for MSU, was generated from these equations using the Mathcad software element as described next.

Figure 6.16 shows the complete Mathcad worksheet needed to compute and plot the sun path for June 21 at MSU (Figure 6.13). Only the date and latitude need be changed to generate and plot the sun path line for another day or location. Since the worksheet represents the kernel needed to construct sun path lines, an examination of the procedure is warranted.

The Julian day is used to compute the declination using Eq. (6-22). The latitude is entered, and the hour angle, hss, is specified in the range from solar noon (0°) to solar midnight (180°). Equation (6-23) is used to calculate the solar altitude angle for every hour angle for the specified day and latitude. Equation (6-24) provides the corresponding solar azimuth angles. Logic is provided to determine solar altitude angles greater than 90°. "hlimit" is the hour angle for which the azimuth angle is equal to ±90° and is determined using Eq. (6-25). The sun path can then be plotted, or values of the altitude and azimuth angles printed, as a function of hour angle. The initial computational results cover solar noon to solar midnight. However, since only the sign of the azimuth angles differs for morning, the complete day's sun path can be generated simply by plotting $-a_s$ for the morning hours.

The solar times corresponding to the azimuth and altitude angles are then extracted for every solar hour (hss = 0, 15, 30, . . . , 180) from the azimuth and alti-

The generation of the sunpath line for a given day as a function of latitude.

$n := 1..365$ The days of the year.

Declination Angle

$$\delta_n := 23.45\sin\left[360\cdot\frac{(n+284)\cdot\pi}{365\cdot 180}\right]$$

The declination angle is the angle between the sun's rays and the zenith (overhead) direction at solar noon on the equator. The declination is dependent on the Earth's position in its orbit around the sun.

Declination for specific day (use Julian date). For June 21, the Julian date is 172.

$\delta D := \delta_{172}$ $\delta D = 23.45$ 21 June Declination angle in degrees for use in sunpath generation.

Input the latitude (in degrees):

$L := 33.455$ Location of Mississippi State University

Establish range variables for days and hours. Degrees to radian conversion:

$hss := 0..180$ $hsp_{hss} := hss$ Hours $dr := \frac{\pi}{180}$

Calculation of sunpath angles following Goswami et al.

$$sin\alpha_{hss} := \sin(L\cdot dr)\cdot\sin(\delta D\cdot dr) + \cos(L\cdot dr)\cdot\cos(\delta D\cdot dr)\cdot\cos(hsp_{hss}\cdot dr)$$ Altitude angle

$\alpha_{hss} := \mathrm{asin}(sin\alpha_{hss})$ $ang_{hss} := \frac{\alpha_{hss}}{dr}$ Altitude angle in degrees

$$sinas_{hss} := \cos(\delta D\cdot dr)\cdot\frac{\sin(hsp_{hss}\cdot dr)}{\cos(\alpha_{hss})}$$ Azimuth angle

Test for azimuth angle > 90 degrees.

Since the principal values of the arcsin are defined for -90 degrees < angle < 90 degrees, logic is needed for any azimuth angle greater than 90 degrees.

$$hlimit := \begin{cases}\left(\mathrm{acos}\left(\frac{\tan(\delta D\cdot dr)}{\tan(L\cdot dr)}\right)\right)\cdot\frac{1}{dr} & \text{if } L > \delta D \\ 0 & \text{otherwise}\end{cases}$$

$hlimit = 48.968$ Hour angle at 90-degree azimuth for given day.

Definition of arcsin function to include azimuth angles > 90 degrees.

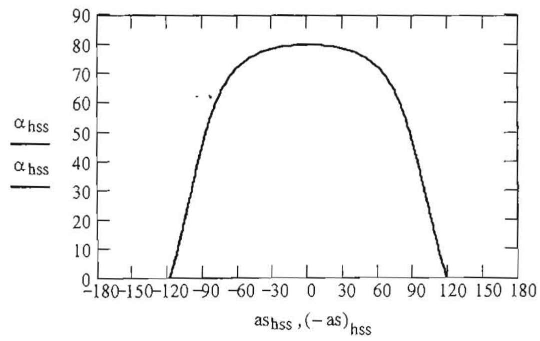
$$as_{hss} := \begin{cases}\left(\pi - \mathrm{asin}(sinas_{hss})\right) & \text{if } hsp_{hss} > hlimit \\ \mathrm{asin}(sinas_{hss}) & \text{otherwise}\end{cases}$$

Change all angles from radians to degrees $\alpha_{hss} := \frac{\alpha_{hss}}{dr}$ $as_{hss} := \frac{as_{hss}}{dr}$

Plot the sunpath taking advantage of the symmetry of the morning and afternoon segments.

(continued on next page)

Figure 6.16 Sun path Mathcad worksheet for a single day.



Establish the lines of constant solar time (hour angle) on the sunpath. Each solar hour corresponds to 15 degrees of hour angle. Thus, 1 PM solar time is 15 degress from solar noon.

