28.117 Refer to Fig. 28-47. Find $B$ at the center point.

The straight segments make no contribution to the field at the center; the curved segments give, by the Biot-Savart law,

$$
\Delta B = \frac{\mu_0 l}{4\pi} \left[ \frac{a(2\pi - \theta)}{a^2} - \frac{b(2\pi - \theta)}{b^2} \right] = \frac{\mu_0 l(a - b)(2\pi - \theta)}{4\pi ab}
$$

28.118 A phonograph record of radius $R$, which carries a uniformly distributed charge $Q$, is rotating with constant angular speed $\omega$. Show that the magnetic field at the center of the disk is given by $B = (\mu_0 \omega Q)/(2\pi R)$.

In Fig. 28-48, we show the record in clockwise rotation (as seen from above) which is standard for phonograph records. The ring of charge $dq$ between radii $r$ and $r + dr$ constitutes a current $di = (dq)/T$, where $T = (2\pi)/\omega$ is the rotation period of the disk. Because the disk is uniformly charged, we have

$$
dq\frac{Q}{\pi R^2} = \frac{2\pi r \, dr}{R^2}, \quad \text{so that} \quad dq = \frac{2Qr \, dr}{R^2}.
$$

Using the result of Prob. 28.90(b), we find that the ring makes a magnetic field contribution

$$
dB = \frac{\mu_0 \omega}{2} \frac{di}{r} = \frac{\mu_0 \omega}{2} \frac{2Qr \, dr}{2\pi r R^2} = \frac{\mu_0 \omega Q}{2\pi R^2} \, dr.
$$

The net field at the center is then

$$
B = \frac{\mu_0 \omega Q}{2\pi R^2} \int_{R_0}^{R} \, dr = \frac{\mu_0 \omega Q}{2\pi R^2}
$$

For $Q > 0$, the field has the direction $-\mathbf{z}$. 

28.5 MORE COMPLEX GEOMETRIES; AMPÈRE'S LAW

28.119 Calculate the magnetic field of a circular current loop at any point on its axis.

Choosing a coordinate system as in Fig. 28-49, and noting that by symmetry the resultant field at \( P \) must be along \( Z \), we have:

\[
dB_z = (dR) \cos \omega \cos \varphi = \left( \frac{\mu_0 I}{4\pi r^2} dl \right) r \sin \varphi = \frac{\mu_0 I a^2}{4\pi r^2} d\varphi = \frac{\mu_0 I a^2}{4\pi (a^2 + z^2)^{3/2}} d\varphi
\]

(1)

Adding all the elements around the loop, we in effect just add all the \( d\varphi \)'s to get

\[
B_z = B = \frac{\mu_0 I a^2}{2(a^2 + z^2)^{3/2}}
\]

(2)

In particular, at the center of the loop \( (z = 0) \), \( B_{\text{center}} = (\mu_0 I)/(2a) \), the well-known result.

28.120 Two circular coils shown in Fig. 28-50(a) have the same number of turns and carry current in the same sense. They have different diameters but subtend the same angle at \( P \). (a) Which coil makes the larger contribution to the magnetic field at \( P \)? (b) If the smaller coil is midway between \( P \) and the larger one, what is the ratio of the larger contribution at \( P \) to the smaller?

\[\text{(a)}\]

\[\text{(b)}\]

Fig. 28-50

(a) Referring to Fig. 28-50(b) and using Eq. (2) of Prob. 28.120, we find that the magnetic field at point \( P \), due to a coil of \( N \) circular loops at the position shown, is given by

\[
B = N\mu_0 I \frac{k^2}{2(z^2 + k^2)^{3/2}}\hat{z}
\]
But \( k/(z^2 + k^2)^{1/2} = \sin \theta \), so
\[
B = \frac{N A_0 i}{2k} \sin^2 \theta
\]
which shows that the coil with the smaller radius makes the larger contribution.

