Multiple choice:

1. D
2. C
3. B
4. B
5. E

Explanation, Question 4:

\[ E = \frac{KQ}{d \sqrt{d^2 + (L/2)^2}} \] (from the equation sheet)

\[ \begin{align*}
E_E &= \frac{32 \sqrt{2^2 + 4^2}}{7 \sqrt{7^2 + 24^2}} = \frac{32 \sqrt{1600}}{7 \sqrt{625}} = \frac{32 \cdot 40}{7 \cdot 25} = 7.31
\end{align*} \]
Problem 6:

Superposition principle: a sphere of charge with a cavity can be thought of as a complete sphere of charge, with a smaller sphere (the size of the cavity) on negative charge "embedded" into it.

If the volume charge density of a sphere is \( q \), then its total charge is \( q \cdot \frac{4}{3}\pi R^3 \) (sphere's radius)³.

The electric field \( E \) of a sphere at distances larger than the sphere radius \( (R) \) is identical with that of a point charge, i.e., \( E = \frac{Q}{r^2} \)

and a force acting on a charge \( q \) located at \( r > R \) from the sphere's center is \( F = \frac{KqQ}{r^2} \).
So, one needs to find the "complete sphere" charge:

\[ Q_1 = q \cdot \frac{4}{3} \pi \left(2a\right)^3 = \frac{32}{3} \pi a^3 \]

The charge on the small "embedded" sphere is

\[ Q_2 = -q \cdot \frac{4}{3} \pi \left(a\right)^3 = -\frac{4}{3} \pi a^3 \]

1) 

So, the force acting on \( q \) is

\[ F = K \frac{Q_1 q}{(4a)^2} + K \frac{Q_2 q}{(3a)^2} \]

\[ = \frac{K q}{a^2} \left( \frac{32}{27} \pi \frac{9}{32} a^3 - \frac{4}{3} \pi a^3 \right) \]

\[ = \frac{K q}{a^2} \left( \frac{32}{27} \pi a^3 - \frac{4}{3} \pi a^3 \right) \]

\[ = \frac{14}{27} K \pi \cdot 9 \cdot q \cdot a = \frac{14}{27} q \cdot 9 \cdot a = \frac{7}{2} q \cdot 9 \cdot a = \frac{7 \cdot 9 \cdot a}{27} \frac{9}{27} \pi \epsilon_0 \]

so, if one gets this

\text{Downward.}

2) 

\[ F = K \frac{Q_1 q}{(4a)^2} + K \frac{Q_2 q}{(5a)^2} = \frac{K q}{a^2} \left( \frac{Q_1 + Q_2}{16} \right) \]

\[ = \frac{K q}{a^2} \left( \frac{32}{3} \pi \frac{9}{3} a^3 - \frac{4}{3} \pi a^3 \right) \]

\[ = \frac{K q}{a^2} \left( \frac{32}{3} \pi a^3 - \frac{4}{3} \pi a^3 \right) \]

\[ = \frac{K q}{a^2} \left( \frac{29}{3} \pi a^3 \right) \]

\[ = \frac{46}{75} \frac{1}{4 \pi \epsilon_0} q \cdot 9 \cdot \pi \cdot 9 \cdot a = \frac{23}{150} q \cdot 9 \cdot a \]

\text{This form is OK.}
Problem 7:

Vertical E component: one can use symmetry arguments - once one knows the solution for a point at distance h from the rod's midpoint:

\[ E_{\text{rod}} = \frac{Q}{dVd^2 + (L/2)^2} \]

In the formula sheet:

Using the equation from the sheet, we get

\[ (E_y)_{\text{from two halves}} = K \frac{2Q}{hV^2 + h^2} \]

Therefore, for:

\[ E_y = \frac{1}{2} (E_y)_{\text{from two halves}} \]

This "trick" works only for \( E_y \), not \( E_x \)!