$solar_1 := \alpha_{15}$ $azi_1 := as_{15}$ $solar_2 := \alpha_{30}$ $azi_2 := as_{30}$

$solar_3 := \alpha_{45}$ $azi_3 := as_{45}$ $solar_4 := \alpha_{60}$ $azi_4 := as_{60}$

$solar_5 := \alpha_{75}$ $azi_5 := as_{75}$ $solar_6 := \alpha_{90}$ $azi_6 := as_{90}$

$solar_7 := \alpha_{105}$ $azi_7 := as_{105}$ $solar_8 := \alpha_{120}$ $azi_8 := as_{120}$

$solar_9 := \alpha_{135}$ $azi_9 := as_{135}$ $solar_{10} := \alpha_{150}$ $azi_{10} := as_{150}$

$solar_{11} := \alpha_{165}$ $azi_{11} := as_{165}$ $solar_{12} := \alpha_{180}$ $azi_{12} := as_{180}$

$solar_0 := \alpha_0$ $azi_0 := as_0$ Solar noon

Add the solar time to the sunpath plot taking advantage of the symmetry..

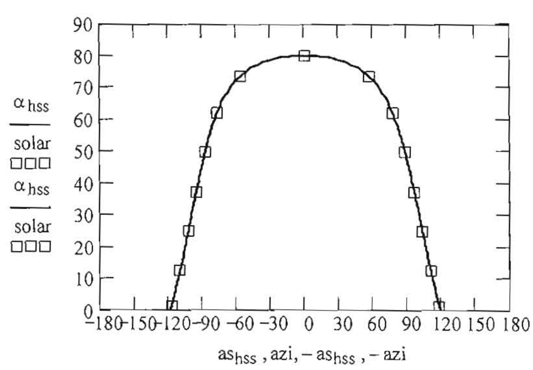


Figure 6.16 *(continued)*

tude angles. The solar times are added to the sun path plot to complete the presentation in Figure 6.13.

Although the sun path for one day is useful for that particular day, a complete understanding of the sun's yearly path at a given location is also needed. Figure 6.17, shows the sun path lines and solar times for seven days spanning a year for MSU. Figure 6.18 illustrates the Mathcad worksheet that is used to generate the sun path for different days of the year. For this example, the 21st of each month was chosen. Because of symmetry, the sun paths for May 21 and July 21, April 21 and August 21, March 21 and September 21, February 21 and October 21, and January 21 and November 21 are the same. Only June 21 and December 21 are lacking symmetry

ds to 15

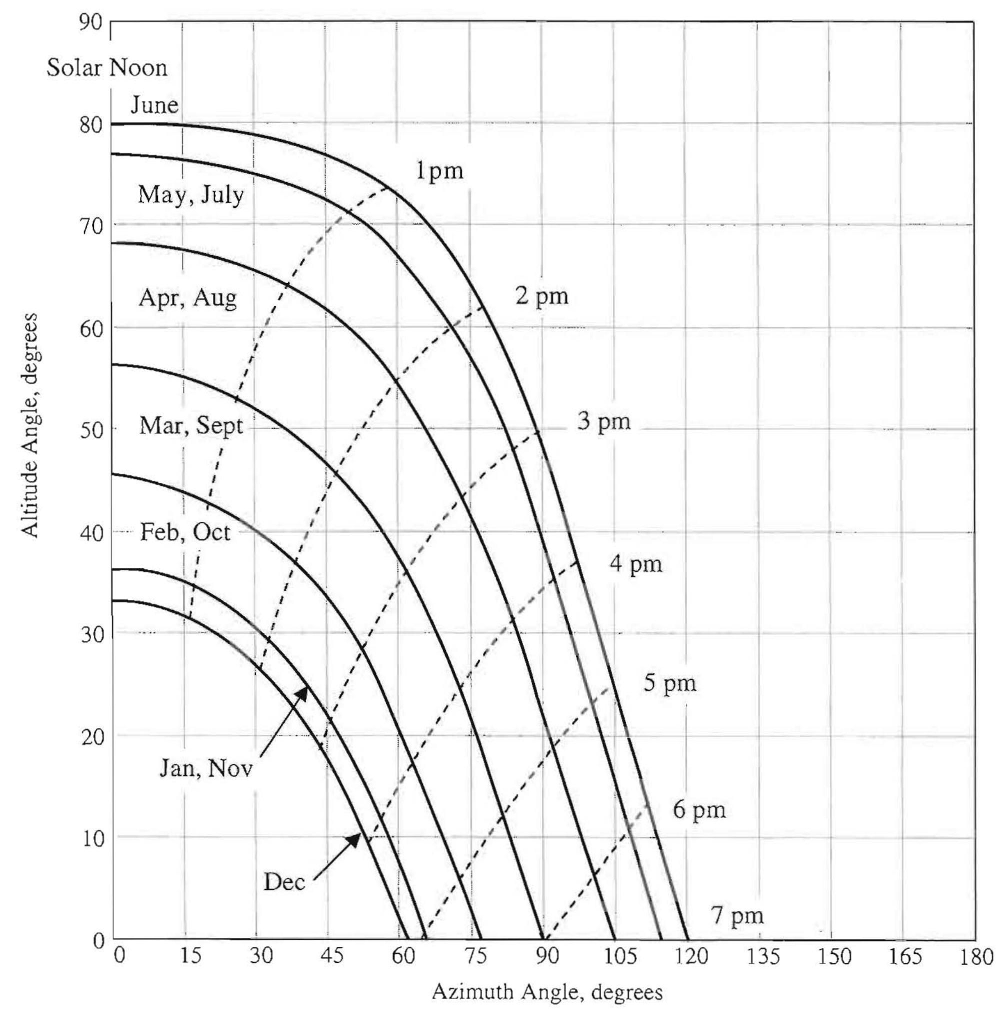


Figure 6.17 Sun path lines for the 21st day of every month for MSU.

The generation of the sunpath chart as a function of latitude.

$n := 1..365$ The days of the year.

