Symmetries & Idealizations Homework 4

Due 10/9/15 @ 4:30 pm

QUIZ:

1. Be able to give the first four nonzero terms of the power series for $\sin z$, $\cos z$, e^z, $\ln(1+z)$, and $(1+z)^p$.

PRACTICE:

2. Write out the first four nonzero terms in the series:

   (a)
   $$\sum_{n=0}^{\infty} \frac{1}{n!}$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n!}$$

3. Write the following series using sigma ($\sum$) notation.

   (a)
   $$1 - 2\,\theta^2 + 4\,\theta^4 - 8\,\theta^6 + \dots$$

   (b)
   $$\frac{1}{4} - \frac{1}{9} + \frac{1}{16} - \frac{1}{25} + \dots$$

4. If you need more practice with sigma ($\sum$) notation, you can get really good practice by going back and forth between the two representations of the standard power series on the memorization page. Power series are used everywhere in physics and it is very important to be able to translate back and forth between the two representations.

REQUIRED:

5. Consider the geometry of $|\vec{\boldsymbol{r}} - \vec{\boldsymbol{r}}'|$.

   (a) Make a sketch of the graph
   $$|\vec{r} - \vec{a}| = 2$$
   for each of the following values of $\vec{a}$:
   $$\begin{aligned} \vec{a} &= \vec{0} && (1) \\ \vec{a} &= 2\hat{\imath} - 3\hat{\jmath} && (2) \\ \vec{a} &= \text{points due east and is 2 units long} && (3) \end{aligned}$$

(b) Derive a more familiar equation equivalent to

$$|\vec{r} - \vec{a}| = 2$$

for arbitrary $\vec{a}$, by expanding $\vec{r}$ and $\vec{a}$ in rectangular coordinates. Simplify as much as possible. (Ok, ok, I know this is a terribly worded question. What do I mean by "more familiar"? What do I mean by "simplify as much as possible"? Why am I making you read my mind? Try it anyway. Real life is not full of carefully worded problems. Bonus points to anyone who can figure out a better way of wording the question that doesn't give the point away.)

(c) Write a brief description of the geometric meaning of the equation

$$|\vec{r} - \vec{a}| = 2$$

6. Recall that, if you take an infinite number of terms, the series for $\sin z$ and the function itself $f(z) = \sin z$ are equivalent representations of the same thing for all real numbers z, (in fact, for all complex numbers z). This is not always true. More commonly, a series is only a valid, equivalent representation of a function for some more restricted values of z. The technical name for this idea is convergence–the series only "converges" to the value of the function on some restricted domain.

   Find the power series for the function $f(z) = \frac{1}{1+z^2}$. Then, using the Mathematica worksheet from class (vfpowerapprox.nb) as a model, explore the convergence of this series. Where does your series for this new function converge? Can you tell anything about the region of convergence from the graphs of the various approximations? Print out a plot and write a brief description (a sentence or two) of the region of convergence.

   Note: As a matter of professional ettiquette (or in some cases, as a legal copyright requirement), if you use or modify a computer program written by someone else, you should always acknowledge that fact briefly in whatever you write up. Say something like: "This calculation was based on a (*name of software package*) program titled (*title*) originally written by (*author*) copyright (*copyright date*).

7. **Note: Do NOT do part c of this problem yet, save it until next week. But you might want to be thinking about it when we start covering electric fields.**

   Consider a series of three charges arranged in a line along the z-axis, charges $+Q$ at $z = \pm D$ and charge $-2Q$ at $z = 0$.

   (a) Find the electrostatic potential at a point P in the x, y-plane at a distance r from the center of the quadrupole.

   (b) Assume $r >> D$. Find the first two non-zero terms of a Laurent series expansion to the electrostatic potential you found in the first part of this problem.

(c) Is it possible to find the electric field at P from your answer to the first part of the problem? If you answered that it is possible, find the electric field. If you answered that it is not possible, explain your answer.

(d) A series of charges arranged in this way is called a linear quadrupole. Why?

(e) Briefly describe in words something you learned from doing this problem that you would like to remember for the future. Make your statement using good scientific writing, as you would in a research paper.