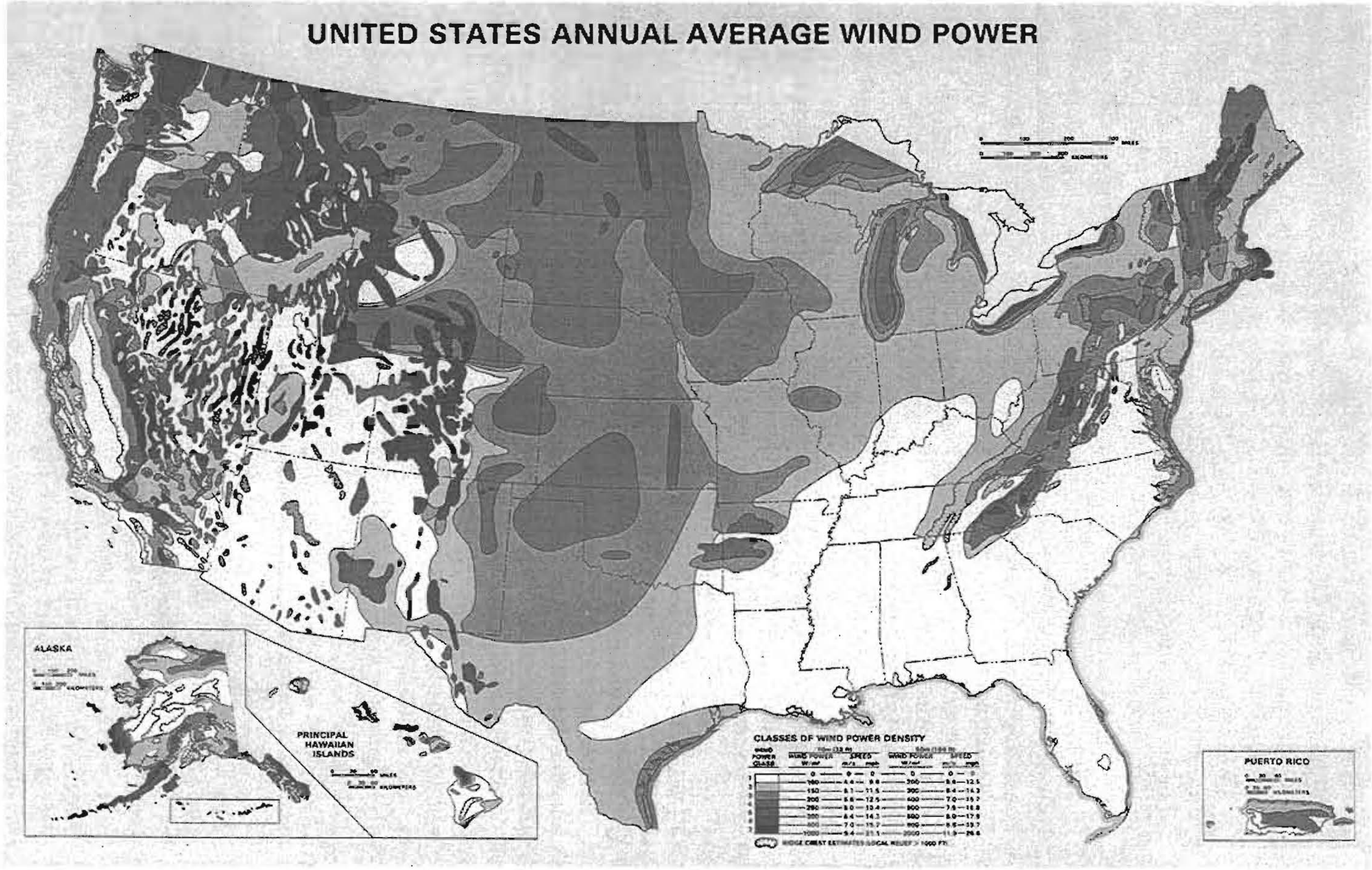


Figure 4.7 United States annual average wind power density (NREL).

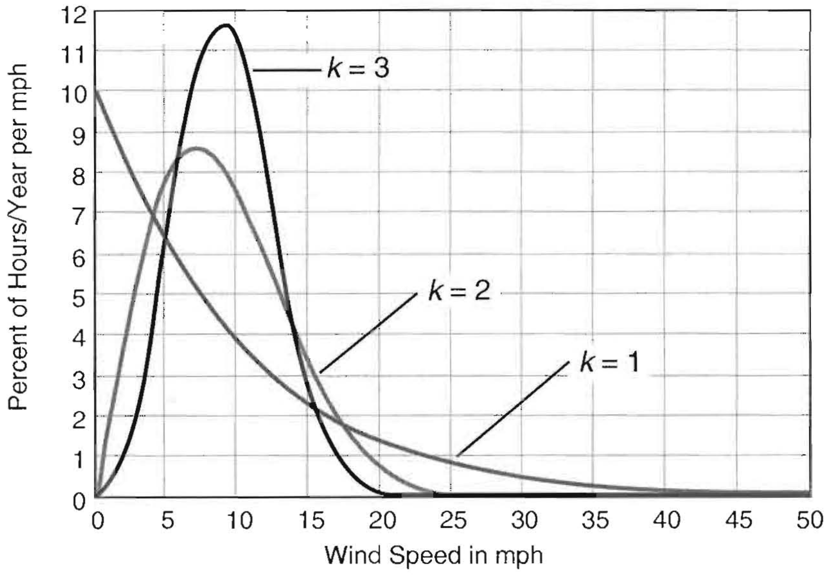


Figure 4.8 Weibull distribution for $c = 10$ mph and various k values.

to assessing the metrics of wind energy? The mode speed represents the most probable speed in a distribution. The mean speed is defined as

$$V_{\text{mean}} = \int_0^{\infty} h(v, k, c) \cdot v \cdot dv \tag{4-19}$$

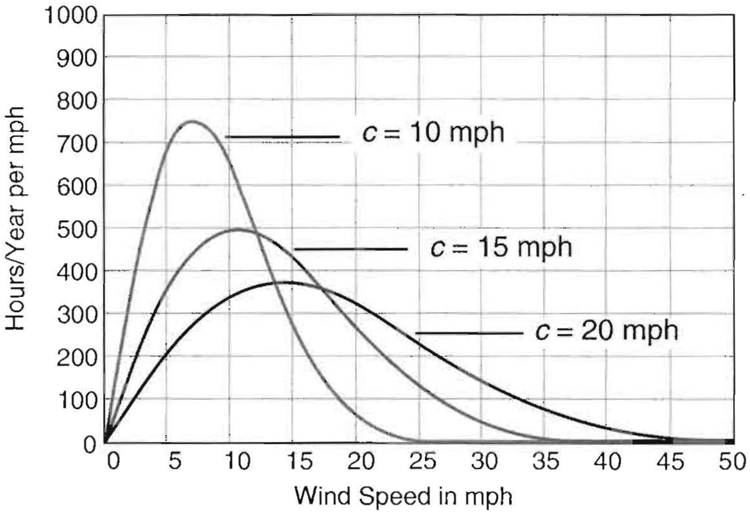


Figure 4.9 Weibull distribution for various c values and $k = 2$.