Declination Angle

$\delta_n := 23.45 \cdot \sin\left[360 \cdot \frac{(n+284)\cdot\pi}{365\cdot 180}\right]$

The declination angle is the angle between the sun's rays and the zenith (overhead) direction at solar noon on the equator. The declination is dependent on the Earth's position in its orbit around the sun.

Declination for specific days

$\delta D_0 := \delta_{355}$ 21 Dec $\delta D_1 := \delta_{21}$ 21 Jan $\delta D_2 := \delta_{52}$ 21 Feb $\delta D_3 := \delta_{80}$ 21 March

$\delta D_4 := \delta_{111}$ 21 April $\delta D_5 := \delta_{141}$ 21 May $\delta D_6 := \delta_{172}$ 21 June

$$\delta D = \begin{pmatrix} -23.45 \\ -20.138 \\ -11.226 \\ -0.404 \\ 11.579 \\ 20.138 \\ 23.45 \end{pmatrix}$$

Declination angle in degrees for use in sunpath generation.

Input the latitude (in degrees):

$L := 33.455$ Latitude of Mississippi State University

Establish range variables for days and hours.

$i := 0..6$ Days of interest

Hours

$hss := 0..180$ $hsp_{hss} := hss$

Degrees to radian conversion:

$dr := \frac{\pi}{180}$

Calculation of sunpath angles following Goswami, Kreith, and Kreider.

$sin\alpha_{hss,i} := \sin(L\cdot dr)\cdot\sin(\delta D_i\cdot dr) + \cos(L\cdot dr)\cdot\cos(\delta D_i\cdot dr)\cdot\cos(hsp_{hss}\cdot dr)$ Altitude angle

$\alpha_{hss,i} := \operatorname{asin}(sin\alpha_{hss,i})$

$ang_{hss} := \frac{\alpha_{hss,6}}{dr}$ Altitude angle in degrees

$sinas_{hss,i} := \cos(\delta D_i\cdot dr)\cdot\frac{\sin(hsp_{hss}\cdot dr)}{\cos(\alpha_{hss,i})}$ Azimuth angle

Test for azimuth angle > 90 degrees.

Since the principal values of the arcsin are defined for -90 degrees < angle < 90 degrees, logic is needed for any azimuth angle greater than 90 degrees.

$hlimit_i := \frac{1}{dr}\cdot\operatorname{acos}\left(\frac{\tan(\delta D_i\cdot dr)}{\tan(L\cdot dr)}\right)$

$hlimit^T = (131.032\ \ 123.709\ \ 107.481\ \ 90.611\ \ 71.936\ \ 56.291\ \ 48.968)$

(continued on next page)

Figure 6.18 Mathcad worksheet for the sun path for MSU.

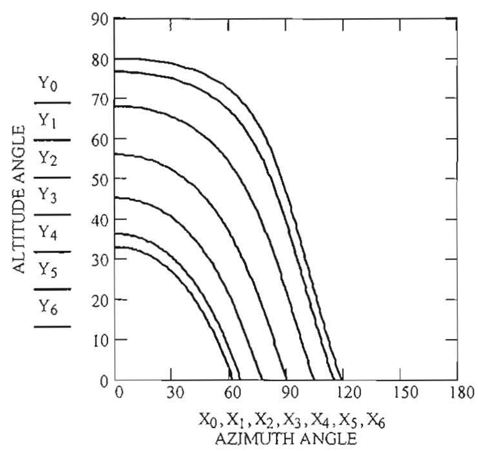
Hour angles at a 90-degree azimuth for a given latitude.

Definition of arcsin function to include azimuth angles > 90 degrees.

$$as_{hss,i} := \begin{cases} (\pi - asin(sinas_{hss,i})) & \text{if } hsp_{hss} > hlimit \\ asin(sinas_{hss,i}) & \text{otherwise} \end{cases}$$

Change all angles from radians to degrees $\alpha_{hss,i} := \frac{\alpha_{hss,i}}{dr}$ $as_{hss,i} := \frac{as_{hss,i}}{dr}$

Redefine $\alpha_{hss,i}$ and $as_{hss,i}$ as Y_i and X_i, respectively. $X_i := as^{\langle i \rangle}$ $Y_i := \alpha^{\langle i \rangle}$



Establish the lines of constant solar time (hour angle) on the sunpath. Each solar hour corresponds to 15 degrees of hour angle. Thus, 1 PM solar time is 15 degrees from solar noon.

$solar1_i := \alpha_{15,i}$ $azi1_i := as_{15,i}$ $solar2_i := \alpha_{30,i}$ $azi2_i := as_{30,i}$

$solar3_i := \alpha_{45,i}$ $azi3_i := as_{45,i}$ $solar4_i := \alpha_{60,i}$ $azi4_i := as_{60,i}$

$solar5_i := \alpha_{75,i}$ $azi5_i := as_{75,i}$ $solar6_i := \alpha_{90,i}$ $azi6_i := as_{90,i}$

$solar7_i := \alpha_{105,i}$ $azi7_i := as_{105,i}$ $solar8_i := \alpha_{120,i}$ $azi8_i := as_{120,i}$

$solar9_i := \alpha_{135,i}$ $azi9_i := as_{135,i}$ $solar10_i := \alpha_{150,i}$ $azi10_i := as_{150,i}$

$solar11_i := \alpha_{165,i}$ $azi11_i := as_{165,i}$ $solar12_i := \alpha_{180,i}$ $azi12_i := as_{180,i}$

