CHAPTER 27
Simple Electric Circuits

27.1 OHM'S LAW, CURRENT, RESISTANCE

<table>
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<th>TABLE 27-1 Resistivities ((\rho)) at 20 °C and Temperature Coefficients ((\alpha)).</th>
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27.1 What is the relation between resistance and resistivity?

The resistance \(R\) of a wire of length \(L\) and cross-sectional area \(A\) is

\[ R = \rho \left( \frac{L}{A} \right), \]

where \(\rho\) is a constant called the **resistivity** and is a characteristic of the material from which the wire is made. For \(L\) in m, \(A\) in m², and \(R\) in Ω, the units of \(\rho\) are Ω · m.

27.2 How does the resistance of a conductor vary with temperature?

If a wire has a resistance \(R_0\) at a temperature \(T_0\), then its resistance \(R\) at a temperature \(T\) is

\[ R = R_0 + \alpha R_0 (T - T_0), \]

where \(\alpha\) is the **temperature coefficient of resistance** of the material of the wire. Usually \(\alpha\) varies with temperature and so a linear relation is applicable only over a small temperature range. The units of \(\alpha\) are K⁻¹ or °C⁻¹.

A similar relation applies to the variation of resistivity with temperature. If \(\rho_0\) and \(\rho\) are the resistivities at \(T_0\) and \(T\) respectively, then

\[ \rho = \rho_0 + \alpha \rho_0 (T - T_0). \]

Table 27-1 lists the resistivities of a number of conductors for \(T_0 = 20\ °C\), as well as temperature coefficients of resistance.

27.3 How are current and current density related?

The rate of flow of electric charge across a given area (within a conductor) is defined as the **electric current** \(I\) through that area. Thus,

\[ I = \frac{dq}{dt}. \]  \hspace{1cm} (A)

The **electric current density** \(\mathbf{J}\) at a point (within a conductor) is a vector whose direction is the direction of flow of charge at that point and whose magnitude is the current through a unit area perpendicular to the flow direction at that point. Thus, the current through an element of area \(dS\), arbitrarily oriented with respect to the flow direction, is given by (see Fig. 27-1)

\[ dI = \mathbf{J} \cdot dS = J \ dA, \]

where \(dA = dS \cos \theta\) is the projection of \(dS\) perpendicular to the flow direction. The total current through a surface \(S\) (e.g., a cross section of the conductor) is then

\[ I = \int_S \mathbf{J} \cdot dS = \int_S J \ dA. \]
27.4 Starting from the standard form of Ohm's law, \( V = IR \), find the relation between \( \mathbf{J} \), the current density, and \( \mathbf{E} \), the electric field in a current-carrying conductor.

We consider a conductor of uniform cross-sectional area \( A \) and length \( L \). The resistance \( R = \rho(L/A) \), where \( \rho \) is the resistivity. The current can be expressed as \( \mathbf{J} = IA \), and the potential drop across the resistor is related to the average electric field by \( V = EL \). Then \( V = IR \) becomes \( EL = IA(\rho(L/A)) \), or \( E = \rho I \). Often one specifies the conductivity, \( \sigma \), instead of the resistivity, where \( \sigma = 1/\rho \). Then \( \mathbf{J} = \sigma \mathbf{E} \). This result can be generalized to an arbitrary conductor in the vector form: \( \mathbf{J} = \sigma \mathbf{E} \), which holds at each point in the conductor.

27.5 How many electrons per second pass through a section of wire carrying a current of 0.7 A?

\( I = 0.7 \text{ A} \) means 0.7 C/s. Dividing by \( e = 1.6 \times 10^{-19} \text{ C} \), the magnitude of charge on a single electron, we get number of electrons per second = \( 0.7/(1.6 \times 10^{-19}) = 4.4 \times 10^{18} \).

27.6 A current of 7.5 A is maintained in a wire for 45 s. In this time (a) how much charge and (b) how many electrons flow through the wire?

(a) \( q = It = (7.5 \text{ A})(45 \text{ s}) = 337.5 \text{ C} \)

(b) The number of electrons \( N \) is given by

\[
N = \frac{q}{e} = \frac{337.5 \text{ C}}{1.6 \times 10^{-19} \text{ C}} = 2.1 \times 10^{21}
\]

where \( e = 1.6 \times 10^{-19} \text{ C} \) is the charge of an electron.

27.7 If 0.6 mol of electrons flow through a wire in 45 min, what are (a) the total charge that passes through the wire, and (b) the magnitude of the current?