Since the wind power is proportional to the cube of the wind speed, the average power density available for collection per unit of swept area is

$$\text{Power}_{\text{avail}} = \int_0^\infty \frac{1}{2} \cdot \rho \cdot h(v,k,c) \cdot v^3 \cdot dv \tag{4-20}$$

Thus, the speed of interest for wind energy is the root-mean-cube speed

$$V_{\text{rmc}} = \sqrt[3]{\int_0^\infty h(v,k,c) \cdot v^3 \cdot dv} \tag{4-21}$$

and the annual average power density available becomes

$$\text{Power}_{\text{avail}} = \frac{1}{2}\rho V_{\text{rmc}}^3 \tag{4-22}$$

Based on the results of Figure 4.6, a reasonable power coefficient value for a modern, well-designed wind turbine is 0.5. The annual average extraction power density can be cast as

$$\text{Power}_{\text{ext}} = \frac{1}{4}\rho V_{\text{rmc}}^3 \tag{4-23}$$

The total energy that can be extracted per year for a given distribution is the integral of Eq. (4-23) for each velocity over all possible velocities, or

$$\text{Energy}_{\text{rmc}} = 0.25 \cdot \rho \int_0^\infty h(v,k,c) \cdot 8760 \cdot v^3 \cdot dv \tag{4-24}$$

The best way to assimilate all this information is via an example problem.

Example 4.2

Find V_{mode}, V_{mean}, V_{rmc}, the power density available distribution, and the power extracted per m^2 for a wind turbine at a site corresponding to a Weibull wind distribution with $c = 15$ m/sec and $k = 1.5$. The air density is 1.225 kg/m^3.

Solution A graphical representation of the Weibull distribution for $k = 1.5$ and $c = 15$ m/sec is presented in Figure 4.10.

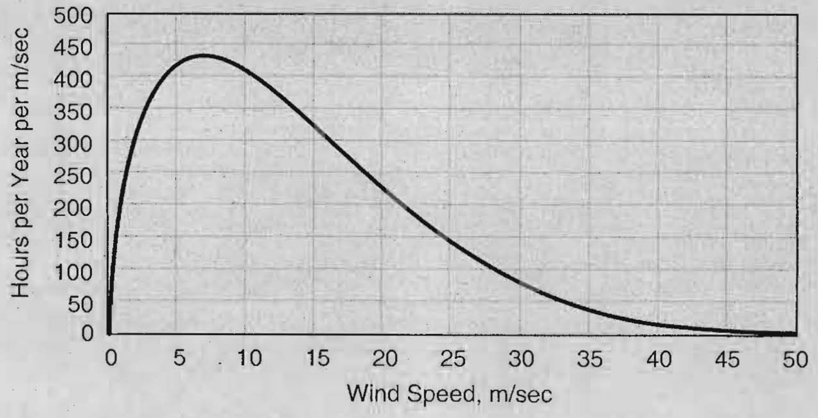
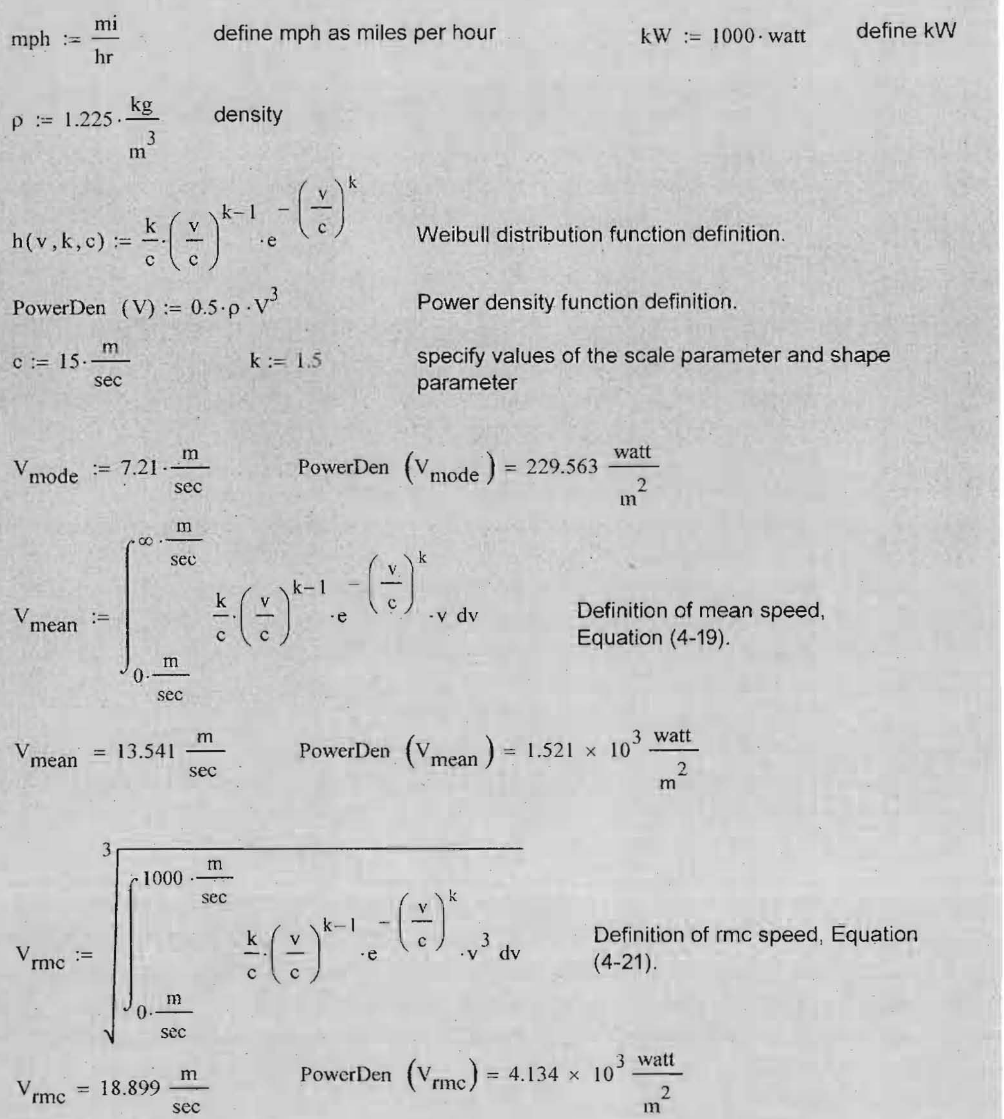


Figure 4.10 Weibull distribution for $k = 1.5$ and $c = 15$ m/sec.

The mode, the most probable wind speed, occurs at 7.21 m/sec. The mean wind speed and the root-mean-cube speed are defined in Eqs. (4-19) and (4-21), respectively. The arithmetic for this example is accomplished in Mathcad, and the Mathcad worksheet is reproduced in Figure 4.11.



$\mathrm{mph} := \dfrac{\mathrm{mi}}{\mathrm{hr}}$ define mph as miles per hour

$\mathrm{kW} := 1000 \cdot \mathrm{watt}$ define kW

$\rho := 1.225 \cdot \dfrac{\mathrm{kg}}{\mathrm{m}^3}$ density

$h(v,k,c) := \dfrac{k}{c} \cdot \left(\dfrac{v}{c}\right)^{k-1} \cdot e^{-\left(\frac{v}{c}\right)^k}$ Weibull distribution function definition.

$\mathrm{PowerDen}\,(V) := 0.5 \cdot \rho \cdot V^3$ Power density function definition.

$c := 15 \cdot \dfrac{\mathrm{m}}{\mathrm{sec}}$ $k := 1.5$ specify values of the scale parameter and shape parameter

$V_{\mathrm{mode}} := 7.21 \cdot \dfrac{\mathrm{m}}{\mathrm{sec}}$ $\mathrm{PowerDen}\left(V_{\mathrm{mode}}\right) = 229.563\ \dfrac{\mathrm{watt}}{\mathrm{m}^2}$

$$V_{\mathrm{mean}} := \int_{0 \cdot \frac{\mathrm{m}}{\mathrm{sec}}}^{\infty \cdot \frac{\mathrm{m}}{\mathrm{sec}}} \frac{k}{c} \cdot \left(\frac{v}{c}\right)^{k-1} \cdot e^{-\left(\frac{v}{c}\right)^k} \cdot v\, dv$$

Definition of mean speed, Equation (4-19).