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Figure 6.18 *(continued)*

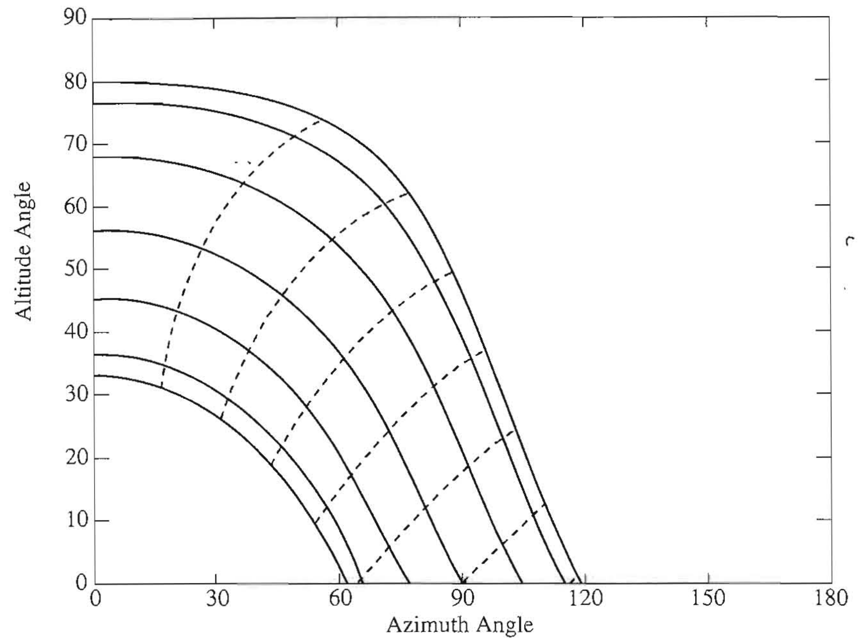

Figure 6.18 *(continued)*

months. On the multi-day sun paths, to avoid cluttered presentations, only the afternoon portions are presented since symmetry provides the mornings. The seven sun path lines representing the 21st day of each month provide a relatively complete picture of the variations in the sun's path (Ptolemaic view) over the year for a given location. The variations in length of day, in solar azimuth sunset (and sunrise) angles, and in solar altitude angle at noon are evident on inspection of the figure. Additionally, the azimuth angles swept per hour can be determined.

An examination of Figure 6.18 is appropriate. The solar declinations are computed for each of the seven days, and the altitude and azimuth angles are generated for each solar hour angle for each day. The same logic as used in Figure 6.16 for a single day is used in Figure 6.18 for each day. Thus, the altitude and azimuth angles are arrays rather than vectors (in the nomenclature of Mathcad).

Obviously, sun path lines could be presented for a wide range of latitudes. In particular, a sun path at a higher latitude is worth examining. Consider Figure 6.19, which is for a latitude above the Arctic Circle. Such latitudes are characterized by the phenomenon "the sun never sets" during some summer months. The figure, for latitude 80° (north), illustrates that the sun does not set from April through August. Even in the summer, the maximum solar altitude angle is only 33.45°, which is relatively low on the horizon. However, if the sun never sets during some summer months, that means the sun never rises during some winter months. For the latitude of 80° (north), the sun barely rises (has a positive solar altitude angle) during March and September, and, as the sun path lines indicate, the sun does not rise during October through February. All in all, Figure 6.19 portrays the sun's path at this high

