PH 451: Capstone in Quantum Mechanics

Homework 7

Due 2/27/09

1. In a small group activity in class, we found the 8 by 8 matrix for the correction to the hydrogen Hamiltonian due to spin-orbit coupling in the uncoupled basis of states. Diagonalize this matrix explicitly. Do you need to diagonalize the whole matrix at one go?

2. Review the calculation of the spin orbit and relativistic corrections for the hydrogen atom (Griffiths, Section 6.3) and be certain you understand the origin of Equations 6.57, 6.65, and 6.67. You don't have to turn anything in, but you must understand it.
   Now, armed with your complete understanding, calculate the size of the following (for hydrogen): (for a-d, tabulate your results and give answers in three forms: theoretical in terms of $\alpha^n mc^2$, in *eV* or *meV*, and in *GHz*)

a) The energy difference between the n = 1 and n = 2 states BEFORE any perturbations were considered.

b) The correction to the n = 1 and n = 2 states due to spin-orbit coupling. Note that the formula we derived in class is problematic for $\ell = 0$. Show that if you set $j = \ell + \frac{1}{2}$ and then use $j = \frac{1}{2}$, the problem goes away. (This is the Darwin term we talked about, but go ahead and call it spin-orbit here.)

c) The correction to the n = 1 and n = 2 states due to the relativistic term.

d) The total correction to these states, *i.e.*, the fine structure correction.

e) What wavelength resolution must your detector have to be able to able to resolve the two lines in the n = 2 to n =1 transition? Be careful here. When you include the correction, you will find that it is very small compared to the unperturbed value. Be sensible about how to include the effects.

f) Is it important to use the reduced mass of the electron in your calculations or is it OK to use the free mass?

(More questions on the back.)

3. Derive the expression for the energy of the hydrogen atom levels in a strong external magnetic field. Proceed as follows:

(i) If the applied magnetic field is "strong", what is it "strong" compared to? Give a numerical estimate.

(ii) If the external field is strong, we can ignore (for the moment) the spin-orbit (S-O) and relativistic effects. Consider the Zeeman Hamiltonian $\hat{H}_{Zeeman} = \frac{e}{2mc} B_{ext} (L_z + 2S_z)$ as a perturbation of the field-free Hamiltonian (excluding S-O and relativistic terms), and calculate the correction to the energy levels. (Why must you use the $|n, \ell, m_\ell, s, m_s\rangle$ basis?)

(iii) Now consider the spin-orbit and relativistic effects as perturbations to the field-free plus Zeeman Hamiltonians and show that these fine structure corrections result in:

$$E_{fs} = \frac{mc^2\alpha^4}{2n^3}\left\{\frac{3}{4n} - \left[\frac{\ell(\ell+1) - m_\ell m_s}{\ell(\ell+\frac{1}{2})(\ell+1)}\right]\right\} \text{(at least for states where } \ell \neq 0\text{)}$$

(iv) Draw an energy level diagram for the 2*p* states of hydrogen, showing how the degeneracy is lifted in the presence of a strong external field.