(b) If \( k_2 = 2k_1 \), then we find
\[
\frac{B_2}{B_1} = \frac{k_2}{k_1} = 2
\]

28.121* A spherical shell of radius \( R \) carries a surface charge of uniform density \( \sigma \) and spins on its axis with frequency \( f \). (a) Section the surface into rings concentric to the axis and show that the current carried by each ring of width \( R \, d\theta \) is \( 2\pi \sigma R^2 \sin \theta \, d\theta \). (b) Show that the spinning charge generates the following field at the center of the sphere: \( B = (4\pi \mu_0 \sigma R^2)/3 \).

(a) The area of the spherical ring at polar angle \( \theta \) is \( dA = 2\pi R^2 \sin \theta \, d\theta \), and the total charge on it is \( \sigma \, dA \). The current in the ring is then \( dI = (\sigma \, dA)/T = (\sigma \, dA) f = 2\pi R^2 \sigma \sin \theta \, d\theta \). (b) To find \( B \) at the center we add the contributions of all rings using Eq. (2) of Prob. 28.119, with \( a = R \sin \theta \) and \( (a^2 + z^2)^{1/2} = R \). The contribution of the ring at polar angle \( \theta \) is \( dB = [\mu_0 dI(R \sin \theta)^2]/2R^2 = \pi \mu_0 \sigma R^2 \sin^2 \theta \, d\theta \). Integrating \( \sin^2 \theta \, d\theta \) from 0 to \( \pi \) yields \( \frac{1}{3} \), so \( B = (4\pi \mu_0 \sigma R^2)/3 \).

28.122 (a) In Fig. 28-51, \( AB \) is a finite length of wire carrying current \( i \). Find the field at \( P \). (b) Deduce the results for an infinite straight wire.

![Fig. 28-51](image)

(a) The Biot–Savart law states that the contribution \( dB \) to the field at \( P \) (due to a wire segment \( dx \) centered at \( x \)) is given by
\[
\frac{dB}{dB} = \frac{\mu_0 i}{4\pi} \frac{dx}{r^2} \times (\hat{x} - \hat{z}a) = -\frac{2\mu_0 i a \, dx}{4\pi r^2}
\]
From the figure, we have \( \tan \theta = -a/x \), so \( x = -a \cot \theta \) and \( dx = (a \csc^2 \theta) \, d\theta \). Furthermore, \( r = a/\sin \theta = a \csc \theta \). Thus Eq. (1) can be rewritten as
\[
\frac{dB}{dB} = -\frac{2\mu_0 i (a^2 \csc^2 \theta) \, d\theta}{4\pi a^3 \csc^3 \theta} = -\frac{2\mu_0 i}{4\pi a} \sin \theta \, d\theta
\]
(2)
The resultant field must be found by integrating over the segments from point \( A \) (where \( \theta = \pi - \alpha_1 \)) to point \( B \) (where \( \theta = \pi - \alpha_2 \)):
\[
B = -\frac{2\mu_0 i}{4\pi a} \int_{\pi - \alpha_1}^{\pi - \alpha_2} \sin \theta \, d\theta = -\frac{2\mu_0 i}{4\pi a} \left[ -\cos \theta \right]_{\pi - \alpha_1}^{\pi - \alpha_2} = -\frac{2\mu_0 i}{4\pi a} \left[ -\cos (\pi - \alpha_2) + \cos (\pi - \alpha_1) \right] = -\frac{2\mu_0 i}{4\pi a} \left( \cos \alpha_2 - \cos \alpha_1 \right)
\]
(3)
since \( \cos (\pi - \alpha) = -\cos \alpha \). Equation (3) is the desired result. (b) In the case of an infinite straight wire, we have \( \alpha_1 = \pi \) and \( \alpha_2 = 0 \), so that \( \cos \alpha_1 = -1 \) and \( \cos \alpha_2 = 1 \). Then \( B = -\frac{2\mu_0 i}{(2\pi a)} \), as expected.