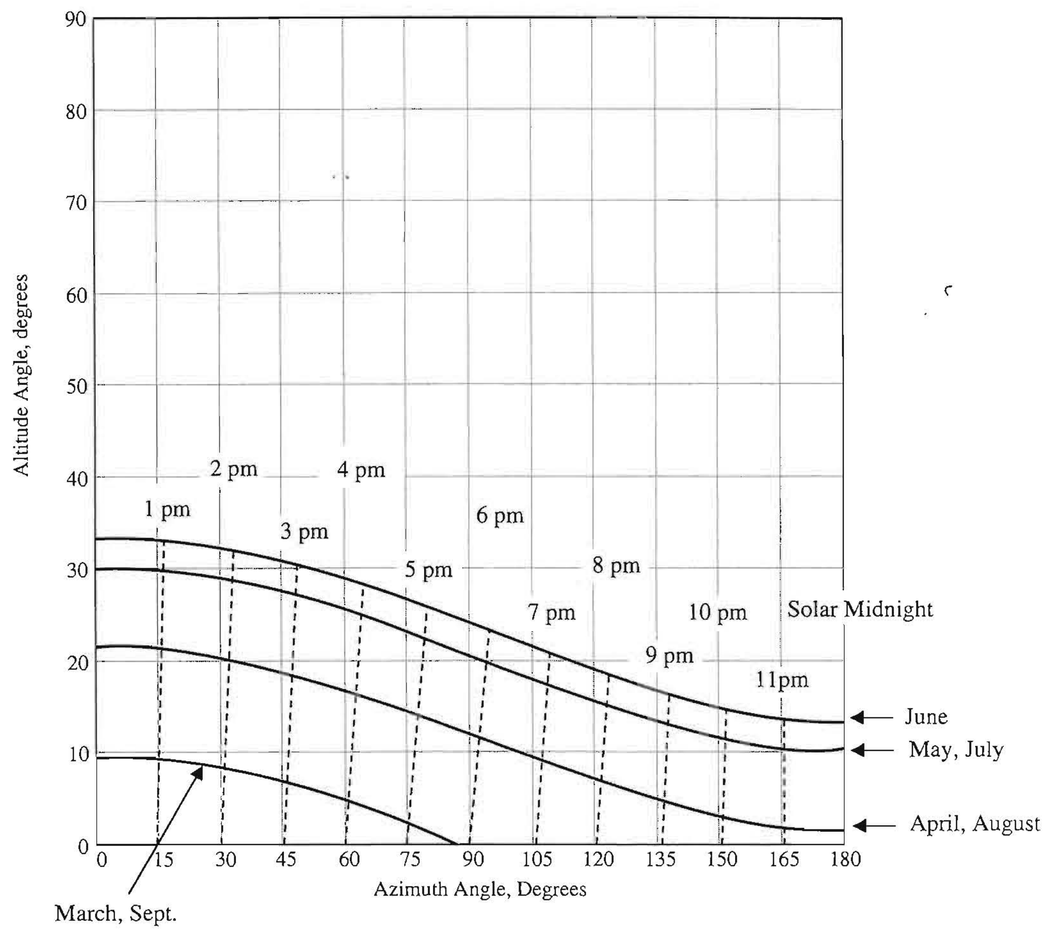

Figure 6.19 Sun path lines for latitude above the Arctic Circle.

latitude as skirting the horizon in the summer and sinking below the horizon during the winter.

Additional information and details on Mathcad procedures described herein can be found in Hodge (2003).

6.5 THE SOLAR ENERGY DATABASE

For any quantitative considerations of solar energy applications, hourly, monthly, and yearly information about the climate, including irradiation (the solar "insolation"), is needed. The United States Department of Energy's National Renewable Energy Laboratory (NREL) at Golden, Colorado, provides an astonishingly wide range of climatic and energy engineering information, much of which is available on its website, www.rredc.nrel.gov.

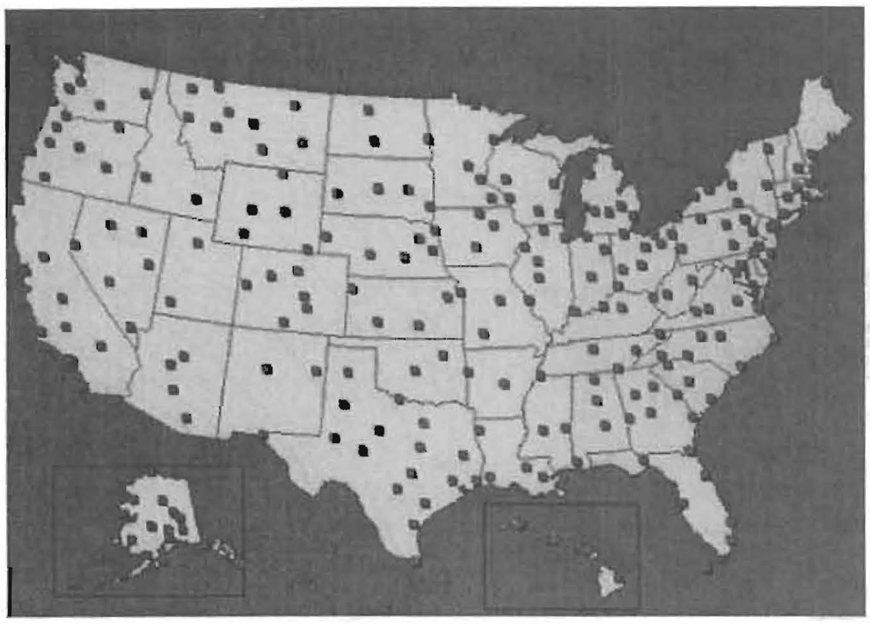

Figure 6.20 NWS sites for which NSRDB data are available (NCDC).

The National Solar Radiation Data Base (NSRDB) contains 30 years (1961–1990) of solar radiation and supplementary meteorological data from 237 National Weather Service (NWS) sites in the United States, plus sites in Guam and Puerto Rico. Figure 6.20 shows the sites for which NSRDB data are available.

The following data products are available on the NREL website:

1. Daily statistics files (monthly averages of daily totals)
2. Hourly data files
3. *Solar Radiation Data Manual for Buildings* [30-year (1961–1990) average of solar radiation and illuminance for each month]
4. *Solar Radiation Data Manual for Flat-Plate and Concentrating Collectors*
   a. Averages of solar radiation for the 360 months from 1961 to 1990
   b. Thirty-year (1961–1990) average of solar radiation for each month.
5. Typical Meteorological Year 2 (TMY2) files
6. United States solar radiation resource maps

The Typical Meteorological Year 2 (TMY2) data sets are derived from the 1961–1990 National Solar Radiation Data Base. Because they are based on the NSRDB, these data sets are referred to as TMY2 to distinguish them from the TMY data sets, which were based on older data. The TMY and TMY2 data sets cannot be used interchangeably because of differences in time (solar versus local), formats, elements, and units. The TMY2 data sets contain hourly values for solar radiation and meteorological elements for a one-year period. Their intended use is for computer simulations of solar energy conversion systems and building systems to facilitate performance comparisons of different system types, configurations, and locations in the United States and its territories. Since TMY2 data sets represent typical rather than extreme conditions, they are not suited for designing systems to meet the worst-case conditions occurring at a given location. The TMY2 data sets and manual were produced by the NREL Analytic Studies Division under the Resource Assessment Program, which is funded and monitored by the U.S. Department of Energy Office of Solar Energy Conversion. The data contained in the NREL website will be used in the following chapters.

