1 Common Power Series

The following power series for common functions are used so often in physics, that you should make the extra effort to memorize them.

\[
\sin(z) = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n+1}}{(2n+1)!} \quad \text{valid } \forall z
\]

\[
= z - \frac{z^3}{3!} + \frac{z^5}{5!} - \frac{z^7}{7!} + \ldots
\]

\[
\cos(z) = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!} \quad \text{valid } \forall z
\]

\[
= 1 - \frac{z^2}{2!} + \frac{z^4}{4!} - \frac{z^6}{6!} + \ldots
\]

\[
e^z = \sum_{n=0}^{\infty} \frac{z^n}{n!} \quad \text{valid } \forall z
\]

\[
= 1 + z + \frac{z^2}{2!} + \frac{z^3}{3!} + \frac{z^4}{4!} + \ldots
\]

\[
\ln(1 + z) = \sum_{n=1}^{\infty} (-1)^{n+1} \frac{z^n}{n} \quad \text{valid for } |z| < 1
\]

\[
= z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \ldots
\]

\[
(1 + z)^p = \sum_{n=0}^{\infty} \frac{p!}{n!(p-n)!} z^n \quad \text{valid for } |z| < 1
\]

\[
= 1 + pz + \frac{p(p-1)}{2!} z^2 + \frac{p(p-1)(p-2)}{3!} z^3 + \ldots
\]

You may not know the meaning of \( p! \) if \( p \) is not a positive integer. If necessary, just use the second line of the power series for \((1+z)^p\) instead of the first line. (The factorial function can, in fact, be extended to be a valid function on the domain of all complex numbers, except the negative integers; the process is called analytic continuation. If you are curious, look up “gamma function” in any good mathematical methods text.)