Wind energy, like solar energy, has captured much media attention recently. Indeed, wind energy has exhibited the most rapid growth of all renewable energy sources the last few years. As shown in Figure 1.9, wind energy accounted for 4 percent of the renewable energy used in the United States in 2007. Wind energy is also an increasingly important part of the energy mix in western Europe. The purposes of this chapter are to develop the fundamental principles of wind energy and to impart a quantitative understanding of wind energy.

Devices to harvest wind energy are available in many different configurations. A number of possible configurations are illustrated in Figures 4.1 and 4.2. Fundamental designations of a wind energy device include the horizontal-axis wind turbine (HAWT) shown in Figure 4.1 and the vertical-axis wind turbine (VAWT) in Figure 4.2. The designation depends simply on the axis of rotation; HAWT devices rotate in the horizontal plane and VAWT devices rotate in the vertical plane. HAWTs are more common than VAWTs, but horizontal devices must have a mechanism—a yaw control—to keep them pointed into the wind. VAWTs, on the other hand, do not need a yaw control.

Many of the wind energy devices illustrated in Figures 4.1 and 4.2 are speculative; they have little or no demonstrated functionality and are not available commercially. The most common configurations for the HAWT are the two- and three-bladed turbines, the windmill, and the sail wing. The two- and three-bladed wind turbines can be either upwind or downwind, with upwind being the most common. In terms of installed kilowatts, the total capacity of the HAWTs greatly exceeds that of the VAWTs. Figure 4.3 presents photographs of horizontal- and vertical-axis wind turbines; Figure 4.3(a) shows a large HAWT suitable for commercial power generation, and the turbine in Figure 4.3(b) is a much smaller device, suitable for a residence. A detailed examination of the characteristics of commercially available horizontal wind turbines will be provided later in this chapter.
4.1 Introduction

Indeed, sources of purposes of to impart configurations. and 4.2. HAWT in devices. HAWTs anism—a hand, do specula- available

Figure 4.1 Horizontal-axis wind turbine taxonomy (Kreith and West 1997).
Chapter 4 Wind Energy

**Figure 4.2 Vertical-axis wind turbine taxonomy (Kreith and West 1997).**

- **Primarily Drag-Type**
  - Savonius
  - Multi-Balanced
  - Plates
  - Cupped

- **Primarily Lift-Type**
  - Darrieus
  - Delta Darrieus
  - Girouette
  - Turbine

- **Combinations**
  - Savonius-Darrieus
  - Split Savonius
  - Magneto
  - Aeolli

- **Others**
  - Deflector
  - Sunlight
  - Venturi
  - Confined Vortex
4.2 Fundamental Concepts

The Savonius and the Darrieus are the most common versions of the VAWT, with most of the remaining vertical configurations either in the experimental/developmental or speculative stage. However, even the Savonius and the Darrieus configurations have not been extensively commercialized. VAWTs are generally classified as drag or lift devices, based on their operating principle. The Savonius rotor is a drag device since the drag of the wind on the "cups" generates the torque on the axis. The Darrieus is classified as a lift device since the shaft torque results primarily from lift on the blades. One indication of the dominance of the horizontal-configuration wind turbines is that on the National Renewable Energy Laboratory (NREL) website, only one photograph (depicting a Darrieus) of a vertical-axis wind turbine is presented in the renewable-energy photographic section. Figure 4.3(c), reproduced from the NREL website, is a photograph of a Darrieus wind turbine, a FloWind Corporation 17 EHD, taken in 1995.

4.2 FUNDAMENTAL CONCEPTS

As a prelude to discussing wind turbine operation, some details on the components of a wind turbine are needed. Figure 4.4 is a schematic illustrating the important parts of a typical HAWT. The tower is mounted to a base, and on top of the tower is the nacelle, which contains the gearbox, controls, and generator subsystems. The rotor is attached to the gearbox and generator by a shaft. The rotor diameter and the swept area of the blades are indicated, as is the hub height.