The NREL website also provides a useful assessment of the potential for solar energy in the United States. Figure 6.21 shows the average daily direct solar energy incident on a perpendicular surface tracking the sun path for the United States. The desert southwest consistently receives 6.5 to 8.5 $kWh/m^2 \cdot day$, while the southeast,

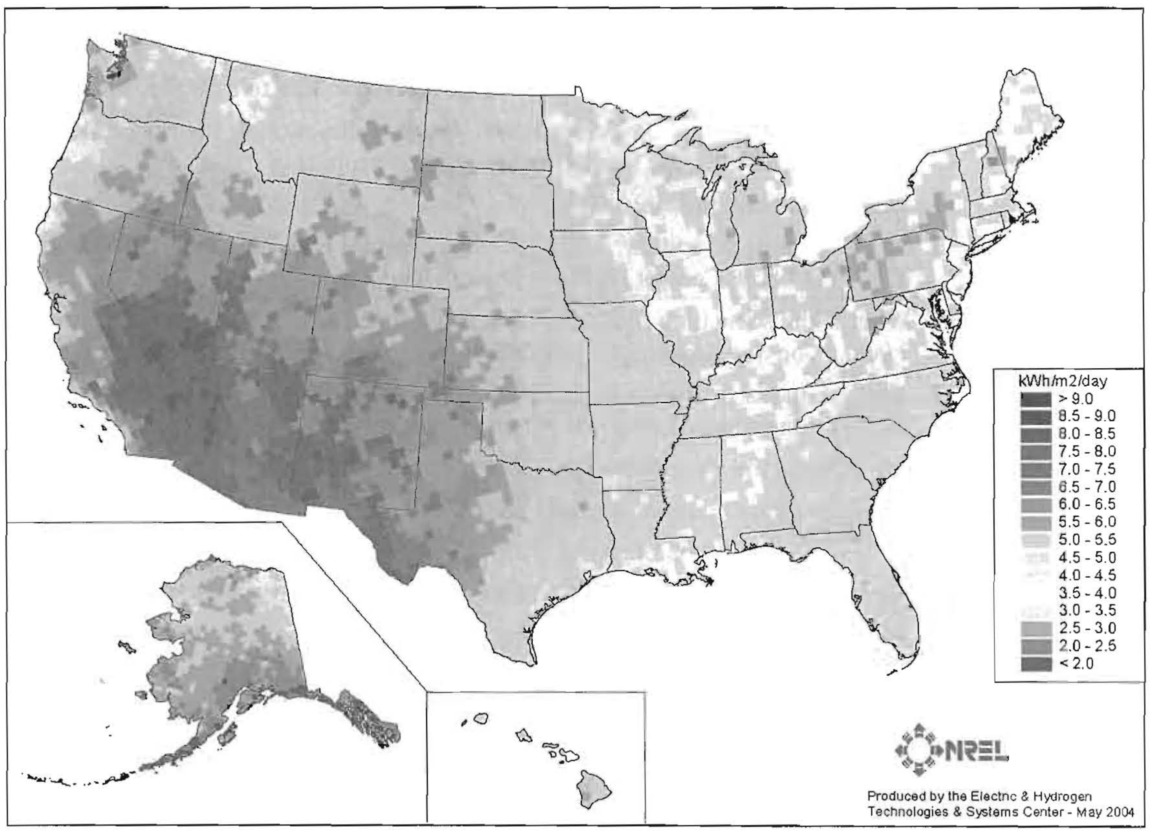

Figure 6.21 Solar potential for the United States (NREL).

because of cloud cover and humidity effects, receives about 4.0 $kWh/m^2 \cdot day$. Much of the northeast solar energy reception is 3.0 $kWh/m^2 \cdot day$ or less. Even though the southwest is the most favored region, solar energy systems are viable in other parts of the United States.

6.6 CLOSURE

This chapter has provided a review of radiation heat transfer, examined sun path lines, and identified sources of solar energy engineering data. With these topics as a basis, solar energy engineering principles can now be considered. The next three chapters discuss, respectively, active solar application, passive solar applications, and photovoltaic solar concepts.

REVIEW QUESTIONS

1. What is the visible spectrum?
2. Sketch and label the Planck distribution corresponding to a 5800 K blackbody.
3. Explain the differences between the spectral directional, the spectral hemispherical, and the total hemispherical surface properties.
4. Under what circumstances is $\alpha_\lambda = \varepsilon_\lambda$? What is required for $\alpha = \varepsilon$?
5. At 35° N latitude and 80° W longitude, the actual time on June 21 (Julian date of 172) is 1 p.m. Central Daylight Savings Time. Find the solar time.
6. What is the solar time for a clock time (the time read on a clock or a watch) of 2:00 p.m. on September 27 in a location specified by 35:25:30 N and 82:30:00 W?
7. The sun paths in Figure R6.7, are for 33.5° N latitude, and the location is at 88.5° W longitude. Answer the following questions:
   (a) What is the altitude angle at solar noon on June 21?
   (b) How many hours of daylight are there in the shortest day?
   (c) How many hours of daylight are there in the longest day?
   (d) At solar noon on June 21, how long a shadow would a 100-ft flagpole cast?
   (e) If the latitude were increased to 50° N latitude, would the hours of daylight for December 21 be more or less?
8. At 35° N latitude and 88° W longitude, the solar time on January 30 is 11 a.m. Find the actual (clock) time.
9. At 55° N latitude and 98° W longitude, the solar time on the 180th day of the year is 11 a.m. Find the local time (Central Daylight Savings Time).