$V_{\mathrm{mean}} = 13.541\ \dfrac{\mathrm{m}}{\mathrm{sec}}$ $\mathrm{PowerDen}\left(V_{\mathrm{mean}}\right) = 1.521 \times 10^3\ \dfrac{\mathrm{watt}}{\mathrm{m}^2}$

$$V_{\mathrm{rmc}} := \sqrt[3]{\int_{0 \cdot \frac{\mathrm{m}}{\mathrm{sec}}}^{1000 \cdot \frac{\mathrm{m}}{\mathrm{sec}}} \frac{k}{c} \cdot \left(\frac{v}{c}\right)^{k-1} \cdot e^{-\left(\frac{v}{c}\right)^k} \cdot v^3\, dv}$$

Definition of rmc speed, Equation (4-21).

$V_{\mathrm{rmc}} = 18.899\ \dfrac{\mathrm{m}}{\mathrm{sec}}$ $\mathrm{PowerDen}\left(V_{\mathrm{rmc}}\right) = 4.134 \times 10^3\ \dfrac{\mathrm{watt}}{\mathrm{m}^2}$

(continued on next page)

Figure 4.11 Mathcad solution for Example 4.2.

$\mathrm{Power}(v) := 0.5\cdot\rho\cdot h(v,k,c)\cdot v^3$ Power density available with Cp = 1.0.

$v := 0..\,50\cdot\frac{m}{sec}$ wind velocity range

$$\mathrm{Energy} := \int_{0\cdot\frac{m}{sec}}^{1000\cdot\frac{m}{sec}} 0.25\cdot\rho\cdot\left[\frac{k}{c}\cdot\left(\frac{v}{c}\right)^{k-1}\cdot e^{-\left(\frac{v}{c}\right)^{k}}\right]\cdot 8760\frac{hr}{yr}\cdot v^3\,dv$$

$$\mathrm{Energy} = 1.811\times 10^{4}\,kW\cdot\frac{hr}{yr\cdot m^2}$$

Figure 4.11 *(cont.)*

At the start of the worksheet, mph and kW are defined in Mathcad variables, and the density (specified in the problem statement) is indicated. Functions are defined for the Weibull distribution, $h(v, k, c)$, and the power density, PowerDen(V). The shape and scale parameters from the problem statement are inserted into the worksheet. The mode speed is 7.21 m/sec, and the power density is calculated as 230 W/m^2. The mean speed is computed using Eq. (4-19) to be 13.54 m/sec with a power density of 1521 W/m^2. The root-mean-cube (rmc) speed is 18.9 m/sec [using Eq. (4-21)], and the power density at the rmc speed is 4134 W/m^2. The use of Mathcad, with its ability to carry units in computations, simplifies the arithmetic in this example. Since the cubic power of the wind speed is specified in the average power density expression, care should always be exercised to differentiate between the mode, mean, and rmc speeds, as the power density manifests significant variation depending on which value is used.

The energy density available for collection over a year per unit area of wind turbine is specified in Eq. (4-24) since $h(v, k, c)\cdot 8760$ represents the hours per year at a given wind speed from the distribution. Figure 4.12 presents the energy density per year per m^2 per m/sec for the wind speed distribution specified in the problem statement. The mode for the annual energy available is near 27 m/sec. Although the number of hours per year at 27 m/sec is small (far from the mode, as illustrated in Figure 4.10), the product of this wind speed and the number of hours per year is a maximum for the distribution because of the cubic functional dependence on wind speed.

An estimate of the annual energy extracted for $Cp = 0.5$ for a given wind speed distribution is provided by Eq. (4-20). Under the assumption of $Cp = 0.5$, the annual energy extracted for this distribution is $18{,}110\,\frac{\mathrm{kW\,hr}}{\mathrm{yr\,m^2}}$.

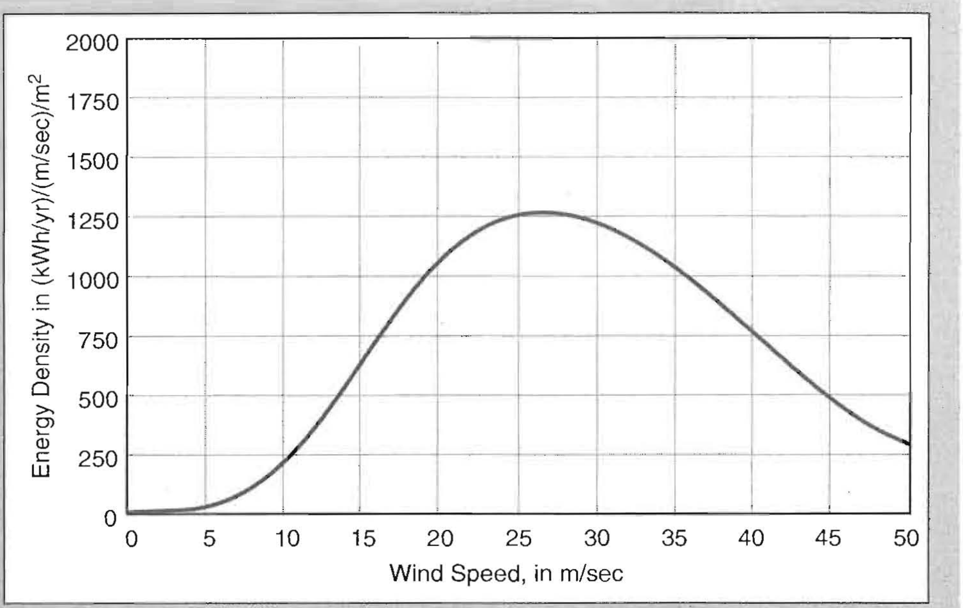


Figure 4.12 Annual energy-available distribution for Example 4.2.

Section 4.2 delineated the performance characteristics of wind turbines at specified speeds. This section has described the wind speed distribution and presented the statistics of the wind speed distribution. The next section assimilates the material in these two sections and addresses the operation and control of wind turbines as a function of wind speed.

4.4 WIND TURBINE OPERATION

What should the operating strategy be for a wind turbine as a function of wind speed? The answer is *not* to operate the wind turbine at the maximum power coefficient. Why not operate the wind turbine at the maximum *Cp*? Operation at the maximum *Cp* for all wind speeds would maximize the energy extracted, but factors such as generator capacity, structural requirements, and safety preclude such operation. The maximum-speed range will occur for only a few hours for a given wind speed distribution. Thus, sizing a generator for an input corresponding to the maximum-speed range would result in an oversized generator that would operate at maximum output only a few hours per year. And if the maximum *Cp* were used, the advance ratio, Ω, would have to be maintained constant. As the wind speed increased, the rotor rotation rate would have to increase to maintain a constant *Cp*. Since the radial stresses in a rotor are proportional to the rotation speed, operating at high wind speeds and a constant Ω would require a structurally robust wind turbine. Additionally, as the wind speed increases, safety and structural integrity become of increasing concern.

But no matter what operating strategy is used, a wind turbine must contain a controller to implement the strategy and mechanical elements to respond to the controller. Figure 4.13, from the NREL website, graphically illustrates typical HAWT features. The cutaway of the nacelle shows the gearbox and generator as well as other elements needed for operation. The blades and tower are also shown. The yaw motor and yaw drive are used to keep the plane of the blade oriented into the wind. The pitch mechanism on the blades adjusts the angle of the blades (the pitch) with respect to the wind direction in order to control the power extracted from the wind. The purpose of the brake is to slow down or completely stop the rotor. All of these elements are needed to implement the control strategy.

