Prep 2-3

*Suggested finish date:* Friday, April 21

The formats (type, length, scope) of these Prep problems have been purposely created to closely parallel those of a typical exam (indeed, these problems have been taken from past exams). *To get an idea of how best to approach various problem types (there are three basic types), refer to these sample problems.*
1. a. Which units are equivalent to electric field magnitude? Select any/all that are true, but show your reasoning/derivation for each.
   (i) N·C
   (ii) kg·m/(C·s²)
   (iii) J/(C·m)
   (iv) N/C

b. The drawing to the right shows 3 point charges at the corners of a rectangle.
   (i) Find the net electric field at the location of the -10 nC charge that is caused by the other two charges.

   (ii) Find the net force (magnitude and direction) on the -10 nC charge.
1. c. A uniform electrical field of $1.30 \times 10^5$ N/C has a direction identical to the positive $x$-axis. A proton and an electron are placed into this field. The electron is placed at the origin (0,0); the proton is placed 1.50 cm to the right of the electron. Find the magnitude and direction of the net force on the electron.

d. A uniform electric field is present throughout a set of $x$-$y$ axes (coordinates given in m). The field strength is $E$ N/C (where $E$ is a known value) and its direction is the same as the positive $y$-axis. An electron is located at the origin. Find the net electric force (magnitude and direction) on a proton located at the point (2,0). To make things simpler, just express your answer in terms of $e$, $E$, and $k$ (in other words, don’t plug in numerical values for $e$ and $k$).

e. A uniform electric field is present throughout a set of $x$-$y$ axes (coordinates given in meters). The field strength is $E$ N/C (where $E$ is a known value) and its direction angle is $45^\circ$, measured conventionally (counter-clockwise from the positive $x$-axis). An electron is located at the origin. Find the net electric force (both magnitude and direction) on a proton located at the point (1,–1). To make things simpler, just express your answer in terms of $e$, $E$, and $k$ (in other words, don’t plug in numerical values for $e$ and $k$).
1.  Two electrons are located along the y-axis at points (0, s) and (0, −s).
You may consider the following as known values (so they may be included in your answers): k, \( \varepsilon_0 \), e, s

(i) Write an expression for the electric force magnitude exerted on one electron by the other.

(ii) Write an expression for the net electric field (magnitude and direction) at the point \((r, 0)\) on the positive x-axis. (You may consider \( r \) as a known value, too, so your answer may include \( r, k, \varepsilon_0, e, s \).) Be sure to use diagrams if they help!

(iii) Referring to the field expression from part b, how you would calculate the value of \( r \) where that field strength is a maximum? You do not have to actually do the calculation; just fully explain your reasoning and the procedure.
2. a. (i) Use equation 23.7 (p. 633) to solve the problem below.
   (ii) Then solve the same problem using equation 23.8 instead.

   An electric dipole with charges \( \pm 1.00\text{nC} \) and charge separation 2.00mm is located (centered) at the origin, with the charges located on the y-axis. Find the electric field produced by this dipole at the point (0, 10 cm).

   \[
   \vec{E}_{\text{axis}} = \frac{2kqys}{\left(y - \frac{s}{2}\right)^2 \left(y + \frac{s}{2}\right)^2} \hat{j}
   \]

   \[
   = \frac{2(8.99 \times 10^9)(1 \times 10^{-9})(0.10)(0.002)}{(0.10 - 0.001)^2 (0.10 + 0.001)^2} = 36.0 \text{ N/C}
   \]

   \[
   \vec{E}_{\text{axis}} \approx \frac{2kqs}{y^3} = \frac{2(8.99 \times 10^9)(1 \times 10^{-9})(0.002)}{(0.10)^3} = 36.0 \text{ N/C}
   \]

b. An electric dipole with charges \( \pm q \) and charge separation \( s \) is located (centered) at the origin, with the charges located on the y-axis.

