1. (Goswami 18.1) Calculate the first-order corrections to the energy and wave function of the one-dimensional harmonic oscillator ground state due to the anharmonic perturbation $Cx^3$.

2. (Goswami 18.5 & Liboff 13.7) A particle of mass $m$ is in an infinite potential well that is perturbed as shown at right.
   a) Calculate the first-order energy shift of the $n$th state.
   b) Calculate the second-order energy shift of the ground state.

3. (Griffiths 6.9) Consider a quantum system with just three states and a Hamiltonian given by

   \[ H = V_0 \begin{pmatrix} 1 - \epsilon & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{pmatrix} \]

   where $V_0$ is a constant and $\epsilon$ is a small number ($\epsilon \ll 1$).
   a) Write down the eigenvectors and eigenvalues of the unperturbed Hamiltonian ($\epsilon = 0$).
   b) Solve for the exact eigenvalues of the complete Hamiltonian. Expand each of them as a power series in $\epsilon$, up to second order.
   c) Use first and second-order nondegenerate perturbation theory to find the approximate eigenvalue for the state that grows out of the nondegenerate eigenvector of the unperturbed Hamiltonian. Compare with the exact result from (b).
   d) Use degenerate perturbation theory to find the first-order corrections to the two initially degenerate eigenvalues.