Goos–Hänchen induced vector eigenmodes in a dome cavity

David H. Foster,* Andrew K. Cook, and Jens U. Nöckel

1Deep Photonics Corporation, Corvallis, Oregon 97333, USA
2Department of Physics, University of Oregon, Eugene, Oregon, 97403, USA
*Corresponding author: noeckel@darkwing.uoregon.edu

Received February 23, 2007; revised April 12, 2007; accepted April 26, 2007; posted May 2, 2007 (Doc. ID 80390); published June 14, 2007

We demonstrate numerically calculated electromagnetic eigenmodes of a 3D dome cavity resonator that owe their shape and character entirely to the Goos–Hänchen effect. The V-shaped modes, which have purely TE or TM polarization, are well described by a 2D billiard map with the Goos–Hänchen shift included. A phase space plot of this augmented billiard map reveals a saddle-node bifurcation; the stable periodic orbit that is created in the bifurcation corresponds to the numerically calculated eigenmode, dictating the angle of its “V.” A transition from a fundamental Gaussian to a TM V mode has been observed as the cavity is lengthened to become nearly hemispherical. © 2007 Optical Society of America

OCIS codes: 140.4780, 260.5430, 260.2110, 230.5750.

The Goos–Hänchen (GH) effect [1] is an apparent parallel translation of the reflected beam away from its geometrically expected position, arising in oblique reflection at dielectric interfaces or layered structures when the reflection phase shift is a function of the incident angle. It can be observed in single- or multiple-scattering experiments, with planar [1] or curved [2] surfaces; however, the most sensitive measurements rely on optical resonators [3,4]. The GH effect can manifest itself indirectly in spectral shifts of the cavity modes [5], but it is desirable to make direct spatially resolvable observations to distinguish unambiguously between a lateral shift in beam position and the ubiquitous alteration of effective cavity length due to finite penetration depth [6]. This was achieved in [3], which also contains a discussion of the experimental status of the GH effect.

Here we present and analyze theoretically a class of eigenmodes in a nonparaxial dome cavity, the “V modes” (Fig. 1), showing a distinct spatial (V-shaped) structure. We find that the GH effect is responsible for the creation and stability of these modes, determining their characteristic opening angle by simple geometry.

We consider a perfectly conducting concave spherical mirror, $M_2$, facing a planar thin film (distributed Bragg reflector), denoted by $M_1$; see Fig. 1(a). The electromagnetic eigenmodes are calculated by the basis expansion methods described in [7]. We first described the V modes in [7], but no physical explanation for their appearance has been put forward until now.

To provide quantitative results, we fix some parameters: the radius of $M_2$ is $R=60\ \mu m$, and the length of the resonator, measured from the front surface of $M_1$ to the zenith of $M_2$, is $L=R-z_1$, where we will focus on the range $0.3\leq z_1\leq 0.7\ \mu m$. This means the resonator is nearly hemispherical, and increasing mirror offset $z_1$ makes the cavity more paraxial. The layer structure of $M_1$ is $IAAAA(BA)^{22}O$, where $I$ denotes the vacuum region inside the cavity, each $A$ denotes a layer of $n=3.52$ material (AlGaAs) of quarter-wave thickness at $570\ nm$, each $B$ denotes a layer of $n=3.0$ material (AlAs), also of quarter-wave thickness at $570\ nm$, and $O$ denotes the vacuum region outside the cavity.

Axial symmetry allows one to label all cavity modes by a total angular momentum quantum number $m$; the complex electric and magnetic fields $E\phi$, $E_x$, $B\phi$, $B_x$, and $B_z$ (in cylinder coordinates) are proportional to $\exp(im\phi)$. In this paper, we present data only for modes with $m=\pm 1$. The vectorial eigenmodes typically found in an axially symmetric cavity are in general neither TE nor TM. Here, TE refers to fields whose angular spectrum contains only $s$-polarized plane waves with respect to the Bragg mirror; TM denotes $p$ polarization.

Figure 1(b) shows a V mode having wavelength $\lambda=755.44\ nm$ at $z_1=0.4\ \mu m$. The contrast-enhanced plot shows the $y-z$ slice of the physical $E_x$ field ($z$ is
the dome axis). Azimuthally, the electric field here is proportional to \(\exp[i(\pm \phi - o\ell)]\), which may be combined to make the “linearly” polarized versions \(\cos/sin(\phi)\exp(-i o\ell)\). The V mode is almost completely \(s\)-polarized (TE). Figure 1(c) shows the TE polar angle distribution of plane waves, \(|\psi_p(\theta_p)|^2\), for this mode; \(\theta_p\) is the angle of incidence of a plane wave with respect to the cavity axis. The TM admixtures are negligible on the scale of the plot.

Averaging \(\theta_p\) against this distribution yields \(\langle \theta_p \rangle\) \(=53.6^\circ\), which one can identify as the “opening angle” of the V-shaped beam. Similar modes have been observed that are entirely TM. We also found TE modes that exhibit a transverse node in each side of the V. This leads us to introduce the notation \(\text{VTM}_m^p\) and \(\text{VTE}_n^p\) for these V modes, where \(m\) is the total angular momentum and \(n\) is the number of nodes in the transverse wave function of each side of the V. The mode in Fig. 1 is then \(\text{VTE}_0^0\).

