PH 451: Capstone in Quantum Mechanics

Homework 4

Due 1/30/09

1. (Griffiths 2.13) A particle in the harmonic oscillator potential starts out in the state

$$\psi(x,0) = A\left[3\psi_0(x) + 4\psi_1(x)\right]$$

   a) Find A.

   b) Construct $\psi(x,t)$ and $|\psi(x,t)|^2$.

   c) Find $\langle x \rangle$ and $\langle p \rangle$. Check that Ehrenfest's theorem (Eqn. 1.38 Griffiths) holds for this case.

   d) If you measured the energy of this particle, what values might you get, and with what probabilities?

2. (Griffiths 3.34) A harmonic oscillator is in a state such that measurement of the energy would yield either $\hbar\omega/2$ or $3\hbar\omega/2$, with equal probability. What is the largest possible value of $\langle p \rangle$ in such a state? If it assumes this maximal value at time t = 0, what is $\psi(x,t)$?

3. A particle is in a state that has orbital angular momentum quantum numbers $\ell = 1; m_\ell = 0$.
   a. What is the physical meaning of these numbers? Use the Maple worksheet links on the syllabus to make a plot of the angular probability distribution of this state.
   b. Suppose you measure the orbital angular momentum component along the x-direction. What might you measure? And with what probability?
   c. Suppose such a measurement yields one unit of angular momentum along the x-direction. Use the Maple worksheet links to make a plot of the angular probability distribution of this new state.
   d. You subsequently measure the orbital angular momentum component along the z-direction. What might you measure? And with what probability?