

PH 451: Capstone in Quantum Mechanics

Homework 3

Due 1/23/09

1. (Goswami 7.8) (Think about a simple application of the quantum harmonic oscillator result.) Consider a proton as a bound oscillator with a natural frequency of 3×10^{21} Hz. What is the energy of its ground state? its first excited state? What is its classical oscillation amplitude?
2. (Goswami 7.5) What are the energy levels of a particle of mass m moving in the one-dimensional potential well defined by:

$$V(x) = \begin{cases} \frac{1}{2} m \omega^2 x^2 & \text{for } x < 0 \\ \infty & \text{for } x > 0 \end{cases}$$

(Hint: You don't need to do a lengthy calculation.)

3. (Goswami 7.A4) Suppose a particle is in the ground state of the oscillator potential

$$V_1(x) = \frac{1}{2} m \omega_1^2 x^2$$

Suddenly the potential changes to

$$V_2(x) = \frac{1}{2} m \omega_2^2 x^2$$

What is the probability that the particle will be in the ground state of the new potential?

4. (Griffiths 2.41) A particle in the harmonic oscillator potential starts out in the state

$$\psi(x, 0) = A \left[1 - 2\sqrt{\frac{m\omega}{\hbar}} x \right]^2 e^{-\frac{m\omega}{2\hbar} x^2}$$

for some constant A .

- a) What is the expectation value of the energy?
- b) At some later time T the wave function is

$$\psi(x, T) = B \left[1 + 2\sqrt{\frac{m\omega}{\hbar}} x \right]^2 e^{-\frac{m\omega}{2\hbar} x^2}$$

for some constant B . What is the smallest possible value of T ?