The power available from a wind of speed $V$ with mass flow rate $\dot{m}$ sweeping an area $A$ is

$$\text{Power} = \frac{1}{2} \dot{m} V^2 \quad (4-1)$$
But the mass flow rate is $m = \rho AV$, so the power available becomes

$$\text{Power} = \frac{1}{2} \rho AV^3$$  \hspace{1cm} (4-2)

Equation (4-2) is very important, as it demonstrates that the power available from the wind is proportional to the area swept by a wind turbine and the cube of the wind speed. With a cubic power relationship, it is not surprising that the ideal location for wind turbines is where wind speeds are high. Wind resource information will be presented in Section 4.3. The fundamental question relating to Eq. (4-2) is, how much of the available power can be extracted from the wind? The analysis of Betz is the traditional approach used to answer this question.

The Betz analysis uses an actuator disk approach. In actuator theory, all energy transfer takes place in the plane of the actuator, and only energy transformation takes place upstream and downstream of the actuator. Actuator disk analysis is illustrated in Figure 4.5. The figure depicts three distributions: pressure, velocity, and cross-sectional...
flow area. Each will be examined in turn. The actuator disk is represented by the blades in the pressure and velocity distributions and by a dotted line in the cross-sectional area distribution. The actuator area, the area swept by the blades, is \( A = \frac{\pi}{4} D^2 \), where \( D \) is the rotor diameter. Far upstream from the actuator, at the point denoted as (e), the velocity is the wind velocity—c in this analysis. At the actuator disk, the velocity has been reduced to \( c(1 - a) \), and at the outlet, indicated as (o), the velocity is \( c(1 - b) \). As the velocity is reduced from c to \( c(1 - a) \) upstream of the actuator, the static pressure increases from \( P_2 \) to \( P_1 \).

When energy is extracted from the flow at the actuator, the pressure discontinuously

---

Figure 4.5 Actuator disk schematic with pressure, velocity, and area distributions illustrated.
decreases to $P_2$. Downstream of the actuator, as the velocity continues to decrease, the pressure increases from $P_2$ to $P_o$. Far downstream of the actuator, atmospheric pressure is recovered, so that $P_o = P_e$, the atmospheric pressure. Since the velocity decreases from the entrance to the outlet, the cross-sectional flow area must increase.

Cast in terms of the variables in the figure, the power available, Eq. (4-2), becomes

$$\text{Power available} = \frac{1}{2} \rho A c^3 \tag{4-3}$$

Upstream and downstream of the actuator, the Bernoulli equation applies and can be written as

$$P_e + \frac{1}{2} \rho c^2 = P_1 + \frac{1}{2} \rho c^2 (1 - a)^2$$
$$P_2 + \frac{1}{2} \rho c^2 (1 - a)^2 = P_0 + \frac{1}{2} \rho c^2 (1 - b)^2 \tag{4-4}$$

Solving for the pressure differences yields

$$\frac{P_e - P_1}{\rho} = \frac{1}{2} c^2 [1 - (1 - a)^2 - 1]$$
$$\frac{P_2 - P_0}{\rho} = \frac{1}{2} c^2 [(1 - b)^2 - (1 - a)^2] \tag{4-5}$$

Adding the two expressions and using $P_e = P_o$, the expression for the change in pressure across the actuator becomes

$$\frac{P_e - P_1 + P_2 - P_o}{\rho} = \frac{P_2 - P_1}{\rho} = \frac{1}{2} c^2 [(1 - b)^2 - (1 - a)^2 + (1 - a)^2 - 1]$$
$$= \frac{1}{2} c^2 [1 - (1 - b)^2 - 1] \tag{4-6}$$