(a) The number \( N \) of electrons in 0.6 mol is

\[
N = (0.6 \text{ mol})(6.02 \times 10^{23} \text{ electrons/mol}) = 3.6 \times 10^{23} \text{ electrons}
\]

\[
q = Ne = (3.6 \times 10^{23})(1.6 \times 10^{-19} \text{ C}) = 5.78 \times 10^{4} \text{ C}
\]

(b) \( t = (45 \text{ min})(60 \text{ s/min}) = 2.7 \times 10^{3} \text{ s} \)

\[
I = \frac{q}{t} = \frac{5.78 \times 10^{4} \text{ C}}{2.7 \times 10^{3} \text{ s}} = 21.4 \text{ A}
\]

27.8 An electron gun in a TV set shoots out a beam of electrons. The beam current is 10 \( \mu \text{A} \). How many electrons strike the TV screen each second? How much charge strikes the screen in a minute?

Let \( n_e \) = number of electrons per second. \( n_e = I/e = (1.0 \times 10^{-5} \text{ C/s})/(1.6 \times 10^{-19} \text{ C}) = 6.3 \times 10^{13} \text{ electrons per second} \). The charge \( Q \) striking the screen obeys \( |Q| = IT = (10 \mu \text{C/s})(60 \text{ s}) = 600 \mu \text{C} \). Since the charges are electrons, the actual charge is \( Q = -600 \mu \text{C} \).

27.9 In the Bohr model, the electron of a hydrogen atom moves in a circular orbit of radius \( 5.3 \times 10^{-11} \text{ m} \) with a speed of \( 2.2 \times 10^{6} \text{ m/s} \). Determine its frequency \( f \) and the current \( I \) in the orbit.
Each time the electron goes around the orbit, it carries a charge $e$ around the loop. The charge passing a point on the loop each second is

$$I = ef = (1.6 \times 10^{-19} \text{ C})(6.6 \times 10^{15} \text{ s}^{-1}) = 1.06 \text{ mA}$$

Note that the current flows in the opposite direction to the electron, which is negatively charged.

27.10 A typical copper wire might have $2 \times 10^{21}$ free electrons in 1 cm of its length. Suppose that the drift speed of the electrons along the wire is 0.05 cm/s. How many electrons would pass through a given cross section of the wire each second? How large a current would be flowing in the wire?

$$I = \frac{Q}{t} = (2 \times 10^{21})(0.05) = 1 \times 10^{20} \text{ electrons per second}.$$  

27.11 What is the current through an 8-$\Omega$ toaster when it is operating on 120 V?

This is an application of Ohm’s law: $V = IR$, 120 V = $I(8\Omega)$, and $I = 15 \text{ A}$.  

27.12 What potential difference is required to pass 3 A through 28 $\Omega$?

$$V = IR = (3 \text{ A})(28 \Omega) = 84 \text{ V}$$  

27.13 Determine the potential difference between the ends of a wire of resistance 5 $\Omega$ if 720 C passes through it per minute.

First we determine the current, $I = \frac{Q}{t}$, or $I = 720 \text{ C/60 s} = 12 \text{ A}$. Then use Ohm’s law, $V = IR$, or $V = (12 \text{ A})(5 \Omega) = 60 \text{ V}$.  

27.14 A copper bus bar carrying 1200 A has a potential drop of 1.2 mV along 24 cm of its length. What is the resistance per m of the bar?

From Ohm’s law, applied to 24 cm of the bar, $V_{24} = IR_{24}$, or $(1.2 \times 10^{-3} \text{ V}) = (1200 \text{ A})R$, and $R_{24} = 1 \mu \Omega$. By proportion, $R_{100} = (100/24)R_{24} = 4.2 \mu \Omega$.  

27.15 A current of 3.0 A flows down a straight metal rod that has a 0.20-cm diameter. The rod is 1.5 m long, and the potential difference between its ends is 40 V. Find (a) current density and (b) field in the rod, and (c) resistivity of the material of the rod.

(a) $J = I/A = 3/(\pi \times 10^{-3}) = 9.55 \times 10^2 \text{ A/m}^2$;  
(b) $E = V/d = 40/1.5 = 27 \text{ V/m}$; and (c) since $J = E/\rho$, $\rho = 2.8 \times 10^{-2} \Omega \cdot m = 28 \mu \Omega \cdot m$.  

27.16 A 0.20-mm-diameter copper wire is sealed end to end to a 5.00-mm-diameter iron rod, and a current is sent lengthwise through them. If the current in the copper is 8.0 A, what are (a) the current and current density in the iron, and (b) the current density in the copper?

(a) Since charge must be conserved, $I_{Cu} = I_{Fe} = 8.0 \text{ A}$, and $J_{Cu} = I/A = 8.0/[\pi(5.0 \times 10^{-3})^2/4] = 407 \text{ kA/m}^2$.  
(b) By inverse proportion, $J_{Cu} = (5.00/0.20)^2J_{Fe} = 255 \text{ MA/m}^2$.  

