

Symmetries & Idealizations Homework 1

Due 9/30/15 @ 4:30 pm

QUIZ:

1. For each of the following complex numbers z , find z^2 , $|z|^2$, and rewrite z in exponential form, i.e. as a magnitude times a complex exponential phase:

$$z_1 = i,$$

$$z_2 = 2 + 2i,$$

$$z_3 = 3 - 4i.$$

PRACTICE:

2. If you are unfamiliar with Mathematica and you have not already done so, go through the following tutorials:

<http://www.wolfram.com/broadcast/screencasts/handsonstart/>

More detailed information can be found at:

<http://www.wolfram.com/support/learn/>

WolframAlpha - a Mathematica-based computation engine. Enter equations into the box in the window and get instant results. A remarkably powerful piece of software. Can be found at:

<http://www.wolframalpha.com/>

3. Make sketches of the following functions, by hand, all on the same axes. Briefly describe, using good scientific writing that includes both words and equations, the role that the number two plays in the shape of each graph:

$$y = \sin x \tag{1}$$

$$y = 2 + \sin x \tag{2}$$

$$y = \sin(2 + x) \tag{3}$$

$$y = 2 \sin x \tag{4}$$

$$y = \sin 2x \tag{5}$$

REQUIRED:

4. Use Euler's formula $e^{i\theta} = \cos \theta + i \sin \theta$ and its complex conjugate to find formulas for $\sin \theta$ and $\cos \theta$. In your physics career, you will often need to read these formula "backwards," i.e. notice one of these combinations of exponentials in a sea of other symbols and say, "Ah ha! that is $\cos \theta$." So, pay attention to the result of the homework problem!

5. For the following expressions, determine the complex conjugate, square, and norm. Plot and clearly label each on an Argand diagram.
- (a) $z_1 = 4i - 3$
 - (b) $z_2 = 5e^{-i\pi/3}$
 - (c) $z_3 = -8$
 - (d) In a few full sentences, explain the geometric meaning of the complex conjugate and norm.
6. Express $e^{i\omega t/2} - e^{-i3\omega t/2}$ in rectangular ($x + iy$) and exponential ($re^{i\theta}$) forms. (ωt is real & $\sin(\omega t) > 0$)