The ultimate purpose of a wind turbine control strategy is to regulate the power output of the turbine as a function of wind speed and direction. Additionally, the control protocol must ensure safe operation over all wind conditions. Patel (2005) suggests that the power output versus wind speed characteristic of a wind turbine can be viewed as being composed of several regions. Figure 4.14, adapted from Patel, illustrates a typical power output as a function of wind speed and delineates the various operating regimes and conditions. The ordinate variable is the percentage of generator output.

The first condition is the cut-in speed of the system. Below the cut-in wind speed, the system component efficiencies are so low that running the system is not worthwhile. Once the cut-in speed is reached, the system is operated in a constant-*Cp* region. In the constant-*Cp* region, the turbine extracts the maximum power from the wind, but the power extracted is less than the rated input to the generator. The rotor

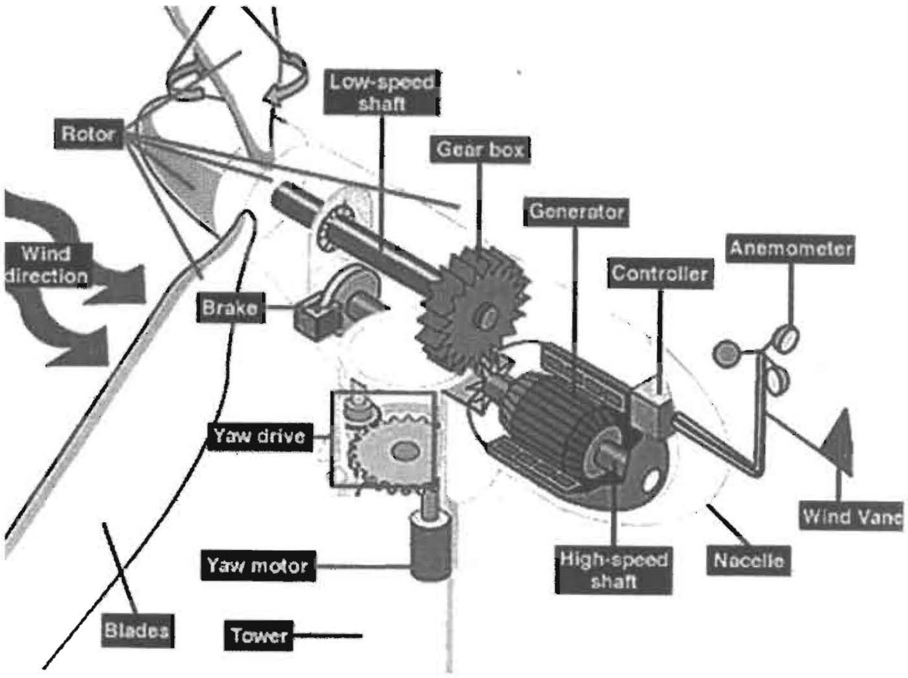


Figure 4.13 HAWT nacelle components and features (NREL).

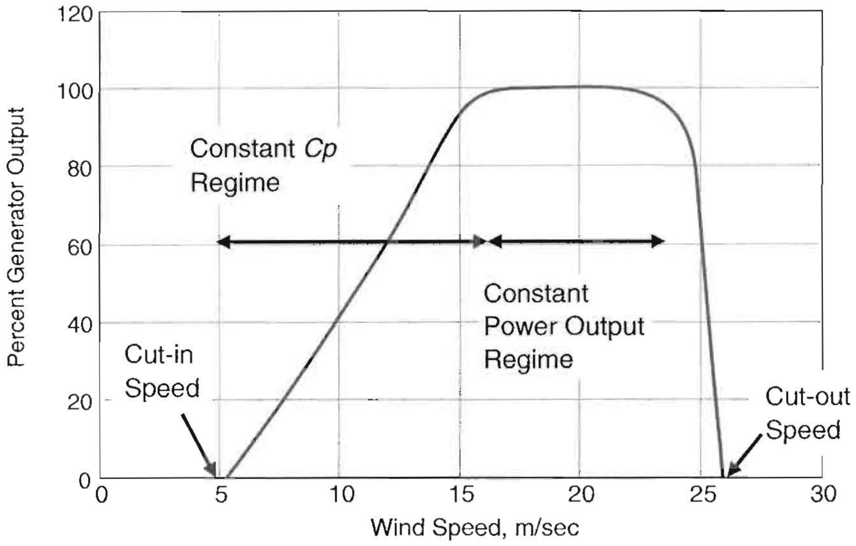


Figure 4.14 Typical regimes of turbine speed control.

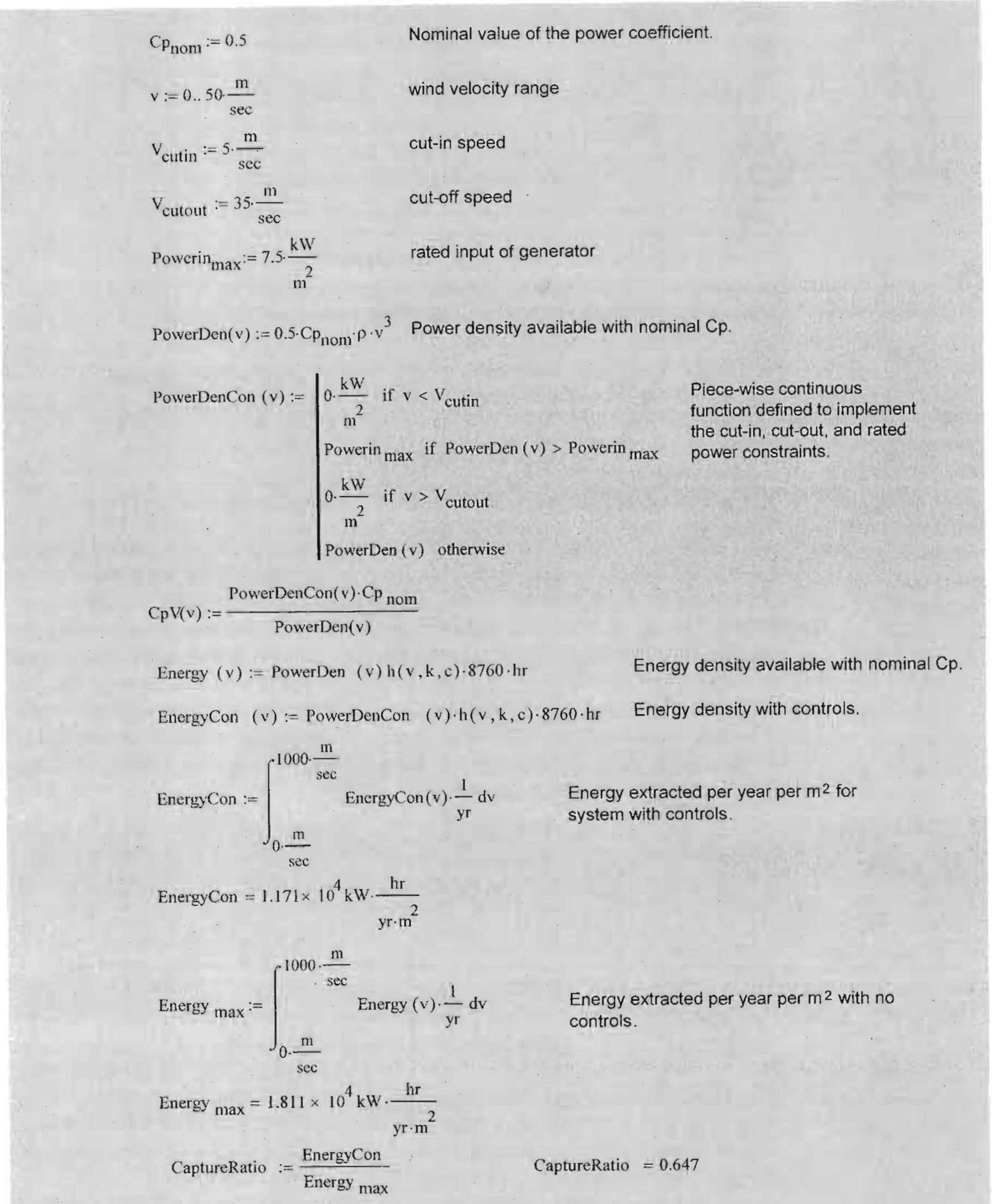
speed is varied so that the advance ratio is maintained near the maximum *Cp* value. When the wind speed is sufficiently high, the power extracted by the rotor exceeds the rated input of the generator. In this regime, the constant power output regime, the system is made to produce the rated output of the generator by operating the turbine at a *Cp* lower than the maximum *Cp*. The cut-out speed is the wind speed beyond which operation would damage the system. For speed in excess of the cut-out, the rotor pitch is set to unload the rotor, and the rotor is locked with the brake. The total energy that can be extracted from a given wind distribution is reduced by the rated input of the generator and the cut-in and cut-out speeds. Example 4.2 will be extended to illustrate these effects.

