Symmetries & Idealizations Homework 3

Due 10/7/15 @ 4:30 pm

QUIZ:

1. Make sure that you have memorized the following identities and can use them in simple algebra problems:

$$\begin{aligned} e^{u+v} &= e^u\, e^v \\ \ln u\, v &= \ln u + \ln v \\ u^v &= e^{v \ln u} \end{aligned} \tag{1}$$

PRACTICE:

2. For each case below, find the total charge.
   (a) A positively charged (dielectric) spherical shell of inner radius a and outer radius b with a spherically symmetric internal charge density $\rho(\vec{\boldsymbol{r}}) = \alpha\, 3e^{(kr)^3}$
   (b) A positively charged (dielectric) cylindrical shell of inner radius a and outer radius b with a cylindrically symmetric internal charge density $\rho(\vec{\boldsymbol{r}}) = \alpha\, \frac{1}{r}\, e^{kr}$.

REQUIRED:

3. When physicists calculate the value of a physical quantity from an equation, they pay particular attention to the units involved. A force of 2 is ill-defined, a force of 2 Newtons is clear. When physicists want to check the plausibility of an equation, without worrying exactly about which set of units will be used (e.g. Newtons vs. pounds vs. dynes), they often look at the "dimensions" of the physical quantities involved. "Dimension" refers to the powers of the basic physical quantities: length (L), time (T), mass (M), and charge (C), that make up the physical quantity. For example, since force is mass times acceleration,the dimensions of force are ML/T^2. Find the dimensions of electrostatic potential energy. Also, find the dimensions of electrostatic potential.

4. In this course, two of the primary examples we will be using are the force due to gravity and the force due to an electric charge. Both of these forces vary like $1/r^2$, so they will have many, many similarities. Most of the calculations we do for the one case will be true for the other. But there are some extremely important differences:
   (a) Find the value of the electric potential energy of a system consisting of a hydrogen nucleus and an electron separated by the Bohr radius. Find the value of the gravitational potential energy of the same two particles at the same radius. Use the same system of units in both cases. Compare and the contrast the two answers.

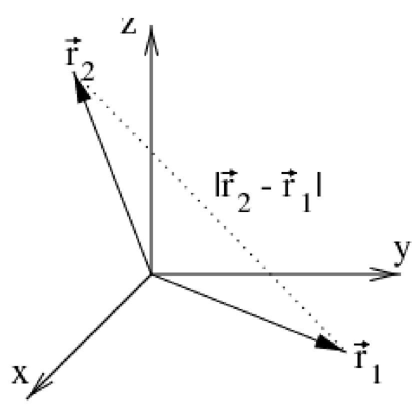
(b) Find the value of the electric potential due to the nucleus of a hydrogen atom at the Bohr radius. Find the gravitational potential due to the nucleus at the same radius. Use the same system of units in both cases. Compare and contrast the two answers.

(c) Think of and briefly discuss at least one other fundamental difference between electromagnetic and gravitational systems. Hint: Why are we bound to the earth gravitationally, but not electromagnetically?

5. Use integration to find the total mass of ice cream in a packed cone (both cone and hemisphere of ice cream on top).

6. The distance $|\vec{r}_2 - \vec{r}_1|$ between the point $\vec{r}_1 = (x_1, y_1, z_1)$ and the point $\vec{r}_2 = (x_2, y_2, z_2)$ is a coordinate-independent, physical and geometric quantity. But, in practice, you will need to know how to express this quantity in different coordinate systems.

(a) Find the distance $|\vec{r}_2 - \vec{r}_1|$ between the point $\vec{r}_1 = (x_1, y_1, z_1)$ and the point $\vec{r}_2 = (x_2, y_2, z_2)$ in rectangular coordinates.



(b) Show that this same distance written in cylindrical coordinates is:

$$|\vec{r}_2 - \vec{r}_1| = \sqrt{r_2^2 + r_1^2 - 2r_1 r_2 \cos(\phi_2 - \phi_1) + (z_2 - z_1)^2}$$

(c) Show that this same distance written in spherical coordinates is:

$$|\vec{r}_2 - \vec{r}_1| = \sqrt{r_2^2 + r_1^2 - 2r_1 r_2 \left[\sin\theta_2 \sin\theta_1 \cos(\phi_2 - \phi_1) + \cos\theta_2 \cos\theta_1\right]}$$

(d) Now assume that $\vec{r}_1$ is in the x-y plane. Simplify the previous two formulas.