   (i) Derive an expression for \( \vec{E}_{\text{dipole.bisector}} \), the total electric field value (both x- and y-components) at a distance \( r \) along the x-axis (i.e. in the plane that bisects the dipole).

   \[
   +q \cdot \vec{E} + \vec{E}_- = \left[ \frac{kq}{d^2} \cos q, -\frac{kq}{d^2} \sin q \right]
   \]

   \[
   \vec{E}_- = \left[ -\frac{kq}{d^2} \cos q, -\frac{kq}{d^2} \sin q \right]
   \]

   \[
   \vec{E}_{\text{total}} = \left[ 0, -2\frac{kq}{d^2} \sin q \right]
   \]

   \[
   \frac{\sin q}{d} = \frac{s/2}{d}
   \]

   \[
   d = \sqrt{\left(s/2\right)^2 + r^2}
   \]

   Thus

   \[
   \vec{E}_{\text{total}} = \left[ 0, -\frac{2kqs}{d^3/2} \right] = \left[ 0, -\frac{kqs}{\left(s/2\right)^2 + r^2}^{3/2} \right]
   \]

   Or

   \[
   \vec{E}_{\text{dipole.bisector}} = -\frac{kqs}{r^2 + (s/2)^2}^{3/2} \hat{j}
   \]

   (ii) Show that for small values of \( r \ (r \ll s) \), \( \vec{E}_{\text{dipole.bisector}} \approx \vec{E}_{\text{dipole.midpoint}} \) (i.e. the E-field at the origin).

   When \( r \) goes to zero, the above expression becomes \( -\frac{kqs}{(s/2)^3} \hat{j} \).

   This agrees with the result of HW1, item 6c, that \( \vec{E}_{\text{dipole.midpoint}} = \frac{8kq}{s^2} \).

   (iii) Show that for large values of \( r \ (r \gg s) \), \( \vec{E}_{\text{dipole.bisector}} \approx -\frac{1}{4\varepsilon_0} \left\{ \frac{qs}{r^3} \right\} \hat{j} \)

   When \( r \gg s \), \( r^2 + (s/2)^2 \) approaches \( r^2 \), so \( \vec{E}_{\text{dipole.bisector}} \) indeed approaches \( -\frac{kqs}{r^3} \hat{j} \).

   c. Evaluate (T/F/N) the following statement. Justify your answer fully with any valid mix of words, drawings and calculations.

   In the drawing at right (all lengths in nm), if the two charges are located at (0, 1.00) and (0, -1.00), then \( E(0, 126) \approx -E(100, 0) \).
3.  a. A wire’s linear net charge density as a function of the distance from the origin is given as \( \lambda = cx^2 \), where \( x \) is the distance from the origin and \( c \) is a positive constant. The wire extends from the origin to \( x = d \). What is the total net charge on the wire?

b. A small glass bead has a net charge of +6.00 nC and is located along the perpendicular bisector, 4.00 cm from the midpoint, of a thin, uniformly charged glass rod that is 10.0 cm long.
   (i) If the rod repels the bead with a force magnitude of 840\( \mu \)N, what is the total net charge on the rod?

   (ii) Assuming the result of part (i), if the bead were then moved to a location 4.00 m from the midpoint of the rod, with what force magnitude would the rod then repel it?

c. Two infinite lines of charge, each with the same uniform linear charge density \( \lambda \), lie along the \( x \)-and \( y \)-axes, respectively (so they intersect at the origin). What is the electric field strength (magnitude) at point \((x, y)\)?

d.  (i) Textbook problem 23.43a, page 655.

   (ii) Textbook problem 23.43b, page 655.
4. a. (i) Using the results of textbook Example 26.4, show that the on-axis electric field of a thin ring of radius $R$ and (uniformly distributed) net charge $Q$ has the expected result when $z \ll R$ and when $z \gg R$.

(ii) Find the location and magnitude of the maximum on-axis electric field of a thin ring of radius $R$ and (uniformly distributed) net charge $Q$.

b. A pair of thin 10.0-cm-diameter rings of charge face each other (they are parallel, with their central axes aligned), 20.0 cm apart. Each ring has a net charge of +20.0 nC, distributed uniformly.

(i) Find the electric field magnitude at the midpoint between the rings.