27.17 A copper wire of 3.0-mm$^2$ cross-sectional area carries a current of 5.0 A. Find the magnitude of the drift velocity for the electrons in the wire.

We have

$$J = \frac{I}{A} = \frac{5.0 \text{ A}}{3.0 \times 10^{-6} \text{ m}^2} = 1.67 \times 10^6 \text{ A/m}^2$$

The drift velocity is given by

$$v = \frac{J}{ne} = \frac{1.67 \times 10^6 \text{ A/m}^2}{n(1.60 \times 10^{-19} \text{ C})} = \frac{1}{n}(1.04 \times 10^{25} \text{ m}^{-2} \cdot \text{s}^{-1})$$

where $n$ is the number of charge carriers per unit volume. To find $n$ we must find the number of copper atoms
per unit volume. Assuming one free electron per atom, \( M = 63.5 \text{ kg/kmol} \), and \( \rho = 8920 \text{ kg/m}^3 \), we have

\[
n = \frac{(6.02 \times 10^{23} \text{ atoms/kmol})(8920 \text{ kg/m}^3)}{63.5 \text{ kg/kmol}} = 8.5 \times 10^{28} \text{ atoms/m}^3
\]

Substituting in the expression for the drift velocity gives \( v = 0.12 \text{ mm/s} \).

27.18 As shown in Fig. 27-2, a metal rod of radius \( r_1 \) is concentric with a metal cylindrical shell of radius \( r_2 \) and length \( L \). The space between rod and cylinder is tightly packed with a high-resistance material of resistivity \( \rho \). A battery having a terminal voltage \( v_t \) is connected as shown. Neglecting resistances of rod and cylinder, derive expressions for (a) the total current \( I \), (b) the current density \( J \) and the electric field \( E \) at any point \( P \) between rod and cylinder, and (c) the resistance \( R \) between rod and cylinder.

![Fig. 27-2](image)

(a) Assuming radial flow of charge between rod and cylinder, we have at \( P \)

\[
J = \frac{I}{2\pi r L} \quad \text{and} \quad E = \frac{\rho I}{2\pi r L}
\]

with both \( J \) and \( E \) in the direction of \( r \). Then, by definition of the potential,

\[
dv = -E \cdot ds = -E dr = -\frac{\rho I}{2\pi r L} \frac{dr}{r}
\]

and so, noting the polarity of \( v_t \),

\[
-v_t = \int_{r_1}^{r_2} dv = -\frac{\rho I}{2\pi L} \int_{r_1}^{r_2} \frac{dr}{r} = -\frac{\rho L}{2\pi L} \ln \left( \frac{r_2}{r_1} \right)
\]

Solving for \( I \),

\[
I = \frac{2\pi L v_t}{\rho \ln \left( \frac{r_2}{r_1} \right)}
\]

(b) From \( a \),

\[
J = \frac{I}{2\pi r L} = \frac{v_t}{\rho r \ln \left( \frac{r_2}{r_1} \right)} \quad \text{and} \quad E = \frac{\rho I}{r \ln \left( \frac{r_2}{r_1} \right)}
\]

(c) From Ohm's law,

\[
R = \frac{v}{I} = \frac{\rho \ln \left( \frac{r_2}{r_1} \right)}{2\pi L}
\]

27.19 Compute the resistance of 180 m of silver wire having a cross section of 0.3 mm². (Assume that \( t = 20^\circ \text{C} \).)

Resistance is given by \( R = \rho (L/A) \). From Table 27-1 we have \( \rho \) for silver at \( 20^\circ \text{C} = 1.6 \times 10^{-8} \text{ } \Omega \cdot \text{m} \). Then \( R = (1.6 \times 10^{-8} \text{ } \Omega \cdot \text{m}) (180 \text{ m}) / (0.3 \times 10^{-6} \text{ m}^2) = 9.6 \text{ } \Omega \).

27.20 How long a piece of aluminum wire 1 mm in diameter is needed to give a resistance of 4 \( \Omega \)? (Assume \( t = 20^\circ \text{C} \)).
27.21 A 20-cm-long copper tube has an inner diameter of 0.85 cm and an outer diameter of 1.10 cm. Find its electric resistance when used lengthwise.

\[ R = \rho (L/A) \]  
\[ \text{The cross-sectional area is } \pi [(1.10^2 - 0.85^2)]/(4 \times 10^{-6}) = 3.83 \times 10^{-3} \text{ m}^2; \quad \text{then with } L = 0.20 \text{ m and } \rho = 1.7 \times 10^{-8} \text{ from Table 27-1, } R = 89 \mu \Omega. \]