28.123 Use the result of Prob. 28.122 to show that the magnetic field at the center of a square frame of side \( l \) illustrated in Fig. 28-52(a), and carrying current \( i \), is \( [(\mu_0 i)/2\pi l]2\sqrt{2} \).
Figure 28-52(b) shows a square loop of side $l$, with angles labeled as in Fig. 28-51, so that the contribution to the magnetic field at $C$ from the upper side can be evaluated. According to the result of Prob. 28.122, the magnetic field due to the upper side has magnitude

$$B_1 = \frac{\mu_0 i}{4\pi a} (\cos \alpha_2 - \cos \alpha_4) \quad (1)$$

With $a = l/2$, $\alpha_1 = 135^\circ$, and $\alpha_2 = 45^\circ$, Eq. (1) yields

$$B_1 = \frac{\mu_0 i}{4\pi (l/2)} \left[ \frac{\sqrt{2}}{2} - \left( -\frac{\sqrt{2}}{2} \right) \right] = \frac{\mu_0 i \sqrt{2}}{2\pi l} \quad (2)$$

By applying the right-hand rule and considering the symmetry of the arrangement, we conclude that the net magnetic field $B_n$ at $C$ due to the square loop is directed up out of the page and has a magnitude

$$B_n = 4B_1 = \frac{2\sqrt{2} \mu_0 i}{\pi l} \quad (3)$$

**28.124** A length $L$ of wire carrying current $i$ is to be bent into a circle or a square, each of one turn. In which case is $B$ at the center of the figure greater? What is the ratio of $B_s$ (greater) to $B_c$ (smaller)?

- A square of perimeter $L$ has side $l = L/4$, so Eq. (3) of Prob. 28.123 yields

$$B_s = \frac{2\sqrt{2} \mu_0 i}{\pi L} \quad \text{so} \quad B_s = 3.60 \mu_0 i \quad (4)$$

A circle of circumference $L$ has radius $r = L/(2\pi)$. Then,

$$B_c = \frac{\mu_0 i}{2\pi r} = \frac{\mu_0 i}{2[L/(2\pi)]} = \frac{\pi \mu_0 i}{L} = 3.14 \mu_0 i$$

Therefore the field is stronger at the center of the square than it is at the center of the circle (of equal perimeter). The ratio is

$$\frac{B_s}{B_c} = \frac{8\sqrt{2} \mu_0 i}{\pi L} = 8\sqrt{2} = 1.15 \quad (5)$$

**28.125** Show that $B$ at point $P$ on the axis of a solenoid of finite length equals $(\mu_0/2)ni(\cos \alpha_2 - \cos \alpha_1)$, the angles being defined in Fig. 28-53(a) and $n$ being the number of turns per unit length.

- We denote the radius of the solenoid by $k$ and we use the coordinates $x$ and $\alpha$ as indicated in Fig.
28-53(b). From Prob. 28.119 and the superposition principle, we find the on-axis field due to a (flat) circular coil with \(dN\) turns and carrying current \(i\):

\[
BH = \frac{\mu_0 il^2}{2} \frac{dN}{(z^2 + k^2)^{3/2}}
\]

(1)

For a solenoid, \(dN = n\,dz\) since \(n\) is the number of turns per unit length. As can be seen from Fig. 28-53(b), we have \(z = k/\tan \alpha\), so that

\[
dz = \frac{-k\,d\alpha}{\sin^2 \alpha} \quad \text{and} \quad (z^2 + k^2)^{3/2} = k^3(\cos^2 \alpha + 1)^{3/2} = k^3 \csc \alpha \csc \alpha
\]

Since all contributions to the net field at \(P\) are along the same line, we may integrate Eq. (1) to find

\[
B = \int_{\alpha_1}^{\alpha_2} \frac{\mu_0 il^2}{2} \frac{n\,dz}{(z^2 + k^2)^{3/2}} = \frac{\mu_0 il^2}{2} \int_{\alpha_1}^{\alpha_2} \frac{-k\,d\alpha}{\alpha(\csc \alpha \csc \alpha)}
\]

\[
= \frac{\mu_0 il^2}{2} \left[ \frac{-\sin \alpha}{\alpha} \right]_{\alpha_1}^{\alpha_2} = \frac{\mu_0 il^2}{2} \left( \cos \alpha_2^* - \cos \alpha_1^* \right)
\]

(2)

28.126 Refer to Prob. 28.125. What is the value of \(B\) at point \(A\) on the axis of the solenoid just outside the end?