The change in pressure, $P_1 - P_2$, is thus

$$P_1 - P_2 = \frac{1}{2} \rho c^2 [1 - (1 - b)^2] \tag{4-7}$$

The axial thrust across the actuator is

$$T = (P_1 - P_2) A - \frac{1}{2} \rho A c^2 [1 - (1 - b)^2] \tag{4-8}$$

But the axial thrust is equal to the change in axial momentum,

$$T = m [c - c(1 - b)] = \rho A c (1 - a) [c - c(1 - b)] = \rho A c^2 (1 - a)b \tag{4-9}$$

Equating Eqs. (4-8) and (4-9) and solving yields

$$\frac{1}{2} \rho A c^2 [1 - (1 - b)^2] = \rho A c^2 (1 - a)b$$
$$a = \frac{b}{2} \tag{4-10}$$
4.2 Fundamental Concepts

Equation (4-10) demonstrates that the change in velocity upstream of the actuator plane is equal to the change in velocity downstream of the actuator plane. In many actuator disk analyses, the velocity at the actuator plane is taken as the average of the far upstream velocity, \(c\), and far downstream velocity, \(c(1-b)\)—an equivalent result.

The rate of kinetic energy change, expressed in terms of \(c\) and \(a\), is

\[
E_k = \frac{1}{2} \rho Ac(1 - a)[c^2 - c^2(1 - b)^2] = \frac{1}{2} \rho Ac(1 - a)[c^2 - c^2(1 - 2a)^2] \tag{4-11}
\]

The maximum rate of change of kinetic energy, the maximum power extracted, will occur when the derivative of \(E_k\) with respect to \(a\) is set equal to zero, or

\[
\frac{dE_k}{da} = 0 = (1)(1 - a)^2 + 2a(1 - a)(-1)
\]

Then, using \(a = \frac{1}{3}\) in Eq. (4-11) yields the expression for maximum power extracted:

\[
\text{Power}_{\text{max}} = \frac{8}{27} \rho Ac^3 \tag{4-13}
\]

The power coefficient is defined as the power extracted divided by the available power of the wind stream or

\[
C_p = \frac{\text{Power}_{\text{ext}}}{\frac{1}{2} \rho AV_{\text{wind}}^3} \tag{4-14}
\]

The maximum value of the power coefficient, the Betz limit, then becomes

\[
C_{p_{\text{max}}} = \frac{8}{27} \frac{\rho Ac^3}{\frac{1}{2} \rho Ac^3} = \frac{16}{27} = 0.5926 \tag{4-15}
\]

The Betz limit represents the maximum value of the power coefficient and defines the maximum power that can be extracted from a given wind stream. The power coefficient is perhaps the most important single metric used in characterizing a wind turbine. Equation (4-14) is usually rewritten to specify the power extracted in terms of the power coefficient:

\[
\text{Power}_{\text{ext}} = \frac{1}{2} C_{p_{\text{max}}} AV_{\text{wind}}^3 \tag{4-16}
\]

If the power coefficient is the most important metric for wind turbines, then Figure 4.6 is arguably the most referenced figure in wind turbine engineering. This figure presents the expected range of power coefficients for well-designed wind
turbines of different configurations. The abscissa, the x axis, is the advance ratio, which is the rotor tip speed divided by the wind speed, or

\[ \Omega = \frac{r \Omega}{V_{wind}} \]  

(4-17)

where \( r \) is the rotor radius and \( \omega \) is the rotor rotational rate.

The Betz limit is shown in the figure. The Glauert ideal \( C_p \) is the result of a more detailed analysis that explicitly involves the advance ratio. Lying below and to the right of the Glauert ideal are the expected \( C_p \) values for different wind turbine configurations. The American multi-blade and the Dutch windmill have the lowest \( C_p \) values. In many recent publications, the values for the American multi-blade and the Savonius configurations have been interchanged, with the result that the Savonius \( C_p \) values are smaller than those for the American multi-blade. The values presented in Figure 4.6 are correct. As pointed out by Johnson (2001), such an error will take years to correct. In addition to the Savonius and the American multi-blade, typical power coefficient values for the Darrieus and the modern, high-speed two-blade HAWT are presented. The HAWT shows relatively high \( C_p \) values over a rather wide range of advance ratios.
A 27-mph wind at 14.7 psia and a temperature of 70°F enters a two-bladed wind turbine with a diameter of 36 ft. Calculate (a) the power of the incoming wind, (b) the theoretical maximum power that could be extracted, (c) a reasonable value for attainable power, (d) the rotor speed, in RPM, required for part (c), and (e) the torque for part (c).