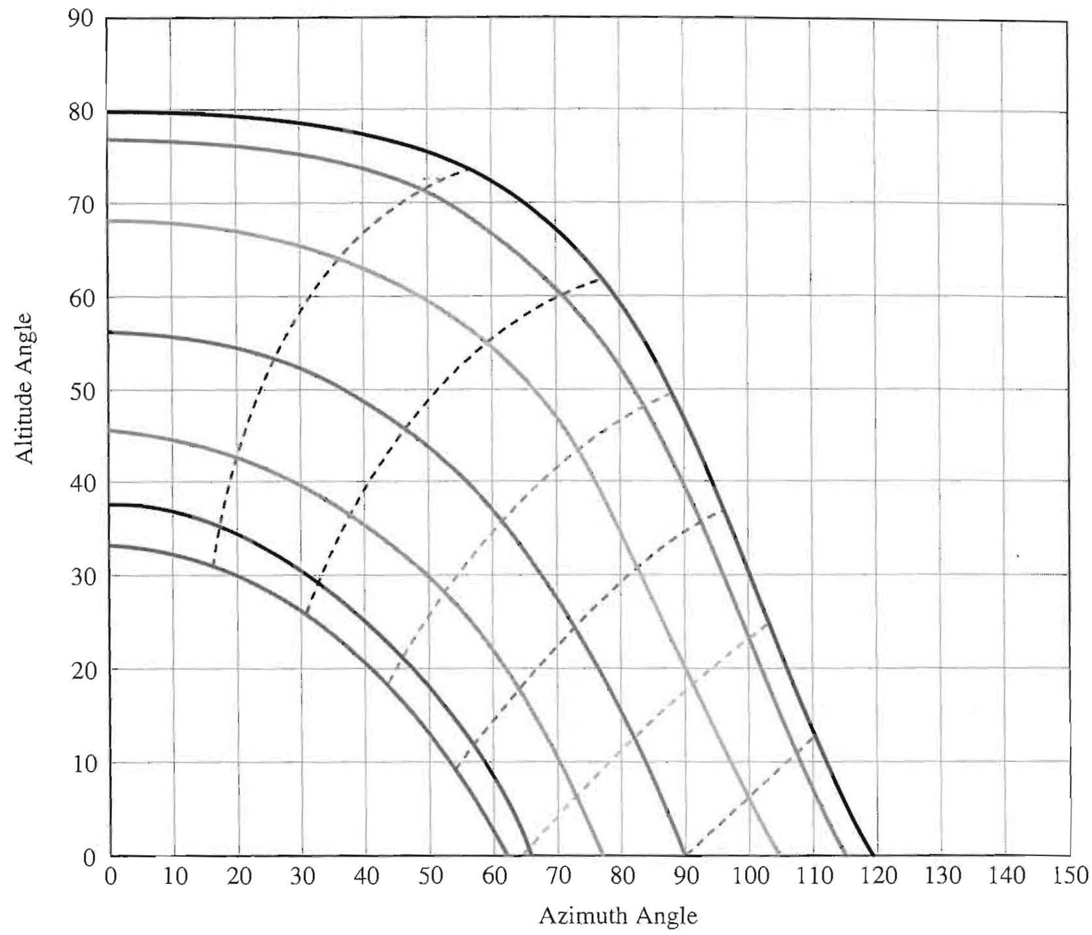

Figure R6.7 Sunpath line with solar times

EXERCISES

1. A radiator on a solar-powered satellite must dissipate the heat being generated with the satellite by radiating the heat into space. The radiator surface has a solar absorptivity of 0.5 and an emissivity of 0.95. What is the surface temperature when the required dissipation is 1500 W/m^2 for each of the following two conditions?
   (a) The radiator is facing the sun, and the solar irradiation is 1353 W/m^2.
   (b) The radiator is shielded from the sun, and the solar irradiation is negligible.

2. A contractor must select a roof covering material from the two diffuse ($\varepsilon_\lambda = \alpha_\lambda$) roof coatings whose spectral characteristics are presented in Figure P6.2.
   (a) Which of the materials would result in a lower roof temperature?
   (b) Which is preferred for summer use?
   (c) Which is preferred for winter use?
   (d) Sketch a spectral distribution that would be ideal for summer.
   (e) Sketch a spectral distribution that would be ideal for winter.

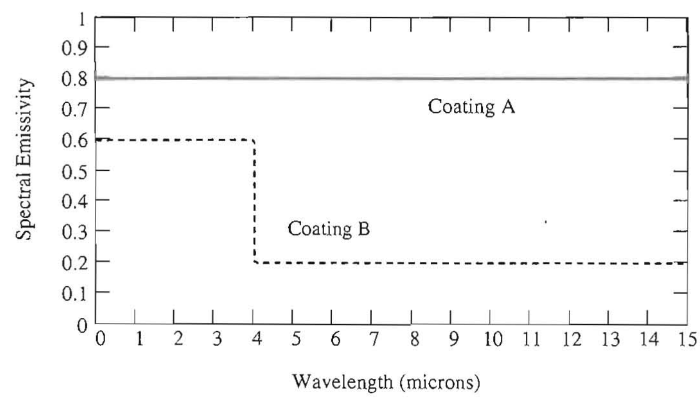

Figure P6.2

3. Two special coatings are available for use on an absorber plate for a flat-plate solar collector. The coatings are diffuse ($\varepsilon_\lambda = \alpha_\lambda$) and are characterized by the spectral distributions illustrated in Figure P6.3.