We let \(L\) denote the length of the solenoid. If \(P\) coincides with \(A\), the angles are \(\alpha_1 = \alpha_2 = \pi/2\) and \(\alpha_2 = \tan^{-1}(k/L) = \cos^{-1}(L/\sqrt{L^2 + k^2})\). Therefore the field at \(A\) is

\[
B_A = \frac{\mu_0 il}{2} \left( \frac{L}{\sqrt{L^2 + k^2}} - 0 \right) = \frac{\mu_0 il}{2} \sqrt{L^2 + k^2}.
\]

28.127 In Prob. 28.125 what is the value of \(B\) at \(C\) on the axis in the center (see Fig. 28-53)?

If \(P\) coincides with \(C\), then

\[
\cos \alpha_1 = \frac{-L/2}{\sqrt{(L/2)^2 + k^2}} \quad \text{and} \quad \cos \alpha_2 = \frac{L/2}{\sqrt{(L/2)^2 + k^2}}
\]

Therefore the field at \(C\) is

\[
B_C = \frac{\mu_0 il}{2} \left[ \frac{L/2}{\sqrt{(L/2)^2 + k^2}} - \frac{-L/2}{\sqrt{(L/2)^2 + k^2}} \right] = \frac{\mu_0 il}{2} \left[ \frac{L}{\sqrt{L^2 + k^2}} \right] = \frac{\mu_0 il}{\sqrt{L^2 + k^2}}.
\]

28.128 Show that the result in Prob. 28.125 for \(B\) inside a long solenoid agrees with the usual formula.

As \(L \to \infty\), then for any point inside the solenoid, the angles \(\alpha_1\) and \(\alpha_2\) approach \(\pi\) and 0, respectively. Hence Eq. (2) implies

\[
B \to \frac{\mu_0 il}{2} \left[ \cos 0 - \cos (\pi) \right] = \mu_0 il
\]

28.129 Helmholtz coils (Fig. 28-54) are sometimes used to obtain a nearly uniform magnetic field in situations where a solenoid would be impractical. Verify that on the axis, midway between the coils, the first three derivatives of \(B\) with respect to \(x\) vanish.

![Fig. 28-54](image)

By Prob. 28.119, the field is given along the \(x\) axis by

\[
B(x) - \frac{\mu_0 il^2}{2} \left[ (a^2 + \left(\frac{a}{2} + x\right)^2)^{-3/2} + (a^2 + (\frac{a}{2} - x)^2)^{-3/2} \right]
\]
As this is an even function of \( x \), \( B'(0) = B''(0) = 0 \). Moreover,

\[
B'(x) = 6 \mu_0 \mu_0 I x \left[ \left( a^2 + \left( \frac{a}{2} + x \right)^2 \right)^{\alpha - 1} (a + x) - \left( a^2 + \left( \frac{a}{2} - x \right)^2 \right)^{\alpha - 1} (a - x) \right]
\]

which shows that \( B'(0) = 0 \). Consequently, near \( O \), we have \( B(x) \approx B_0 + B_0 x^2 \), an almost-constant function.

28.130 A multilayer coil is to be wound on a fixed form with the wires in contact. See the cross section in Fig. 28-55. The insulation is thin, and the space provided is to be filled. How will the strength of the magnetic field produced depend on the diameter \( d \) of the wire chosen if the power consumed by the coil is fixed?

Since all geometrical factors are fixed, the field is proportional to \( N i \), where \( N \) is the total number of turns and \( i \) is the current. The number of turns in each layer is proportional to \( 1/d \), and the number of layers is also proportional to \( 1/d \). Therefore \( N \propto 1/d \).

In this case, \( i^2 R \) is a constant, where \( R \) is the electric resistance. Hence \( i \) is proportional to \( R \). But \( R \) is proportional to \( 1/d^2 \) for any given conducting material. Since \( l \propto 1/d, l/d^2 \propto 1/d^4 \), so \( i \propto R \). Therefore \( B = Ni \propto (1/d^4)(d^2) = d^2 \). That is, at constant power, \( B \) is independent of \( d \).

![Fig. 28-55](image)

28.131* State Ampère's circuit law. How is it related to the Biot–Savart law?