27.22 A bar of copper having mass 1.5 kg is to be drawn into a wire having resistance 250 \( \Omega \) at 20°C. Determine the length \( L \) and diameter \( d \) of the wire. The density of copper is \( 8.9 \times 10^3 \) kg/m³.

\[ \text{From mass} = \text{density} \times \text{vol, we get } (8.9 \times 10^3) L A = 1.5. \]  
\[ \text{On the other hand, Table 27-1 gives for } R_{\text{25C}}(1.72 \times 10^{-8})(L/A) = 250. \]  
\[ \text{Solving these two equations simultaneously, } L = 1.565 \text{ km} \quad \text{and} \quad A = 0.1077 \text{ mm}^2. \]  
\[ \text{But} \quad A = \pi d^2/4, \quad \text{from which } d = 0.37 \text{ mm}. \]

27.23 A copper wire is 20 m long and 0.254 mm in diameter. Compute its resistance.

\[ R = \rho (L/A) \]  
\[ = (1.7 \times 10^{-8}) \left( \frac{20}{\pi(0.000127)^2} \right) = 6.7 \Omega \]

27.24 Compute the resistivity of copper wire in \( \Omega \cdot \text{circular mils/ft} \) (traditional U.S. units).

\[ \text{The area of a circle in circular mils is defined as the square of the diameter of the circle expressed in mils, where } 1 \text{ mil} = 0.001 \text{ in} = 2.54 \times 10^{-3} \text{ m}. \]  
\[ \text{Thus,} \quad 1 \text{ circular mil} = \frac{\pi (2.54 \times 10^{-3})^2}{4} \text{ m}^2 \quad \text{and so} \quad 1 \text{ circular mil/ft} = \frac{\pi (2.54 \times 10^{-3})^2/4}{(12 \text{ in})(2.54 \times 10^{-2} \text{ m/in})} = 1.65 \times 10^{-6} \text{ m}. \]

Then, from Table 27-1,

\[ \rho_{\text{Cu}} = (1.7 \times 10^{-8} \Omega \cdot \text{m}) \frac{1 \text{ circular mil/ft}}{1.65 \times 10^{-6} \text{ m}} = 10.3 \Omega \cdot \text{circular mil/ft} \]

27.25 A coil of wire has a resistance of 25.00 \( \Omega \) at 20°C and a resistance of 25.17 \( \Omega \) at 35°C. What is its temperature coefficient of resistance?

\[ R = R_0[1 + \alpha(T - T_0)], \quad \alpha = \Delta R/(R_0 \Delta T), \quad \text{with } \Delta R = R - R_0 = 0.17 \Omega \quad \text{and} \quad \Delta T = T - T_0 = 15 \text{°C}. \]  
\[ \alpha = (0.17)/(25.00 \times 15) = 4.5 \times 10^{-4} \text{°C}^{-1}. \]

27.26 A metal wire of diameter 2 mm and of length 300 m has a resistance of 1.6424 \( \Omega \) at 20°C, and 2.415 \( \Omega \) at 150°C. Find the values of \( \alpha, R_0, \rho_0 \), where the zero subscript refers to 0°C, and \( \rho_{\text{25C}} \). Identify the metal.

\[ R_{\text{25C}} = 2.415 = R_0(1 + \alpha 150) \]  
\[ R_{\text{25C}} = 1.6424 = R_0(1 + \alpha 20) \]

Solving these relations simultaneously, \( \alpha = 3.9 \times 10^{-3} \text{°C}^{-1} \) and \( R_0 = 1.5236 \Omega \).

From \( R_0 = \rho_0 (L/A) \),

\[ 1.5236 = \frac{\rho_0 (300)}{\pi (2 \times 10^{-3})^2/4} \quad \text{or} \quad \rho_0 = 1.596 \times 10^{-8} \Omega \cdot \text{m} \]

Then,

\[ \rho_{\text{25C}} = \rho_0 (1 + \alpha 20) = (1.596 \times 10^{-8})(1 + (3.9 \times 10^{-3})(20)) = 1.72 \times 10^{-8} \Omega \cdot \text{m} \]

Table 27-1 indicates that the metal is copper.