We will do \( E_y \) again by integration, then \( E_x \)
Charge on $dx$ is $dQ = \lambda \cdot dx$; 
\[ dE = \frac{K \cdot dQ}{\sqrt{x^2 + h^2}} \]
\[ = \frac{K \lambda dx}{\sqrt{x^2 + h^2}} \]
This is the contribution of the charge
in an element $dx$ to $E$ field at $P$.

1) Contribution to vertical $E$ component:

\[ dE_y = dE \sin \theta \quad \text{and} \quad \sin \theta = \frac{h}{\sqrt{x^2 + h^2}} \]

So:
\[ dE_y = \frac{K \lambda h \, dx}{\sqrt{x^2 + h^2}} \Rightarrow E_y = K \lambda h \int_{-L}^{x} \frac{dx}{\sqrt{x^2 + h^2}} \]
\[ = K \lambda h \left[ \frac{-L}{h \sqrt{x^2 + h^2}} - 0 \right] \]
\[ = \frac{K \lambda \cdot L}{h \sqrt{x^2 + h^2}} \quad \text{Final res. for $E_y$} \]

2) On the same token:

\[ dE_x = dE \cdot \cos \theta \quad \text{and} \quad \cos \theta = \frac{x}{\sqrt{x^2 + h^2}} \]

Therefore:
\[ dE_x = \frac{K \lambda x \, dx}{\sqrt{x^2 + h^2}} \Rightarrow E_x = K \lambda \int_{-L}^{x} \frac{x \, dx}{\sqrt{x^2 + h^2}} = K \lambda \left[ \frac{1}{\sqrt{x^2 + h^2}} \right]_{-L}^{x} \]
Problem 8:

1) \( F = m \cdot g = 0.01 \text{ kg} \cdot 9.81 \frac{\text{m}}{\text{s}^2} = 0.0981 \text{ N} \)

2) \( F_E = Q \cdot E = 100 \times 10^{-9} \text{ C} \cdot 300000 \text{ N/C} = 3 \cdot 10^{-2} \text{ N} = 0.030 \text{ N} \)

\[ \tan \theta = \frac{Q \cdot E}{m \cdot g} = \frac{0.030 \text{ N}}{0.0981 \text{ N}} = \]

\[ F_2 = \frac{\sqrt{(mg)^2 + (QE)^2}}{mg} \]

\[ \frac{F_2}{F_1} = \frac{\sqrt{(mg)^2 + (QE)^2}}{mg} = \sqrt{1 + \left( \frac{QE}{mg} \right)^2} = \sqrt{1 + \left( \frac{0.030}{0.0981} \right)^2} = 1.0457^* \]

\[ \theta \]

\[ \tan \theta = \frac{Q \cdot E}{m \cdot g} = \frac{0.030 \text{ N}}{0.0981 \text{ N}} = \]

\[ \frac{F_2}{F_1} = \frac{\sqrt{(mg)^2 + (QE)^2}}{mg} = \sqrt{1 + \left( \frac{QE}{mg} \right)^2} = \sqrt{1 + \left( \frac{0.030}{0.0981} \right)^2} = 1.0457^* \]

4) \( T = 2\pi \sqrt{\frac{L}{g}} = 2\pi \sqrt{\frac{mL}{mg}} = 2\pi \sqrt{\frac{mL}{F_{\text{tension}}}} \)

5) \( T_2 = 2\pi \sqrt{\frac{mL}{(mg)^2 + (QE)^2}} = 1.962\text{ s} \pm 1.96\text{ s} \)

\[ T_1 = 2\pi \sqrt{\frac{mL}{mg}} \text{ therefore } \frac{T_2}{T_1} = \sqrt{\frac{mg}{(mg)^2 + (QE)^2}} \]

\[ = \frac{4\sqrt{m^2q^2}}{\sqrt{(mg)^2 + (QE)^2}} = 0.978 \]

*Comment: Rounding to 3 significant digits is not always good at intermediate stages of problem solving - if the results are to be used in next-step calculations.