(ii) Find the electric field magnitude at the center of one of the rings.

(iii) If the rings were re-positioned (turned, moved closer to each other or further away from each other) by a few cm—in some unknown manner—what would be the approximate electric field magnitude at a point 1.00 km away from the midpoint between their centers?

c. A pair of thin 10.0-cm-diameter disks of charge face each other (they are parallel, with their central axes aligned), 20.0 cm apart. Each disk has a net charge of +50.0 nC, distributed uniformly.

(i) Find the electric field magnitude at the midpoint between the disks (on their common axis).

(ii) Find the electric field magnitude at a point on their common axis that is 5.00 cm from one of the disks.

(iii) If the disks were re-positioned (turned, moved closer to each other or further away from each other) by a few cm—in some unknown manner—what would be the approximate electric field magnitude at a point 1.00 km away from the midpoint between their centers?
4. A thin disk of uniformly distributed charge of radius $R$ is placed at a distance $d$ from an infinite line of charge of uniform density $\lambda$, so that the central axis of the disk intersects the line perpendicularly, as shown.

A proton is placed at point P, midway between the line and the disk’s center. The proton is free to move, but it does not. Find the total charge $Q$ of the disk.

You may consider the following as known values (so they may be included in your answer): $R, d, \lambda, k, e_0$. 

\[ Q = \frac{8\pi R^2 e_0 k\lambda}{d - d^2 (\frac{d^2}{4} + R^2)^{-1/2}} \]
5. a. A wire extends along the $x$-axis from $x = -L$ to $+L$.
The wire has a total net charge $+Q$, and its linear charge density is $\lambda = c|x|$, where $c > 0$.
Find an expression for the electric field at $(0, h)$ where $h > 0$.

b. Evaluate (T/F/N) the following statement. Justify your answer fully with any valid mix of words, drawings and calculations.
If a line of positive charge of uniform density 4.00 nC/m begins at the origin and extends for a very long distance along the positive $x$-axis, then the E-field it causes at the point $(0, 1.50 \text{ m})$ has a $y$-component of 24.0 N/C.

6. a. A negative point charge \(-Q\) is located at a distance \(d\) from a large uniformly distributed plane of charge, of density \(\pm \eta\). Find the total E-field (magnitude & direction) midway between the point charge and the plane.


c. Three infinite parallel sheets of charge, A, B and C, are positioned as shown. These are each infinitely thin planes of charge—with no thickness and not located on any material (not on a slab or a conductor). Each sheet carries a uniform surface charge density as shown, and that charge is fixed—not able to move.

A \(+\eta\) | 1 •
---|---
B \(-\eta\) | 2 •
---|---
C \(+3\eta\) | 3 •
---|---

You may consider the following as known values (so they may appear in your answers): \(\eta, D, H, e, m_e, k, \varepsilon_0\)

(i) Give an expression for the electric field magnitude at each of the points 1, 2, 3 and 4. Along with each answer, draw an arrow to indicate the direction of that field.

(ii) Now suppose that an extra electron is placed and held at rest on sheet B, then released to move freely. Write an expression for the time interval, \(\Delta t\), that elapses as it moves to one of the other sheets.
7. a. One mole of excess protons is uniformly distributed over the surface of a sphere of radius 1.00 m. What is the surface charge density on the sphere?

b. A spherical conductor of radius 1.00 m has excess charge on its surface, the density of which is 1.00 C/m². What is the magnitude of the electric field at a point 9.00 m from its surface and 10.0 m from its center?

c. The electric field strength 3.00 cm from the surface of a metal ball of diameter 8.00 cm is 40,000 N/C. Find the net charge (in nC) on the ball.

b. An electron is initially moving to the right at $1.25 \times 10^6$ m/s in a uniform electric field that also points to the right and has a magnitude of 1.50 kN/C. How long (in seconds) does it take for the electron to return to its initial location? (Assume that the only force acting on the electron is the electric force exerted by the field.)


d. Textbook problem 23.52, page 656.