Example 4.3

The system described in Example 4.2 is specified to have a cut-in speed of 5 m/sec, a cut-out speed of 35 m/sec, and a rated generator input of 7.5 kW/m^2. The maximum power coefficient, *Cp*, is 0.5. Determine and plot the following for both the system with no controls and the system controlled to meet the constraints: (a) the power density of the system, (b) the *Cp* versus wind speed required, (c) the energy extraction, and (d) the total energy extracted by the system.

Solution Much of the information needed for the "no controls" part of this problem was developed in Example 4.2. However, since the results are more meaningful if the "no controls" and the controlled versions are compared, both versions will be examined in the solution. Mathcad will be utilized for all the calculations. The Mathcad worksheet for the solution to this problem is given in Figure 4.15.

The conditions and constraints defined in the problem statement are entered in the worksheet. The range of the wind speed is designated by a Mathcad range



$Cp_{nom} := 0.5$ Nominal value of the power coefficient.

$v := 0..50 \cdot \frac{m}{sec}$ wind velocity range

$V_{cutin} := 5 \cdot \frac{m}{sec}$ cut-in speed

$V_{cutout} := 35 \cdot \frac{m}{sec}$ cut-off speed

$Powerin_{max} := 7.5 \cdot \frac{kW}{m^2}$ rated input of generator

$PowerDen(v) := 0.5 \cdot Cp_{nom} \cdot \rho \cdot v^3$ Power density available with nominal Cp.

$$PowerDenCon(v) := \begin{cases} 0 \cdot \frac{kW}{m^2} & \text{if } v < V_{cutin} \\ Powerin_{max} & \text{if } PowerDen(v) > Powerin_{max} \\ 0 \cdot \frac{kW}{m^2} & \text{if } v > V_{cutout} \\ PowerDen(v) & \text{otherwise} \end{cases}$$

Piece-wise continuous function defined to implement the cut-in, cut-out, and rated power constraints.

$$CpV(v) := \frac{PowerDenCon(v) \cdot Cp_{nom}}{PowerDen(v)}$$

$Energy(v) := PowerDen(v)\, h(v,k,c) \cdot 8760 \cdot hr$ Energy density available with nominal Cp.

$EnergyCon(v) := PowerDenCon(v) \cdot h(v,k,c) \cdot 8760 \cdot hr$ Energy density with controls.

$$EnergyCon := \int_{0 \cdot \frac{m}{sec}}^{1000 \cdot \frac{m}{sec}} EnergyCon(v) \cdot \frac{1}{yr}\, dv$$

Energy extracted per year per m^2 for system with controls.

$$EnergyCon = 1.171 \times 10^4 \, kW \cdot \frac{hr}{yr \cdot m^2}$$

$$Energy_{max} := \int_{0 \cdot \frac{m}{sec}}^{1000 \cdot \frac{m}{sec}} Energy(v) \cdot \frac{1}{yr}\, dv$$

Energy extracted per year per m^2 with no controls.

$$Energy_{max} = 1.811 \times 10^4 \, kW \cdot \frac{hr}{yr \cdot m^2}$$

$$CaptureRatio := \frac{EnergyCon}{Energy_{max}}$$

$CaptureRatio = 0.647$

Figure 4.15 Mathcad worksheet for Example 4.3.

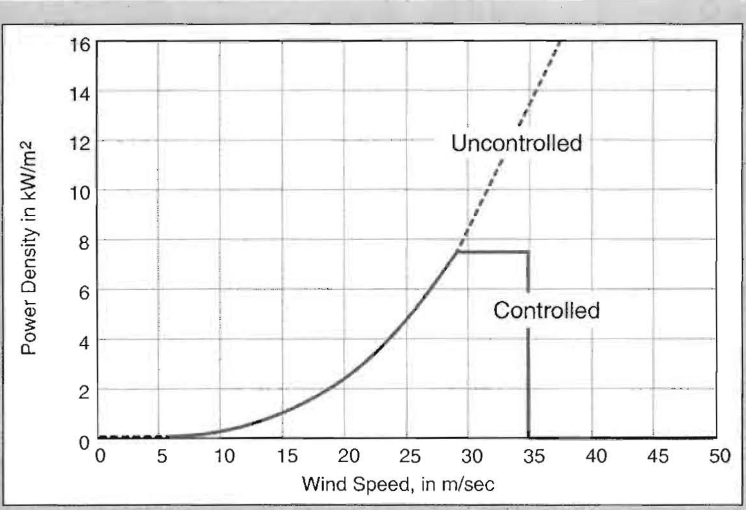


Figure 4.16 Power densities as a function of wind speed and constraints.

variable, $v := 0.50 \cdot \text{m/sec}$. This range variable specifies that the speed, v, is to "range" from 0 to 50 m/sec. Any time v is specified, all of the values in the range are automatically included. The power density for the speed range specified is

$$\text{PowerDen}(v) = 0.5 \cdot Cp_{\text{nom}} \rho v^3 \tag{4-25}$$

The PowerDenCon(v) function is a piecewise continuous function in Mathcad that calculates the power density subject to the cut-in speed, the cut-out speed, and the rated input power of the generator. The first line sets the power density to zero for wind speeds less than the cut-in speed; the second line constrains the power extracted not to exceed the generator input power. The third line implements the cut-out speed constraint, and the last line ensures the constant-Cp regime results. Figure 4.16 presents the power densities for the controlled (constrained) and uncontrolled conditions. The power coefficient, Cp, can be expressed as

$$Cp(v) = \frac{\text{PowerDenCon}(v)}{\text{PowerDen}(v)} \cdot Cp_{\text{nom}} \tag{4-26}$$

and is graphically illustrated in Figure 4.17. The Cp is zero until the cut-in speed is reached and zero for speeds greater than the cut-out speed. The constant-Cp regime is present as is the variable-Cp regime.

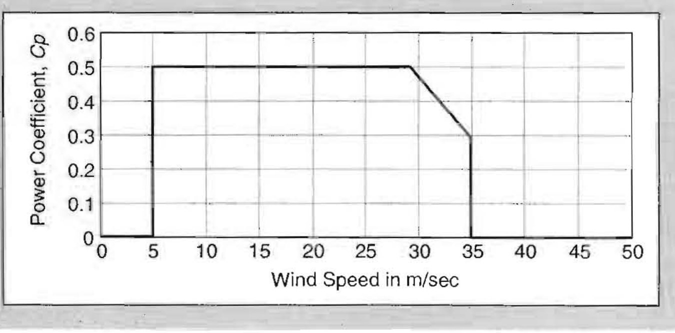


Figure 4.17 Cp as function of wind speed.