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A plot of nine trajectories, each of 1000 bounces, of the map \(T\) for \(z_1=0.55\) and \(s\) polarization (TE) is shown in Fig. 3. The island structure surrounding the stable \(\theta=0\) PO manifests itself in wave physics as the family of paraxial Gaussian modes. The separatrix indicates an unstable PO at \(\theta=37^\circ\), and the second island indicates a stable PO at \(\theta=49^\circ\). As shown in Fig. 2(b), these two POs are created at the same point (this is called a saddle-node bifurcation) as \(z_1\) is lowered to about 0.6 and move apart in \(\theta\) as \(z_1\) is further decreased. As \(z_1\) decreases, the \(\theta=0\) island shrinks, while the new island grows. As it grows, one or more existing wave modes may become distorted until one of them becomes the V mode; the V modes live inside the new island. If the GH effect is turned off in the billiard map (3), there is no new island, and the only PO that exists is the \(\theta=0\) orbit. This leads to the important conclusion that only a ray map augmented by the GH effect predicts the stable cavity modes correctly. This analysis can be applied equally to the \(s\)- or \(p\)-polarized V modes, by inserting the appropriate reflection phase \(\phi_{\text{rip}}\) in the map \(T\).

To characterize the stability conditions for the V modes, we investigate the phase space of the classical ray dynamics. The ray in Fig. 1(a) is a periodic orbit of a billiard map that obeys specular reflection at the top mirror \(M_2\) but receives a GH shift along the \(M_1\) surface, the length of which is given by Eq. (1). To specify this map, we follow a ray through multiple reflections, and just before the \(n\)th bounce at mirror \(M_2\), note its polar coordinate \(\vartheta(n)\) as shown in Fig. 1(a), as well as the component \(p_{\vartheta}^{(n)}\) of the (unit) momentum in the \(\vartheta\) direction. This pair of ray properties is mapped from one reflection at \(M_2\) to the next by the billiard map:

\[
T:(p_{\vartheta}^{(n)}, \vartheta(n)) \mapsto (p_{\vartheta}^{(n+1)}, \vartheta(n+1)),
\]

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At sufficiently large mirror offset \(z_1\), we should be able to make contact with Gaussian-beam theory for the transverse field patterns. In [7,10,11], a dielectric mirror provides a form birefringence that causes the transverse mode patterns of many typical eigenmodes to be symmetric and antisymmetric mixtures of specific pairs of vectorial Laguerre–Gauss modes, causing either TE or TM polarization to dominate the plane-wave spectrum. Starting from paraxial theory, these “mixed” modes can be denoted by the trans-
verse order $N$, the angular momentum $m$, and whether or not they are quasi-TE or quasi-TM:

$$QTE(M_N^m = LG^{m-1}_{(N+|m|)} + LG^{m+1}_{(N+|m|)}$$

for $|m| \geq 1$, $N-|m+1| \in \{0,1,\ldots\}$, and $N-|m-1| \in \{0,1,\ldots\}$. Here $\sigma^\pm$ is the Jones vector for right/left circular polarization and $LG$ denotes the Laguerre–Gauss wave pattern [10]. A detailed discussion of the mixed modes is given in [11]; they, along with pure Gaussians occurring for $|m|=N+1$, comprise the background of eigenmodes against which the V modes appear.

We numerically observe that V modes evolve from some existing mode of this paraxial type as $z_1$ is reduced and the stable island grows. As an example of this metamorphosis, a VTE $VTE_0^1$ mode was created from the predominantly TE mixed mode having the transverse E field, in the paraxial limit, given by $QTE_1 = LG^0_0 \sigma^+ + LG^2_1 \sigma^-$. On the other hand, as shown in Fig. 4, a VTE$_0^1$ mode was created from the fundamental Gaussian mode TEM$_{00}$, which originally has circular polarization. The polarization-dependent stabilization provided by the GH effect thus overcomes the polarization state expected for the paraxial fundamental mode in an axial symmetry cavity. This nonparaxial nature of the V modes underlines the predictive power of the GH picture.

Although the numerical results presented here were limited to $|m|=1$, we have observed VTM$_0^2$, VTE$_0^2$, and VTE$_1^2$, and we expect V modes to generally exist for $|m| \geq 1$. Other dielectric stack designs than the one described here have also been observed to induce V modes.

Patterns reminiscent of the V modes also appear in purely 2D calculations [12] for a circle with a dielectric boundary across its diameter. This roughly corresponds to the limit $z_1 \to 0$ in our model, but neither the relation to the GH effect nor the $z_1$-dependent opening angle of the V modes have been addressed before. Our approach of adding nonspecular corrections to the classical ray mapping $T$ provides a novel starting point on which semiclassical methods such as those of [12] may be built.

The V modes demonstrate that small corrections to specular reflection, such as the GH shift, can have large qualitative effects. Billiard models augmented by nonspecular corrections possess new phase space structures that correspond to the new modes. The V mode in particular is suitable for a laser with two or four entrance/exit channels, yet it economically uses only one curved surface. Interestingly, the spreading angle of each limb of the V modes is smaller than predicted by a Gaussian-beam calculation based on the path length of the V including GH shift. In future work, we will investigate to what extent a stability analysis of the ray map $T$ is able to explain this phenomenon.

This work was supported by NSF grant ECS-0239332.

References