27.27 It is desired to make a 20.0-\( \Omega \) coil of wire which has a zero thermal coefficient of resistance. To do this, a carbon resistor of resistance \( R_1 \) is placed in series with an iron resistor of resistance \( R_2 \). The proportions of iron and carbon are so chosen that \( R_1 + R_2 = 20.00 \Omega \) for all temperatures near 20°C. How large are \( R_1 \) and \( R_2 \)?

\[ \text{We need } R_1(1 + \alpha_1 \Delta T) + R_2(1 + \alpha_2 \Delta T) = 20. \]

Because \( R_1 + R_2 = 20 \) when \( \Delta T = 0 \), we must have \( R_1 \alpha_1 = -R_2 \alpha_2 \), with \( \alpha_1 = -0.5 \times 10^{-3} \) and \( \alpha_2 = 5 \times 10^{-4} \). Solving the two equations \( R_1 + R_2 = 20 \) and \( R_1 = 10R_2 \) simultaneously leads to \( R_1 = 18.18 \) and \( R_2 = 1.82 \Omega \).
27.28 A resistance thermometer measures temperature by the increase in resistance of a wire at high temperature. If the wire is platinum and has a resistance of 10 \( \Omega \) at 20 °C and a resistance of 35 \( \Omega \) in a hot furnace, what is the temperature of the furnace? (\( \alpha \) for platinum is 0.0036 °C\(^{-1}\)).

We assume that \( \alpha \) is constant over the needed temperature range. Then \( \Delta R = \alpha R \Delta t \) leads to \( (35 - 10) = 0.0036(10) \Delta t \). Solving we get \( \Delta t = 25/0.036 = 694 \) °C. Finally 694 + 20 = 714 °C (furnace temperature).

27.29 A 75-W tungsten light bulb has a resistance of 190 \( \Omega \) when lighted and 15 \( \Omega \) when turned off. Estimate the temperature of the filament when the bulb is lighted.

We can make only a very rough estimate since we must use \( \alpha \) from Table 27-1 over far too wide a range. We have

\[
R = R_{20}(1 + \alpha \Delta T) \quad \text{or} \quad \Delta T = \frac{R - R_{20}}{\alpha R_{20}}
\]

Then

\[
T - 20^\circ \text{C} = \frac{(190 - 15) \Omega}{(4.5 \times 10^{-3})^\circ \text{C}(15 \Omega)} = 2590^\circ \text{C}
\]

from which \( T = 2600^\circ \text{C} \).

27.30 A 60-W bulb carries a current of 0.5 A when operating on 120 V. The temperature of its tungsten filament is then 1800 °C. Find the resistance at its operating temperature. Find its approximate resistance at 20 °C.

As in Prob. 27.29, the temperature range is too high for much accuracy. Nonetheless we proceed. Use \( V = IR \) to find \( R = 240 \) \( \Omega \). Then \( R = R_{20}(1 + \alpha \Delta t) \) yields 240 = \( R_{20}[1 + (4.5 \times 10^{-3})(1780)] \), from which \( R_{20} = 26.6 \Omega \).

27.2 RESISTORS IN COMBINATION

27.31 Resistors \( R_1 \), \( R_2 \), and \( R_3 \) are (a) in series and (b) in parallel, as shown in Fig. 27-3(a) and (b). Derive the formula for the equivalent resistance \( R_{eq} \) of each network.

(a) For the series network,

\[
V_{ad} = V_{ab} + V_{bc} + V_{cd} = IR_1 + IR_2 + IR_3
\]

since the current \( I \) is the same in each resistor. Dividing by \( I \),

\[
\frac{V_{ad}}{I} = R_1 + R_2 + R_3 \quad \text{or} \quad R_{eq} = R_1 + R_2 + R_3
\]

since \( V_{ad}/I \) is by definition the equivalent resistance \( R_{eq} \) of the network.

(b) The p.d. across each resistor is the same, whence

\[
I_1 = \frac{V_{ab}}{R_1}, \quad I_2 = \frac{V_{ab}}{R_2}, \quad I_3 = \frac{V_{ab}}{R_3}
\]

Since the line current \( I \) is the sum of the branch currents,

\[
I = I_1 + I_2 + I_3 = \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2} + \frac{V_{ab}}{R_3}
\]

\[\text{Fig. 27-3}\]
Dividing by $V_{ab}$,

$$\frac{I}{V_{ab}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \text{or} \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

since $V_{ab}/I$ is by definition the equivalent resistance $R_{eq}$ of the network.

27.32 Find the resistance equivalent to three resistances in parallel: $R_1 = 12 \, \Omega$, $R_2 = 12 \, \Omega$, and $R_3 = 6 \, \Omega$.

The equivalent resistance $R$ is given by $1/R = 1/R_1 + 1/R_2 + 1/R_3 = \frac{1}{12} + \frac{1}{12} + \frac{1}{6} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$. Thus $R = 3 \, \Omega$.

27.33 What is the resistance between $A$ and $B$ in Fig. 27-4?

For the two resistors in parallel,

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{6} \Rightarrow \frac{2}{6} + \frac{1}{6} = \frac{3}{6} \quad \text{or} \quad R = 2 \Omega$$

This is in series with 8 $\Omega$, so

$$R_{ab} = 2 \Omega + 8 \Omega = 10 \Omega$$

27.34 Three resistances of 12, 16, and 20 $\Omega$ are connected in parallel. What resistance must be connected in series with this combination to give a total resistance of 25 $\Omega$?

