

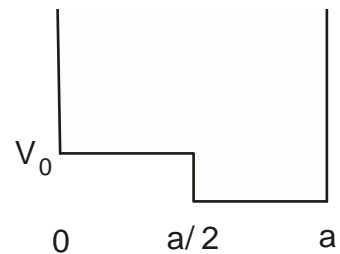
PH 451: Capstone in Quantum Mechanics

Homework 6

Due 2/13/09

1. (Goswami 18.1) Calculate the first-order corrections to the energy and wave function of the one-dimensional harmonic oscillator ground state due to the anharmonic perturbation Cx^3 .
2. (Goswami 18.5 & Liboff 13.7) A particle of mass m is in an infinite potential well that is perturbed as shown at right.

- a) Calculate the first-order energy shift of the n th state.
- b) Calculate the second-order energy shift of the ground state.



3. (Griffiths 6.9) Consider a quantum system with just three states and a Hamiltonian given by

$$H \doteq V_0 \begin{pmatrix} (1-\epsilon) & 0 & 0 \\ 0 & 1 & \epsilon \\ 0 & \epsilon & 2 \end{pmatrix}$$

where V_0 is a constant and ϵ is a small number ($\epsilon \ll 1$).

- a) Write down the eigenvectors and eigenvalues of the unperturbed Hamiltonian ($\epsilon = 0$).
- b) Solve for the exact eigenvalues of the complete Hamiltonian. Expand each of them as a power series in ϵ , up to second order.
- c) Use first and second-order nondegenerate perturbation theory to find the approximate eigenvalue for the state that grows out of the nondegenerate eigenvector of the unperturbed Hamiltonian. Compare with the exact result from (b).
- d) Use degenerate perturbation theory to find the first-order corrections to the two initially degenerate eigenvalues.