Figure 28-56 shows an open surface \( S \) bounded by a closed curve \( C \). Total current \( I \), either concentrated or distributed, crosses \( S \); in the latter case, \( I = \int_s J \cdot ds \), where \( J \) is the current density vector. This current generates a magnetic field which, according to Ampère's law, satisfies \( \oint_c B \cdot ds = \mu_0 I \), where the line integral is around \( C \).

Since both Ampère's law and the Biot–Savart law specify the connection between electric currents and magnetic fields, they must be equivalent (as can be shown mathematically by use of vector calculus).

28.132* A straight wire, along \( Z \), in Fig. 28.57, carries a current \( I \). Find the field at \( P \) due to section \( ab \) only (length, \( l_1 + l_2 \) using (a) the Biot–Savart law, and (b) Ampère's circuit law.

![Fig. 28-57](image)
(a) It is evident that $d\mathbf{B}$ at $P$ due to any current element $I\,dl$ has the direction $-\mathbf{l}$. Hence

$$d\mathbf{B} = \frac{\mu_0 l \sin \theta}{4\pi r^2} \, dz \, (-\mathbf{l})$$

But

$$r^2 = R^2 + z^2 \quad \text{and} \quad \sin \theta = \cos (\theta - 90^\circ) = \frac{R}{r}$$

Thus,

$$d\mathbf{B} = \frac{\mu_0 l R}{4\pi (R^2 + z^2)^{3/2}} \, dz \, (-\mathbf{l})$$

and

$$\mathbf{B} = \frac{\mu_0 l R}{4\pi} \int_{-l_2}^{l_1} \frac{dz}{(R^2 + z^2)^{3/2}} = \frac{\mu_0 l}{4\pi R} \left[ \frac{l_1}{(R^2 + l_1^2)^{1/2}} + \frac{l_2}{(R^2 + l_2^2)^{1/2}} \right] (-\mathbf{l})$$

From the cylindrical symmetry of the problem it is evident that at all points of the circle $x^2 + y^2 = R^2$, $\mathbf{B}$ is tangential and of constant magnitude.

(b) Application of Ampère's law to the circle $x^2 + y^2 = R^2$, together with the symmetry considerations of (a), gives

$$R(2\pi R) = \mu_0 I \quad \text{or} \quad R = \frac{\mu_0 l}{2\pi R}$$

In particular, at point $P$,

$$\mathbf{B} = \frac{\mu_0 l}{2\pi R} \, (-\mathbf{l})$$

(Before you believe this, see Prob. 28.133.)

28.133 Explain the discrepancy between parts (a) and (b) of Prob. 28.132.

Electric current cannot really appear abruptly at point $a$ and disappear abruptly at point $b$; there must be a return path from $b$ to $a$. If the integration in (a) were extended over the entire closed path, a different result for $\mathbf{B}$ would be obtained. Likewise, the problem no longer being cylindrically symmetric, a different result would also be found in (b); and these two new results would agree. For example, suppose that the path closes at infinity. Letting $l_1 \to \infty$ and $l_2 \to \infty$ in (a) and holding $R$ fixed (noting that the return path at $\infty$ can be ignored),

$$\frac{l_1}{(R^2 + l_1^2)^{1/2}} \to 1 \quad \frac{l_2}{(R^2 + l_2^2)^{1/2}} \to 1 \quad \text{and so} \quad \mathbf{B} \to \frac{\mu_0 l}{2\pi R} \, (-\mathbf{l})$$

This result agrees with that of (b), which, for an infinite straight wire, is correct as it stands.

28.134 Figure 28-58 shows the cross section of a long metal rod of radius $R = 30$ mm. The rod carries a current $I = 5$ kA directed out of the paper. Find $B$ at point $P$ $(r = 20$ mm) and at the surface of the rod, assuming that the permeability of the metal may be approximated as $\mu_0$.

Assuming uniform current density over the cross section, the lines of $\mathbf{B}$ will be circles concentric with the surface of the rod. The current through the dashed circle, of area $\pi r^2$, is

$$\frac{\pi r^2}{\pi R^2} I = \frac{r^2}{R^2} I$$

Fig. 28-58
and Ampère's circuital law gives, by symmetry,

\[ B(2\pi r) = \mu_0 \frac{r^2}{R^2} I \quad \text{or} \quad B = \left( \frac{\mu_0 I}{2\pi R} \right) r \]

Substituting numerical values,

\[ B_r = 22.2 \text{ mT} \quad B_{\text{surface}} = \frac{3}{2} B_r = 33.3 \text{ mT}. \]

28.135 By applying Ampère's circuital law to an appropriate contour, show that \( B \) is uniform within a very long solenoid of arbitrary cross section.