The resistance $R$ of the parallel combination is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{12} + \frac{1}{16} + \frac{1}{20} = \frac{20}{240} + \frac{15}{240} + \frac{12}{240} = \frac{47}{240} \quad \text{or} \quad R = 5.11 \Omega$$

Then

$$R_s + R = 25 \quad \text{or} \quad R_s = 25 - 5.11 = 19.89 \Omega$$

27.35 In Fig. 27-5, find the resistance from point $a$ to point $b$.

There are two parallel branches, each of resistance $3/(20) = 60 \Omega$. Thus, $1/R = \frac{1}{60} + \frac{1}{60} = \frac{1}{60}$, and $R = 60 \Omega$.

27.36 An iron wire, of length 2 km, and a copper wire, of length 3 km, are connected in parallel across a source having a terminal voltage of 200 $V$. The diameter of the copper wire is 1 mm; the temperature of the wires is 100 $^\circ C$. If each wire carries the same current, find the current, the diameter of the iron wire, and the electric field strength in each.

We get the resistivities at 20 $^\circ C$ and the temperature coefficients from the Table 27-1. Hence, at 100 $^\circ C$,

$$R_{Ca} = \frac{(1.7 \times 10^{-8})(3000)}{\pi (10^{-6})/4} \left[ 1 + (3.9 \times 10^{-3})(80) \right] = 85.24 \Omega \quad \text{thus} \quad I_{Ca} = \frac{200}{85.24} = 2.34 \, A$$
Since the current is assumed to be the same in each wire,

\[ R_{Fe} = R_{Cu} = 85.24 = \frac{(10 \times 10^{-3})(2000)}{\pi d^2/4} \left[ 1 + (5.0 \times 10^{-3})(80) \right] \]

from which the diameter of the iron wire is \( d = 2.046 \text{ mm} \). The electric fields are

\[ E_{Cu} = \frac{200 \text{ V}}{3000 \text{ m}} = \frac{1}{15} \text{ V/m} \quad E_{Fe} = \frac{200 \text{ V}}{2000 \text{ m}} = \frac{1}{10} \text{ V/m} \]

These values also follow from \( E = \rho I/A \).

27.37 A 50-cm-long metal rod consists of a copper sheath (inner diameter = 2 mm, outer diameter = 3 mm) with an iron core (see Fig. 27-6). What is the resistance of the rod? (Hint: Find the current that would flow through it when the potential difference is \( V \).)

Potential \( V \) across the rod causes \( I \) to flow; by definition \( I = V/R \) but \( I = I_{Cu} + I_{Fe} = V/R_{Cu} + V/R_{Fe} \); \( 1/R = 1/R_{Cu} + 1/R_{Fe} \). Using Table 27-1, \( R_{Cu} = \rho_{Cu}(L/A_{Cu}) = \frac{(1.7 \times 10^{-8})(0.5)}{\pi(1.5^2 - 1.0^2) \times 10^{-6}} = 0.00216 \Omega \). In like manner for Fe, \( R = \rho(L/A) = 0.0159 \Omega \); then \( 1/R = 1/0.00216 + 1/0.0159 \) yields \( R = 1.91 \text{ m}\Omega \).

27.38 Find all the resistances that can be realized with a 6-, a 9-, and a 15-\( \Omega \) resistor in various combinations. Not every combination need use all three resistors.

Figure 27-7 shows all possible combinations and their equivalent resistances.
27.39 Arrange an 8-, a 12-, and a 16-Ω resistor in a combination that has a total resistance of 8.89 Ω.

It is clear that we cannot have any resistance in series with the remaining pair. This observation leads to the solution indicated in Fig. 27-8.

![Fig. 27-8](image)

27.40 Find the equivalent resistance between points a and b for the combination shown in Fig. 27-9(a).

The 3- and 2-Ω resistors are in series and are equivalent to a 5-Ω resistor. The equivalent 5 Ω is in parallel with the 6 Ω, and their equivalent, $R_1$, is

$$\frac{1}{R_1} = \frac{1}{5} + \frac{1}{6} = 0.20 + 0.167 = 0.369 \quad \text{or} \quad R_1 = 2.73 \, \Omega$$

The circuit thus far reduced is shown in Fig. 27-9(b).

The 7- and 2.73-Ω are equivalent to 9.73 Ω. Now the 5, 12 and 9.73 Ω are in parallel and their equivalent, $R_2$, is

$$\frac{1}{R_2} = \frac{1}{5} + \frac{1}{12} + \frac{1}{9.73} = 0.386 \quad \text{or} \quad R_2 = 2.6 \, \Omega$$

This 2.6 Ω is in series with the 9-Ω resistor. Therefore, the equivalent resistance of the combination is $9 + 2.6 = 11.6 \, \Omega$.