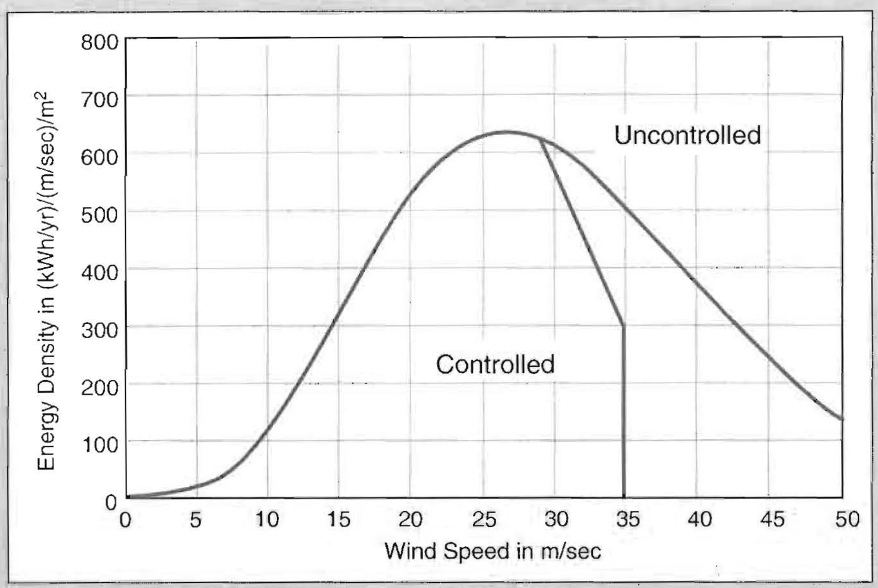
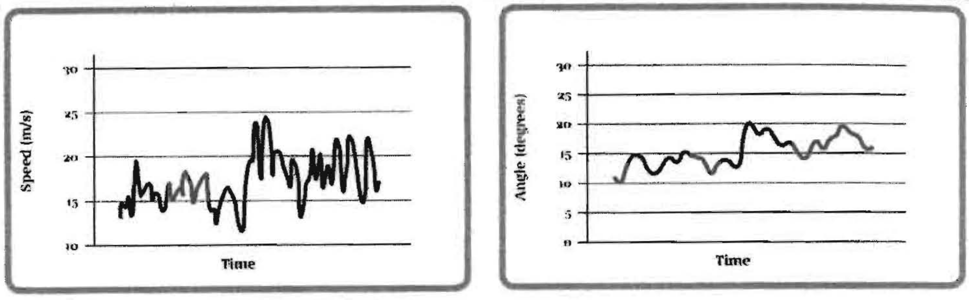


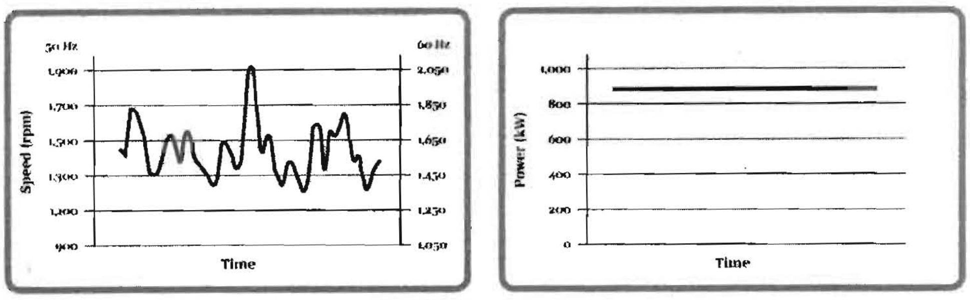
Figure 4.18 Energy densities as a function of wind speed.

The power densities multiplied by the wind speed probability distribution, $h(v, k, c)$, and the number of hours in a year, 8760, yield the energy densities. Figure 4.18 is a representation of the energy densities for the controlled and uncontrolled conditions. This figure is perhaps the most revealing in the solution. The effects of the cut-out speed and the generator input restriction are quite evident in the presentation. Since the areas under the curve represent the total energy extracted, the effects of the control constraints are evident, especially at the higher wind speeds.

The energy densities integrated over all the speeds yield the total energy extracted. For the case of no control, the total energy extracted is 18,110 kWh/yr/m^2, the same result as in Example 4.2. The actual energy extracted corresponds to the case with controls implemented. The actual energy extracted is 11,710 kWh/yr/m^2. The capture ratio is defined as the actual energy extracted divided by the maximum possible energy (no controls) for a given wind speed distribution. For the conditions of this problem, the capture ratio is 0.647; that is, 65 percent of the available energy can be extracted. The only ways to significantly change the capture ratio are to increase the rated input power of the generator and to increase the cut-out speed. For the stated conditions of this example, neither of these strategies is appropriate. Increasing the rated input power would be more expensive and would result in more generator operation at less than the rated input. For example, increasing the rated input of the generator to 10.5 kW/m^2 would result in a capture ratio of 0.682, an increase of only 5.5 percent in actual energy extracted. Increasing the cut-out speed to a value much greater than 35 m/sec would require an enhancement of the structural integrity of the tower, nacelle, and blades.



(a) Wind speed variation (b) Pitch angle variation



(c) Generator speed variation (d) Generator output

Figure 4.19 Vestas V-52 850-kW wind turbine generator output response to wind speed variations (Vestas website).

Vestas Wind Systems, a leading manufacturer of large (MW range) wind turbines, presents an interesting demonstration of the effectiveness of their control strategy for a Vestas V-52 850-kW wind turbine. Figure 4.19 illustrates the response of the generator output for a V-52 wind turbine subject to variations in wind speed.

The random nature of the wind speed variation is shown in Figure 4.19(a). The purpose of the control protocol is to maintain the generator output at a constant value, 850 kW. To do this the blade pitch and the generator speed are modulated. Figures 4.19(b) and (c) illustrate the required modulations in pitch angle and generator speed. The generator output is tracked in Figure 4.19(d). The success of the control protocol in maintaining a constant generator output is evident.

Thus far, this section has addressed the operation of a single wind turbine. However, wind turbine "farms," employing arrays of turbines, are becoming common. For wind turbines employed in arrays, the recommended space is 2–4 rotor diameters facing the prevailing wind and 8–12 rotor diameters parallel to the wind. For more than a single row of wind turbines in an array, the turbine locations in the succeeding rows are staggered. Figure 4.20 provides a schematic illustration of the recommended spacing of wind turbines on wind farms.

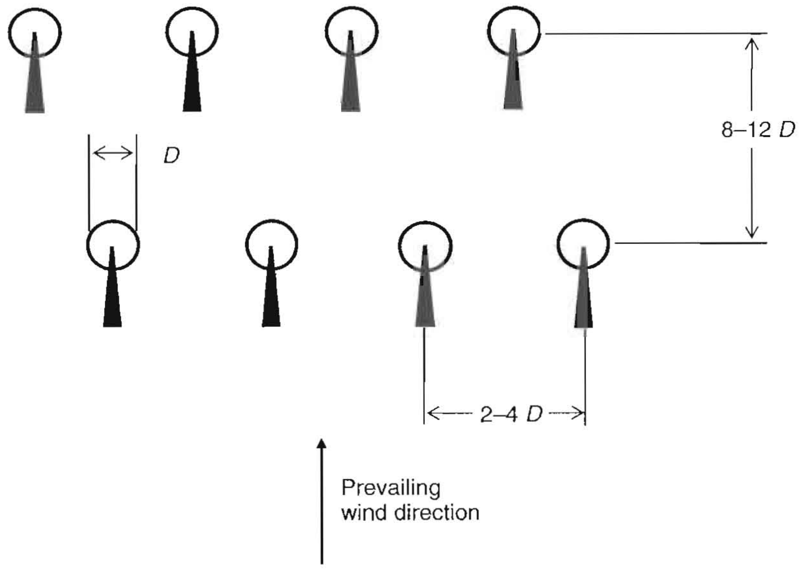


Figure 4.20 Wind turbine arrangement for wind farms.