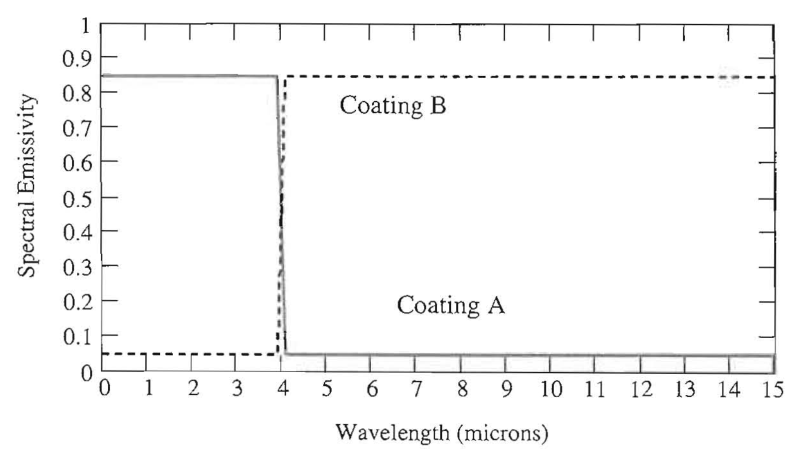

Figure P6.3

(a) If the irradiation incident on the plate is $G = 1000\ \text{W/m}^2$, what is the radiant energy absorbed per m^2 for each surface?

(b) Which coating would you select for the absorber plate? Explain.

4. The spectral absorptivity, α_λ, and the spectral reflectivity, ρ_λ, for a spectrally selective diffuse surface are as shown in Figure P6.4.

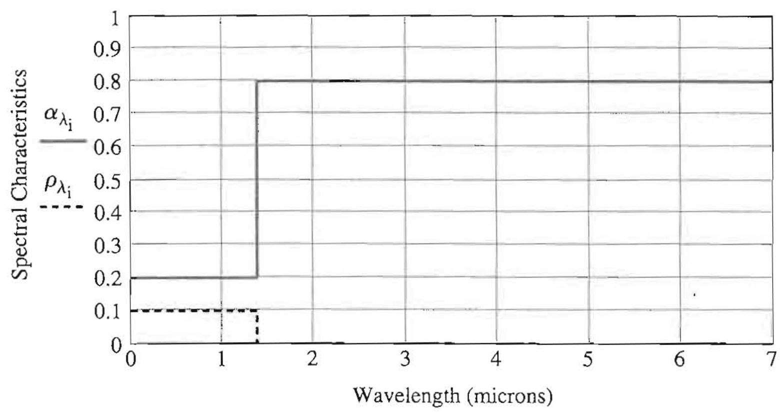

Figure P6.4

(a) Sketch the spectral transmissivity, τ_λ.

(b) If solar irradiation with $G = 750\ \text{W/m}^2$ and the spectral distribution of a blackbody at 5800 K is incident on the surface, determine the fractions of the irradiation that are transmitted, reflected, and absorbed by the surface.

(c) If the temperature of the surface is 350 K, determine the emissivity, ε.

(d) Determine the net heat flux by radiation at the surface of the material.

5. An opaque solar collector surface is 3 m by 1 m and is maintained at 425 K. The surface is exposed to solar irradiation with $G = 800\ \text{W/m}^2$. The surface is diffuse, and its spectral absorptivity is

$$
\begin{aligned}
\alpha_\lambda &= 0 & & 0 < \lambda < 0.5\ \mu\text{m} \\
&= 0.8 & & 0.5\ \mu\text{m} < \lambda < 1.0\ \mu\text{m} \\
&= 0.5 & & 1.0\ \mu\text{m} < \lambda < 2.0\ \mu\text{m} \\
&= 0.3 & & \lambda > 2.0\ \mu\text{m}
\end{aligned}
$$

Determine the absorbed radiation, the emissive power, and the net radiation heat transfer from the surface.

6. An opaque surface, 2 m by 2 m, is maintained at 400 K and is exposed to solar irradiation with $G = 1200\ \mathrm{W/m^2}$. The surface is diffuse and its spectral absorptivity is

$$\alpha_\lambda = 0 \quad 0 < \lambda < 0.5\ \mu\mathrm{m}$$
$$= 0.8 \quad 0.5\ \mu\mathrm{m} < \lambda < 1.0\ \mu\mathrm{m}$$
$$= 0.0 \quad 1.0\ \mu\mathrm{m} < \lambda < 2.0\ \mu\mathrm{m}$$
$$= 0.9 \quad \lambda > 2.0\ \mu\mathrm{m}$$

Determine the absorbed radiation, the emissive power, and the net radiation heat transfer.

7. Develop a figure showing the sun path lines for the latitude of your hometown. On a separate figure show the sun path line for June 21 and indicate the *actual time* (not the solar time) on the June 21 sun path line. How many hours are there from sunrise to sunset?

8. Develop a figure showing the sun path line for the latitude of your hometown on the day of your birth. Indicate the *actual time* (not the solar time) on the sun path line. How many hours are there from sunrise to sunset?

9. Develop a figure showing the sun path lines for the latitude of Washington, DC. Washington's Reagan-National Airport is located at 38° 51′ N, 77° 2′ W.

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