In Fig. 28-59 the dashed breaks allow us to show the end faces of the solenoid. At any point in the interior the field must be parallel to the axis of the solenoid since perpendicular components will be canceled by contributions from the solenoid symmetrically to the left and right of the point. Furthermore, the parallel component must be the same anywhere along a given parallel line since indeed the solenoid is infinitely long. Then we choose a rectangular path, as shown, inside the solenoid, one side, of length \( L \), being along the axis. Since no current is enclosed by the path, and using our earlier conclusions, Ampère's law yields

\[ B_n L - B_s L = 0 \quad \text{or} \quad B_n = B_s. \]

Since this is true for any choice of rectangle, we have our result.

28.136 In the coaxial cable of Fig. 28-60, a straight wire of radius \( a \) carries a current \( I_1 \) along the axis of a metal tube with inner radius \( b \) and outer radius \( c \). The tube carries a current \( I_2 \) in a direction opposite to that in the wire. Find \( B \) for \( a < r < b \). Repeat for \( r > c \).

- Use Ampère's circuital law for \( a < r < b \), \( \mu_0 I_1 = 2\pi B \), so \( B = (\mu_0 I_1)/(2\pi r) = [(2 \times 10^{-7})I_1]/r \). For \( r > c \), one has \( \mu_0 (I_1 - I_2) = 2\pi r B \) from which \( B = \text{const} (I_1 - I_2) = 0 \).

28.137 Repeat Prob. 28.136 if the current in the outer tube is \( I_2 \). Repeat if \( I_1 \) and \( I_2 \) are in the same direction.

- As before, \( B = [(2 \times 10^{-7})I_1]/r \) for \( r < b \) but now for \( r > c \), \( B = [(2 \times 10^{-7})(I_1 + I_2)]/r \).

28.138 Refer to Fig. 28-61. Suppose that the current density in the wire varies with \( r \) according to \( J = kr^2 \), where \( k \) is a constant. Find the value of \( B \) for (a) \( r > a \) and (b) \( r < a \).

- Choose a circular path centered on the conductor and apply Ampère's law. (a) To find the current passing through the area enclosed by the path integrate \( J dA = (kr^2)(2\pi dr) \) from 0 to \( a \) to find \( I = (\pi ka^3)/2 \). The field is then \( B = (\mu_0 I)/2\pi r = (\mu_0 ka^3)/4r \). (b) Integrate \( J dA \) from 0 to \( r \); \( I = (\pi kr^3)/2 \), then \( B = (\mu_0 kr^3)/4 \).
28.139 A long straight metal tube has inner radius \( a \) and outer radius \( b \). It carries lengthwise a current \( I \) spread uniformly over its cross section for \( a \leq r \leq b \). Find the magnetic field in each of the following regions:

- (a) \( r < a \);
- (b) \( a \leq r < b \);
- (c) \( r > b \).

1. Apply the circuit law to circular paths in each region. (a) \( B2 \pi r = \mu_0 (0) \), so \( B = 0 \); no current is enclosed by the path. (b) \( B2 \pi r = \left| \frac{\mu_0 l (r^2 - a^2)}{(b^2 - a^2)} \right| \); only a fraction is enclosed. (c) \( B2 \pi r = \mu_0 l \) giving \( B = \left( \frac{\mu_0 l}{2 \pi r} \right) \); all the current is enclosed.

28.140 Show that fringing of the magnetic field must occur on both sides in Fig. 28-62.

1. We apply Ampère’s law to the rectangular path \( ABCD \). Since the path encloses no current, we find that \( \Sigma = \oint B \cdot dl = 0 \). By hypothesis, \( B \) is perpendicular to sides \( BC \) and \( DA \), and \( B = 0 \) along \( CD \). Therefore \( \Sigma = \oint B \cdot dl \). But \( B \) is antiparallel to side \( AB \) and is nonzero there. This implies that \( \Sigma \neq 0 \), in violation of Ampère's law. Thus our hypothesis that \( B \) drops sharply to zero (outside the region between the pole pieces) must be false.