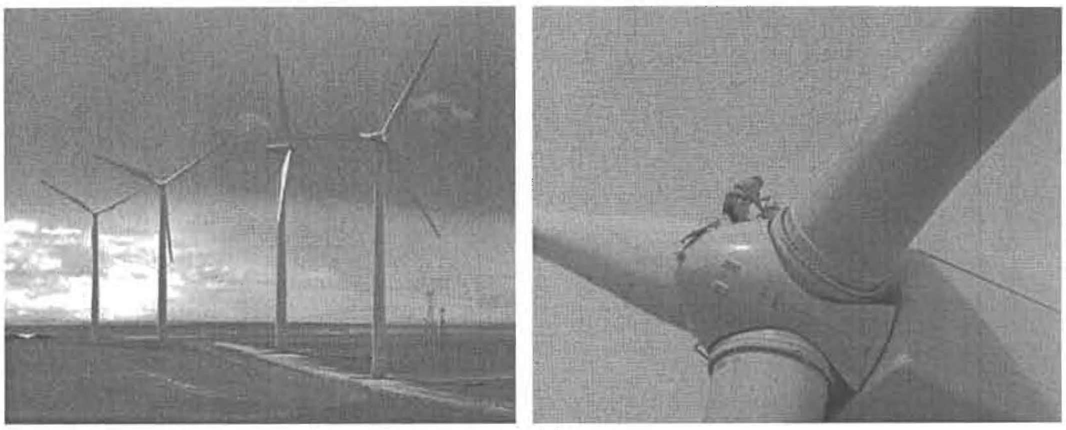
4.5 COMMERCIAL WIND TURBINE EXAMPLES

This section contains some information and technical data on a sampling of wind turbines that are commercially available. The American Wind Energy Association provides a list of U.S. manufacturers of small wind turbines at www.awea.org. A list of manufacturers worldwide for large wind turbines is available at www.ecobusinesslinks.com. The information in this section was obtained from the various company websites and literature and from the NREL database. Additional technical data and price information can be obtained by contacting the individual companies. Examples included herein are for a GE Energy 1.5-MW wind turbine, a Vestas V52 850-kW wind turbine, and a Bergey 10-kW Excel wind turbine. Almost all of the companies included in the small and large wind turbine manufacturers list have websites.

GE Energy 1.5 MW

GE Energy (www.gepower.com) manufactures large wind turbines with nominal outputs of 1.5 MW, 2.5 MW, 3.0 MW, and 3.6 MW. The family of 1.5 MW devices will be examined here. Figure 4.21 contains pictures of GE Energy 1.5 MW wind turbines. Figure 4.21(a) shows the 1.5-MW turbines in a wind farm arrangement. Figure 4.21(b), a view of the nacelle and blade arrangement with a maintenance person, is included because it conveys an idea of the size of a 1.5-MW wind turbine.

The technical information for the turbine is presented in Table 4.2, and the power curve is shown in Figure 4.22.



(a) (b)

Figure 4.21 GE Energy 1.5-MW wind turbine (NREL). (a) Wind farm example. (b) Nacelle with person.

TABLE 4.2 GE Power 1.5-MW specifications

	1.5s	1.5se	1.5sl	1.5sle	1.5xle
Rated capacity (kW)	1500	1500	1500	1500	1500
Cut-in speed (m/sec)	4	4	3.5	3.5	3.5
Cut-out speed (m/sec)	25	25	20	25	20
Rated wind speed (m/sec)	13	13	14	14	12.5
Rotor diameter (m)	70.5	70.5	77	77	82.5
Swept area m^2	3904	3904	4657	4657	5346
Rotor speed (RPM)	12–22.2	12–22.2	11–20.4	11–20.4	10.1–18.7

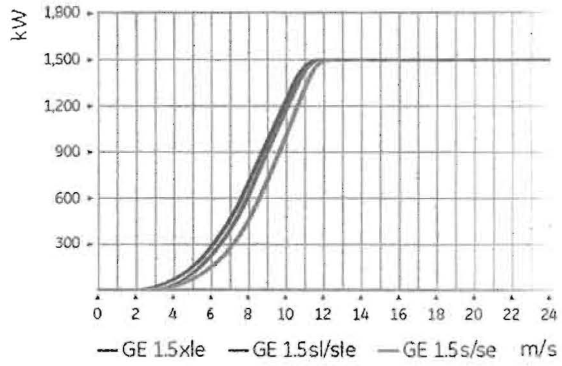


Figure 4.22 Power versus wind speed for a GE 1.5-MW wind turbine (GE Energy).

Figure 4.23 Vestas V52 850-kW wind turbine (Vestas).

Vestas V52 850 kW

The Vestas Wind Systems V52 has a rated output of 850 kW and is a popular wind turbine worldwide. The Vestas website address is www.vestas.com. Vestas reported that in 2005, the company averaged more than 50 installations per week for the year. The effectiveness of the control system for the V52 was discussed in a previous section. Technical information on the V52 850-kW wind turbine is presented in Table 4.3, and a photograph is provided in Figure 4.23. The power output–wind speed performance characteristics are contained in Figure 4.24. The performance characteristics in the figure are parameterized in terms of the sound level.

TABLE 4.3 Vestas V52 850-kW specifications

Rated capacity (kW)	850
Cut-in speed (m/sec)	4
Cut-out speed (m/sec)	25
Rated wind speed (m/sec)	16
Rotor diameter (m)	52
Swept area m^2	2124
Rotor speed (RPM)	14–31.4

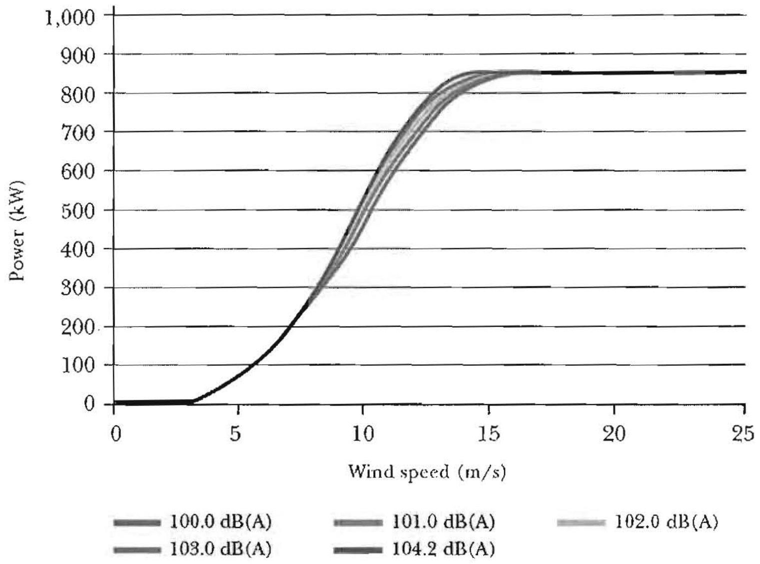


Figure 4.24 Power versus wind speed for a Vestas V52 850-kW wind turbine (Vestas).