**Solution**

The power of the incoming wind is defined by Eq. (4-2):

\[
\text{Power} = \frac{1}{2} \rho AV^3
\]

The thermal equation of state is

\[
P = \rho RT
\]

where \( R \) is the gas constant for air and, in English Engineering units, has the value

\[
R = 53.35 \text{ ft lbf} / \text{ lbf R}
\]

The absolute temperature is required for the equation of state:

\[
70°F = (460° + 70°)R = 530°R
\]

Using the thermal equation of state, the density becomes

\[
\rho = \frac{P}{RT} = 14.7 \text{ lbf} / \text{ in}^2 \times 144 \text{ in}^2 / \text{ ft}^2 = 2028 \text{ lbf} / \text{ ft}^2 = 0.075 \text{ lbf} / \text{ ft}^3
\]

The area swept by the rotor is

\[
A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (36 \text{ ft})^2 = 1018 \text{ ft}^2
\]

The power available from the wind is thus

\[
\text{Power available} = \frac{1}{2} \rho AV^3 = \frac{1}{2} \times 0.075 \text{ lbf} / \text{ ft}^3 \times 1018 \text{ ft}^2 \times \left( \frac{27 \text{ mi}}{h} \times \frac{5280 \text{ ft}}{\text{mi}} \times \frac{h}{3600 \text{ sec}} \right)^3
\]

\[
= 2.37 \times 10^6 \text{ lbf ft sec}^2 / \text{sec}^3 = 73,693 \text{ ft lbf sec} / \sec = 321.74 \text{ ft lbm / sec hp}
\]

\[
= 134 \text{ hp} = 100 \text{ kW}
\]

The power available is 134 hp or 100 kW. The theoretical maximum power that could be extracted is the Betz limit or

\[
\text{Power}_{\text{max}} = 0.5926 \times \text{Power}_{\text{available}} = 0.5926 \times 134 \text{ hp} = 79.4 \text{ hp} = 59.3 \text{ kW}
\]

For a two-bladed HAWT, Figure 4.6 suggests \( C_p = 0.45 \) for an advance ratio, \( \Omega = \omega_0 / V_{\text{oct}} \), of 11. A reasonable value for attainable power from such a wind turbine is

\[
\text{Power}_{\text{act}} = 0.45 \times 134 \text{ hp} = 60.3 \text{ hp} = 45 \text{ kW}
\]
Using the definition of advance ratio, the rotor speed can be cast as

$$\omega = \frac{\Omega \cdot V_{\text{wind}}}{0.5D} = \frac{11 \cdot 39.6}{0.5 \cdot 36} = \frac{24.2}{\frac{1}{\text{sec}}} = 24.2 \text{ Hz}$$

$$= 24.2 \frac{1}{\text{sec}} \cdot \frac{\text{Rev}}{60 \text{ sec}} = 231 \text{ RPM}$$

In order to attain a $C_p$ value of 0.45, a speed of 231 RPM is needed. From Eq. (2-11), the torque becomes

$$\tau = \frac{\text{Power}_{\text{act}}}{\omega} = \frac{60.3 \text{ hp}}{24.2 \text{ Hz}} = 1370 \text{ ft lbf}$$

### 4.3 WIND ENERGY RESOURCES

The previous section treated wind turbine performance in terms of a specified wind speed and pointed out the cubic relationship between speed and power from the wind. Since the wind is variable in terms of speed, direction, and altitude, the results of the previous section need to be extended to account for site-specific wind conditions. Extensive wind data are available at the NREL website, www.nrel.gov. The primary wind resources document is the *Wind Energy Resource Atlas of the United States*. This atlas contains annual average wind resource data as well as regional summaries and extensive explanations and references for the statistics of wind. A unique feature is an assessment of the “certainty” of the wind data. The wind data are rated from 1 (lowest degree) to 4 (the highest degree of certainty).