![Fig. 28-62](image)

28.141 As shown in Fig. 28-63(a), a long, straight metal rod has a very long hole of radius \( a \) drilled parallel to the rod axis. If the rod carries a current \( I \), show that \( B \) has the value (a) \( \left( \frac{\mu_0 I a^2}{(2 \pi c)(b^2 - a^2)} \right) \) on the axis of the rod and (b) \( \left( \frac{\mu_0 I c}{(2 \pi)(b^2 - a^2)} \right) \) on the axis of the hole. (Hint: Superpose the fields from two cylindrical conductors to obtain the required field.)

![Fig. 28-63](image)

1. In the rod \( J = I / [(\pi)(b^2 - a^2)] \). Treat as a solid rod carrying current \( J \pi a^2 \) one way and a second rod with current \( J \pi a^2 \) moving the other way [see Fig. 28-63(b)]. The actual field is the sum of these two current-carrying rods. (a) Only the small rod contributes to the field, so \( B = \left( \frac{\mu_0 J a^2}{(2 \pi c)} \right) \). (b) On the hole axis only the larger rod contributes: \( B = \left( \frac{\mu_0 J c^2}{(2 \pi c)} \right) = \left( \frac{\mu_0 I c}{(2 \pi)(b^2 - a^2)} \right) \).

28.142 For the situation of Prob. 28.141, find the field for a point \( P \) on the line of centers at a radius (a) \( R > b \) from the center of the rod and (b) \( R < b \) but not within the hole.

![Fig. 28-64](image)
28.143 Show that the force per unit length $f$ between two very long parallel wires is $f = -\frac{\mu_0 I_1 I_2}{2\pi r}$, where $I_1$ and $I_2$ are the currents in the wires and $r$ is the distance between the wires. (The minus sign indicates that the wires attract each other when their currents are in the same direction, and that they repel each other when their currents are in opposite directions.)

Refer to Fig. 28-65. The field at a distance $r$ from a long wire carrying a current $I_1$ is $B = \frac{\mu_0 I_1}{2\pi r}$. This field is at right angles to a second wire that is parallel to the first. Therefore, the force on a length $l$ of a second wire at $r$ carrying the current $I_2$ is $F = BI_2l$ and the force per unit length is

$$f = \frac{F}{l} = B I_2 - \frac{\mu_0 I_1 I_2}{2\pi r}$$

Using the right-hand rules for fields and forces, it is seen that the force on wire 2 is repulsive when $I_1$ and $I_2$ have opposite directions and attractive when $I_1$ and $I_2$ have the same direction. This can be expressed in the usual sign convention by

$$f = -\frac{\mu_0 I_1 I_2}{2\pi r} \quad (1)$$

since $f$ will be negative (attractive) when $I_1$ and $I_2$ have the same sign, and will be positive (repulsive) when $I_1$ and $I_2$ have opposite signs.

\[\text{Fig. 28-65}\]

$F_m$ \hspace{1cm} $I_1 = 25 \text{ A}$ \hspace{1cm} $I_2 = 35 \text{ A}$

25 cm

28.144 What is the force per unit length between the wires in Fig. 28-66?

Since the currents have opposite directions, we have, using the results of Prob. 28.143,

$$f = -\frac{\mu_0 I_1 I_2}{2\pi r} = -\frac{\mu_0 (25 \text{ A})(-35 \text{ A})}{2\pi (0.25 \text{ m})} = 7.0 \times 10^{-4} \text{ N/m}.$$

This force is repulsive.