27.41 Suppose that the battery in Fig. 27-10 is 12 V and the resistors are $R_1 = 50 \, \Omega$ and $R_2 = 150 \, \Omega$. (a) What are the currents $I$, $I_1$, and $I_2$? (b) What is the total resistance of the circuit?

(a) $I_1 = \frac{E}{R_1} = \frac{12 \, \text{V}}{50 \, \Omega} = 0.24 \, \text{A}$

$I_2 = \frac{E}{R_2} = \frac{12 \, \text{V}}{150 \, \Omega} = 0.08 \, \text{A}$

$I = I_1 + I_2 = 0.32 \, \text{A}$

![Fig. 27-10](image)
\( \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{50 \, \Omega} + \frac{1}{150 \, \Omega} = \frac{4}{150 \, \Omega} \quad R = 37.5 \, \Omega \)

As a check, \( I = \frac{E}{R} = \frac{12 \, V}{37.5 \, \Omega} = 0.32 \, A \).

27.42 Suppose that the emf of the battery in Fig. 27-10 is 45 V and the resistor \( R_1 = 300 \, \Omega \). (a) What must the resistor \( R_2 \) be in order that the current \( I \) be 0.45 A? (b) What are the currents \( I_1 \) and \( I_2 \)?

(a) The total resistance must be

\[ R = \frac{E}{I} = \frac{45 \, V}{0.45 \, A} = 100 \, \Omega \]

But

\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \quad \text{so} \quad \frac{1}{R_2} = \frac{1}{R_1} - \frac{1}{100 \, \Omega} = \frac{1}{300 \, \Omega} = \frac{2}{300 \, \Omega} \quad \text{or} \quad R_2 = 150 \, \Omega \]

(b)

\[ I_1 = \frac{E}{R_1} = \frac{45 \, V}{300 \, \Omega} = 0.15 \, A \quad I_2 = \frac{E}{R_2} = \frac{45 \, V}{150 \, \Omega} = 0.30 \, A \]

27.43 The three resistors in Fig. 27-11 are \( R_1 = 25 \, \Omega \), \( R_2 = 50 \, \Omega \), and \( R_3 = 100 \, \Omega \). (a) What is the total resistance of the circuit? (b) What are the currents \( I_1 \), \( I_2 \), and \( I_3 \) for a 12-V battery?

(a) The sum of \( R_2 \) and \( R_3 \) in parallel is

\[ \frac{1}{R'} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{50 \, \Omega} + \frac{1}{100 \, \Omega} = \frac{3}{100 \, \Omega} \quad \text{or} \quad R' = 33.3 \, \Omega \]

Since \( R' \) is in series with \( R_1 \), the total resistance of the circuit is \( R = R' + R_1 = 33.3 \, \Omega + 25 \, \Omega = 58.3 \, \Omega \).

(b)

\[ I = \frac{E}{R} = \frac{12 \, V}{58.3 \, \Omega} = 0.206 \, A \]

The potential \( V' \) across \( R_2 \) and \( R_3 \) is \( V' = E - R_1 I = 12 \, V - (25 \, \Omega)(0.206 \, A) = 6.85 \, V \). Therefore,

\[ I_2 = \frac{V'}{R_2} = \frac{6.85 \, V}{50 \, \Omega} = 0.137 \, A \quad I_3 = \frac{V'}{R_3} = \frac{6.85 \, V}{100 \, \Omega} = 0.0685 \, A \]

27.44 The three resistors in Fig. 27-12 are \( R_1 = 80 \, \Omega \), \( R_2 = 25 \, \Omega \), and \( R_3 = 15 \, \Omega \). (a) What is the total resistance of the circuit? (b) What are the currents \( I \) and \( I_2 \), and the voltage across the battery, if \( I_3 = 0.3 \, A \)?

(a) The sum of \( R_2 \) and \( R_3 \) in series is \( R' = R_2 + R_3 = 25 \, \Omega + 15 \, \Omega = 40 \, \Omega \). Since \( R' \) is in parallel with \( R_1 \), the total resistance of the circuit is

\[ \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R'} = \frac{1}{80 \, \Omega} + \frac{1}{40 \, \Omega} = \frac{3}{80 \, \Omega} \quad \text{or} \quad R = 26.7 \, \Omega \]

(b)

\[ E = R_1 I_1 = (80 \, \Omega)(0.3 \, A) = 24 \, V \quad I_2 = \frac{E}{R'} = \frac{24 \, V}{40 \, \Omega} = 0.6 \, A \quad I = I_1 + I_2 = 0.9 \, A \]

(Check: \( I = \frac{E}{R} = \frac{24 \, V}{26.7 \, \Omega} = 0.9 \, A \))

27.45 Find \( I_1 \) and \( I_2 \) for the circuit of Fig. 27-13.
In Fig. 27-14 we show successive reduction of the circuit. From Fig. 27-14(a), \( I = \frac{5}{6} = 0.83 A \); Fig. 27-14(c), \( V_n - (1)(4) = 2.0 V \). Thus (Fig. 27-14(b)) the drop across was 0.6 V and \( I_n = 0.837 A \).