Bergey 10-kW Excel

The previous examples were for relatively large wind turbines used for commercial power generation. The Bergey Wind Power Company (www.bergey.com) manufactures small wind turbines suitable for use in residential and small commercial applications.

Technical information on the Bergey 10-kW Excel wind turbine is presented in Table 4.4, and a photograph is shown in Figure 4.25. The power output–wind speed performance characteristics are given in Figure 4.26.

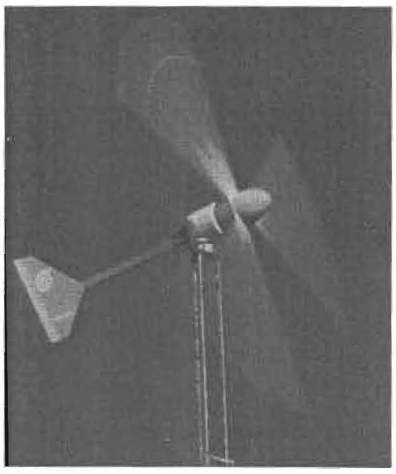


Figure 4.25 Bergey 10-kW Excel wind turbine (Bergey).

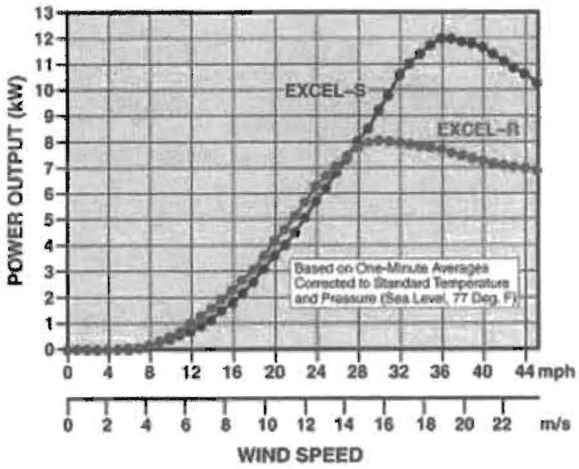


Figure 4.26 Power versus wind speed for a Bergey 10-kW Excel wind turbine (Bergey).

TABLE 4.4 Bergey 10-kW Excel specifications

Rated capacity (kW)	10
Cut-in speed (m/sec)	3.1
Cut-out speed (m/sec)	None (furled at 15.6 m/sec)
Rated wind speed (m/sec)	13.8
Rotor diameter (m)	6.7
Swept area m^2	35.3
Rotor speed (RPM)	14–31.4

Prices for a complete system, including a voltage regulator and a line-commutated inverter, ranged in 2006 from $20,000 to $25,000, depending on options. The Excel is most often installed on a guyed lattice tower, which is available in heights of 18 m (60 ft) to 37 m (120 ft) with prices ranging from $6200 to $9200. Thus, the cost ranges from $2600/kW to $3400/kW, again depending on the options chosen.

The three examples presented here are a relatively small sample of commercially available wind turbines. No recommendation or endorsement of the manufacturer is implied by the inclusion of any of the examples in this section; they are merely presented as samples of commercially available wind turbines.

4.6 CLOSURE

This chapter has explored wind power by developing the operating principles for wind turbines, exploring how the wind speed distribution can be used, addressing how a wind turbine must be controlled, and examining some commercially available wind turbines.

REVIEW QUESTIONS

1. Why is wind speed such an important parameter in determining the power available from the wind?
2. What is the power coefficient?
3. What is the Betz limit?
4. Explain the important variables in the Weibull distribution.
5. Discuss the operating protocol for a wind turbine.
6. Why isn't a wind turbine operated continuously at the maximum power coefficient?
7. Would you recommend constructing a wind turbine at your location? Explain.

EXERCISES

1. A 15-m/sec wind at 101.3 kPa and 20°C enters a two-bladed wind turbine with a diameter of 15 m. Calculate the following:
   (a) The power of the incoming wind
   (b) The theoretical maximum power that could be extracted
   (c) A reasonable attainable turbine power
   (d) The speed in RPM required for part (c)
   (e) The torque for part (c)
2. Compute the power coefficients for a Vestas V52 850-kW (Table 4.3) wind turbine for wind speeds of 10 m/sec and 15 m/sec. What are the advance ratios for these wind speeds? How do the power coefficients compare with the expected values (Figure 4.6)?
3. A General Electric 1.5se wind turbine is used for this exercise. Information on the GE 1.5se is provided in Table 4.2.
   (a) Estimate the power coefficient at the rated speed and at 10 m/sec
   (b) Explain the importance of the result of part (a)
   (c) Estimate the kWh production of a GE 1.5se device in a wind distribution with $c = 10$ m/sec and $k = 2$. Show plots similar to those examined in the chapter
4. A 27-mph wind at 14.65 psia and 70°F enters a wind turbine with a 1000-ft^2 cross-sectional area. Calculate:
   (a) The power of the incoming wind
   (b) The theoretical maximum power that could be extracted
   (c) A reasonable attainable turbine power
   (d) The torque for part (c)

5. Consider two cases: (1) a constant wind velocity twice the mean wind velocity and operating half the time, and (2) a constant wind velocity three times the same mean velocity operating one-third of the time. At all other times the wind velocity is zero. Determine for each case the ratio of the total energy available from the wind to the total wind energy available at the mean velocity continuously.

6. A 15-ft diameter wind turbine operates in a 25 ft/sec wind at 1 atm and 60°F. The turbine is used to pump 60°F water from a 30-ft deep well. How much water (in cubic feet per second) can be pumped if the overall efficiency of the wind-turbine-pump system is 0.25?

7. A wind turbine–generator is designed to attain full-load capacity with a wind velocity of 48 km/h. The rotor diameter is 50 m. If the power coefficient is 0.48 and generator efficiency is 0.85, calculate the rated output for 1 atm and 22°C.

8. The U.S. Department of Energy constructed a Darrieus vertical-axis wind turbine in Sandia, New Mexico. The machine was 60 ft tall and 30 ft in diameter, and swept an area of 1200 ft^2. Estimate the power this device can produce at a wind speed of 20 mph.

9. An early NASA/DOE wind turbine consisted of a 125-ft diameter, two-bladed, horizontal-axis rotor. Maximum power was achieved at a wind speed of 19 mph. For these conditions estimate:
   (a) The power generated in kW
   (b) The rotor speed in RPM
   (c) The velocity downstream of the rotor (V_o)

10. A wind turbine with a rotor diameter of 40 m has a power coefficient of 0.30 in an 8-m/sec wind. The density is 1.2 kg/m^3. The turbine is to be used in a wind farm that is to serve a community of 100,000 (average family size of 4). Each house will require 3 kW. The wind farm will have a turbine spacing of 2.4 rotor diameters perpendicular to the wind and 8 rotor diameters parallel to the wind. The wind farm will have 10 turbines perpendicular to the wind.
    (a) Estimate the power production from one turbine.
    (b) How many turbines will be required in the wind farm for the community?
    (c) Estimate the dimensions of the wind farm.
    (d) How many acres will be required for the wind farm?
    (e) If the average house is on a 0.25-acre lot, how large will the wind farm be in comparison to the community?
    (f) What does this problem imply about wind power feasibility in an urban setting?

11. What is the power coefficient for the Vestas V52 850-kW wind turbine at the rated conditions?

12. What is the power coefficient for the Bergey Excel wind turbine at the rated conditions?

13. The Utopia Wind Turbine Company advertises that its two-bladed, 20-m diameter wind turbine–generator will produce 600 kW in a 15-m/sec wind. The air density is 1.18 kg/m^3. Do you believe its claim? Explain.
14. Repeat Example 4.3 using wind turbine characteristics from a manufacturer's website. If available, use a wind distribution consistent with your location.

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