The wind power density distribution across the United States is very useful for first-order evaluations of candidate locations to determine if wind energy harvesting is feasible. The wind power density is the available average wind power per $m^2$ of wind turbine area. The basis of the distribution classification are the wind power density classes. The wind power density is measured from class 1 (lowest) to class 7 (highest) and is specified at nominal 10-m and 50-m elevations. Table 4.1 presents the wind power density classes and their associated wind speeds.

<table>
<thead>
<tr>
<th>TABLE 4.1 Wind power density classes</th>
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<tr>
<td><strong>Wind Power Class</strong></td>
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Each wind class is bounded by a minimum wind power density and a maximum wind power density. The wind speeds in m/sec and mph corresponding to the minimum and maximum wind power density values are also indicated in Table 4.1. For example, class 4 is defined as a wind power density between 200 W/m² and 250 W/m² or a wind speed range between 5.6 m/sec and 6.0 m/sec, all at a 10-m elevation. At a 50-m elevation, the wind power classes are defined using different power density values than at a 10-m elevation.

The most useful summary figure from the *Wind Energy Resource Atlas of the United States* is the annual average wind power density distribution, reproduced as Figure 4.7. In this figure, the darker the color, the higher the wind class. Much of the Midwest is assessed as wind class 3 or 4, with isolated portions of the mountain west containing embedded regions of class 5 or 6. Except for areas near the coast, the Southeast is not generally suitable for wind energy production. NREL states that wind energy is appropriate only for wind class 3 and above.

The wind power density as illustrated in Figure 4.7 accounts for the yearly variation in wind speed at given locations. To understand the basis of development of information such as that in Figure 4.7, the statistics of wind energy need to be explored.

The probability of occurrence of a given wind speed is expressed by the Weibull distribution,

\[ h(v, k, c) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} \exp\left[-\left(\frac{v}{c}\right)^k\right] \]  

(4.18)

In the Weibull distribution, \( c \) is the scale parameter and \( k \) is the shape parameter. The shape parameter controls the shape of the distribution; the larger the shape factor, the closer the distribution comes to being Gaussian. The scale parameter controls the value of the mode (the most probable speed). The larger the scale parameter, the higher the mode, and the lower the probability of a given speed less than the mode. The shape parameter is dimensionless, and the scale parameter must have the same units as the speed. For wind distributions, the shape parameter value is usually near 2 (Patel 2005).

Figure 4.8 presents the probability distribution, expressed in percentages, for shape parameters of 1, 2, and 3 for a constant value of \( c = 10 \) mph. The shape parameter of 1 yields an exponential distribution that has the highest percentage of hours at zero speed; the shape parameter of 2 results in a distribution skewed toward a higher percentage of occurrence at the lower speeds. The shape parameter of 3 yields a more symmetrical distribution and starts to resemble the Gaussian. The shape factor of \( k = 2 \) provides a generally acceptable match for the wind speed distribution at most sites.

Figure 4.9 illustrates the probability distribution, expressed in hours per year per mph, for scale parameter values of 10 mph, 15 mph, and 20 mph for a constant value of \( k = 2 \). The smaller the value of the scale parameter, the more hours at lower wind speeds. As the value of \( c \) increases, the mode wind speed values increase and the number of hours per year at wind speed higher than the mode increases.

Figures 4.8 and 4.9 provide a visual interpretation of the behavior of the Weibull distribution. However, the real question is, how does the Weibull distribution relate...