27.46 Figure 27-15 shows the three resistors \( R_1 = 5 \Omega \), \( R_2 = 15 \Omega \), and \( R_3 = 25 \Omega \) in four different circuits. For each circuit find the currents \( I_n \), \( I_0 \), and \( I_0 \) in each resistor, and the current \( I \) in the battery.

27.47 It is known that the potential difference across the 6-\( \Omega \) resistance in Fig. 27-16 is 48 V. Determine (a) the entering current \( I_n \), (b) the potential difference across the 8-\( \Omega \) resistor, (c) the potential difference across the 10-\( \Omega \) resistance, (d) the potential difference from a to b. (Hint: The wires connecting c and d can be shank to zero length without altering the currents or potentials.)

27.48 For the circuit shown in Fig. 27-17 find \( R_{eq} \) together with \( I_n \) and \( I_0 \). (Hint: Note that a, b, c, and d are all in the same plane from an electric standpoint. Redraw the diagram to show this.)

27.49 We show the equivalent diagram as well as reduced diagrams in Fig. 27-18. The equivalent resistance is 12 \( \Omega \). From Fig. 27-18(c), \( I_n = \frac{5}{6} = 0.5 A \). The voltage drop across the 18 \( \Omega \) is the same as that across the 6 \( \Omega \) in Fig. 27-18(b), namely, \( V_n = 3 V \). Thus \( I_n = 0.167 A \).
27.3 EMF AND ELECTROCHEMICAL SYSTEMS

27.49 Define: electromotive force, internal resistance, galvanometer.

The electromotive force (emf) of a source is the energy per unit charge converted from chemical or some other form of energy to electric energy. The emf \( \mathcal{E} \) is one volt when each coulomb of charge receives one joule of energy.

The term internal resistance refers to the resistance within an emf source.

A galvanometer is an instrument for measuring very small electric currents; it underlies both voltmeters and ammeters.

27.50 How are electric potential difference and emf related in a circuit?

The potential difference across a battery with negligible internal resistance is equal in magnitude to the emf. As a charge \( q \) passes from the low-voltage (negative) to the high-voltage (positive) terminal through the battery with emf \( \mathcal{E} \), the nonelectric forces do work \( q \mathcal{E} \) on it. If \( q \) is positive, this is positive work and the battery gives up energy. The electric forces, on the other hand, do an equal amount of negative work and the electrostatic system gains electric potential energy. As the charge \( q \) passes around the external circuit back to the negative terminal, this electrostatic energy is depleted and gets replenished by \( q \) going through the battery once more.

27.51 If a battery or other source of emf has internal resistance, then the terminal voltage does not equal the emf when current is flowing. What are the relations between emf and terminal voltage in such situations?

The terminal voltage of a battery or generator when it delivers a current \( I \) is equal to the total electromotive force (emf or \( \mathcal{E} \)) minus the potential drop (or voltage drop) in its internal resistance, \( r \).

1. When delivering current (on discharge), terminal voltage = emf − (voltage drop in internal resistance) = \( \mathcal{E} - IR \).
2. When receiving current (on charge), terminal voltage = emf + (voltage drop in internal resistance) = \( \mathcal{E} + IR \).
3. When no current exists, terminal voltage = emf of battery or generator.

27.52 Find the potential difference between points \( A \) and \( B \) in Fig. 27-19 if \( R \) is 0.7 \( \Omega \). Which point is at the higher potential?

Clearly this is only part of a larger circuit that imposes the 3-A current on the system. We add the voltage difference across the four elements starting at \( A \): \( V_B - V_A = (-6 \, \text{V}) - (3 \, \text{A})(2 \, \Omega) + (9 \, \text{V}) - (3 \, \text{A})(0.7 \, \Omega) = -5.1 \, \text{V} \). \( V_A \) is at the higher potential.

[Equivalently we could add the “voltage drops” from \( A \) to \( B \): \( V_A - V_B = 6 \, \text{V} + (3 \, \text{A})(2 \, \Omega) + (-9 \, \text{V}) + (3 \, \text{A})(0.7 \, \Omega) = 5.1 \, \text{V} \), which is the same result.]