28.145 Consider the three long, straight, parallel wires shown in Fig. 28-67. Find the force experienced by a 25 cm length of wire C.
The fields due to wires $D$ and $G$ at wire $C$ are

\[ B_D = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7} \text{T} \cdot \text{m/A})(30 \text{ A})}{2\pi(0.03 \text{ m})} = 2 \times 10^{-4} \text{T} \]

into the page, and

\[ B_G = \frac{(4\pi \times 10^{-7} \text{T} \cdot \text{m/A})(20 \text{ A})}{2\pi(0.05 \text{ m})} = 0.8 \times 10^{-4} \text{T} \]

out of the page. Therefore the field at the position of wire $C$ is

\[ B = 2 \times 10^{-4} - 0.8 \times 10^{-4} = 1.2 \times 10^{-4} \text{T} \]

into the page. The force on a 25 cm length of $C$ is

\[ F = IL B \sin \theta = (10 \text{ A})(0.25 \text{ m})(1.2 \times 10^{-4} \text{T})(\sin 90^\circ) = 3 \times 10^{-4} \text{ N} \]

Using the right-hand rule at wire $C$ tells us that the force on wire $C$ is toward the right.

For the situation shown in Fig. 28-68, find the force experienced by side $MN$ of the rectangular loop. Also, find the torque on the loop.

Field at $MN$ due to $I_2$ is $(\mu_0 I_2)/(2\pi a)$. The force on $MN$ due to $I_1$ is away from $I_1$, and is $I_1 L B = (\mu_0 I_1 I_2 L)/(2\pi a)$. Forces on the loop due to $I_1$ act in the plane of the loop giving zero torque. These forces compress the loop.

For the situation shown in Fig. 28-68, find the net force on the long, straight wire due to the current in the loop. (Hint: Assume that the action-reaction law is valid.)

From Prob. 28.146, the force on the top element will be $(\mu_0 I_1 L)/(2\pi(a + b))$ directed toward $I_1$. From Newton's third law the force exerted on the straight wire by the loop will be $[(\mu_0 I_1 I_2 L)/(2\pi)]/[1/a - 1/(a + b)]$ and is repulsive.

Explain why the permeability of empty space turns out to be exactly $4\pi \times 10^{-7}$ (in SI units).

Equation (1) of Prob. 28.143 is used to define the ampere, as follows: When $r$ is set at 1 m and $f$ is measured to be $7 \times 10^{-7}$ N/m (a convenient value), for $I_1 = I_2 = I$, then $I = 1 \text{ A}$, exactly. This forces the value $\mu_0 = 4\pi \times 10^{-7}$.

The two infinite plates shown in cross section in Fig. 28-69(a) carry $j$ amperes of current out of the page per unit width of plate. Find the magnetic field at points $P$ and $Q$.

Consider only one plate as shown in Fig. 28-69(b). The field on the two sides is directed as indicated.
Apply the circuit law to the dotted path. Then \(2BL = \mu_0 jL\), which gives

\[ B = \frac{\mu_0 j}{2}. \]  

(1)

Now superpose the fields of two such plates. At \(P\) they cancel and \(B = 0\); at \(Q\) they add, so \(B = \mu_0 j\).

28.150 In Prob. 28.149, find the force per unit area on the lower plate because of the current in the upper plate.

The upward force on a square of side \(L\) of the lower plate is \(F = BIL\), with \(I = jL\). Using Eq. (1) of Prob. 28.149, \(F = \frac{1}{2} \frac{\mu_0 j}{L} L = \frac{\mu_0 j^2 L^2}{2}\). The force per unit area \(= \frac{F}{L^2} = \frac{\mu_0 j^2}{2}\).

28.151 A flat circular coil with 10 loops of wire on it has a diameter of 20 mm and carries a current of 0.5 A. It is mounted inside a long solenoid that has 200 loops on its 250 mm length. The current in the solenoid is 2.4 A. Compute the torque required to hold the coil with its axis perpendicular to that of the solenoid.

Let the subscripts \(s\) and \(c\) refer to the solenoid and coil, respectively. Then \(\tau = N_s I_s A_s B_s \sin 90^\circ\). But \(B_s = \mu_0 n I_s = \mu_0 (N_s / L_s) I_s\), which gives

\[ \tau = \mu_0 N_s N_c I_s (\pi r_c^2) / L_s = (4\pi \times 10^{-7} \text{ H/m})(10)(200)(0.5 \text{ A})(2.4 \text{ A}) \pi (0.01 \text{ m})^2 / (0.25 \text{ m}) \]

\[ \tau = 3.8 \times 10^{-6} \text{ N\cdot m} \]