e. Textbook problem 23.58, page 656.
8. A very long, very wide metal slab carries a total net charge of +990 C. An electron is given an initial velocity of 300,000 m/s in a direction parallel to the slab. A short time (25.0 ns) later, the electron strikes the slab. Its impact velocity makes an angle of 60° with the surface of the slab. Find the total surface area of the metal slab.


c. Textbook problem 24.9, p 683.


g. Evaluate (T/F/N) each statement. Justify your answers with a valid mix of words, drawings and calculations.
   (i) Refer to the drawing at right. A uniform E-field is directed as shown through a planar rectangular surface \( \Delta A \), at an angle of 20.0° measured as shown. If \( \Phi_E = 3762 \text{ N}\cdot\text{m}^2/\text{C} \), and \( E = [-229, -100, 0] \text{ N/C} \), then \( \Delta A \approx 44.0 \text{ m} \).

(ii) In the drawing at right, a hollow box contains a point charge of +10.0 nC in its central cavity. The box’s outer surface carries a net charge of +25.0 nC. No net charge exists in the interior of the box material (i.e. within the grey area). There is no information about charge on the inner cavity surface. A Gaussian surface (dotted line) entirely encloses the box. If the box is an insulator, then the net flux through the Gaussian surface must be greater than if the box is a conductor in electrostatic equilibrium.
10. a. A certain Gaussian surface has a electric flux $\Phi_1$. Another Gaussian surface, with a smaller total surface area, that is completely enclosed within the first one has a smaller electric flux $\Phi_2$. What physical arrangement of field lines (and/or charges) and Gaussian surfaces could satisfy these conditions? Use a diagram to show this clearly.


11. a. The electric flux is 300 N·m$^2$/C through each of two opposing faces of a cube (2 cm on a side). The flux through each of the other faces is 300 N·m$^2$/C. How much net charge is inside the cube?

b. Textbook problem 24.34, p 684.

c. Find an expression for the charge distribution of the previous item (prob. 24.34).

11. e. A net charge of +30 nC is placed inside a hollow metal cube that measures 5.00 cm on each side. The cube is allowed to come to electrostatic equilibrium. Estimate the electric force magnitude that would be exerted on an electron located just outside the box (i.e. very near its surface). State any assumptions you need to make for your estimation.
11. f. Object A is a very long, cylinder of radius \( R \), carrying net positive charge with a volumetric distribution of \( \rho = cr^2 \). Object B has the same dimensions as A, but it is made of conductive material. What amount of charge must B carry per 1 meter of its length so that each cylinder creates the same E-field at a point 1 \( \mu \)m outside its surface? (You’re not given numeric values for this problem, so after showing all work, give your final answer as an expression containing \( c \) and \( R \), as necessary.)


12. f. Positive charge has a spherically symmetric distribution described by the density function

\[ \rho = 12r \]

(where \( \rho \) has units of C/m\(^3\), and \( r \) is in m).

The sphere of charge has an outer radius of 1.50 m and is bisected by a Gaussian surface (so that the center of the sphere is located on the Gaussian surface; half of the sphere is inside the surface, half outside).

The Gaussian surface is shaped like a large brick, with dimensions 4 m \( \times \) 4 m \( \times \) 6 m, as shown.

Point A is located 0.250 m from the outermost charge on the sphere.
Point B is located on the far right-hand face of the Gaussian surface.

(i) T/F/N? The E-field magnitude at point A is at least 11.7 times greater than at point B. **Justify your answer fully with any valid mix of words, drawings and calculations.**

(ii) T/F/N? Some of the charge produces both positive and negative flux through the Gaussian surface. **Justify your answer fully with any valid mix of words, drawings and calculations.**

(iii) Find the total (net) electric flux, \( \Phi_E \), through the entire Gaussian surface.
12. g. Textbook problem 24.48, p 685.
Taking the smaller Gaussian surface now:

\[ \text{But for the rest of the surface—the round ends—} \]

\[ \text{For the curved "barrel" portion of our cylindrical Gaussian surface,} \]

Taking the larger Gaussian surface first:

\[ \text{by definition, normal to the surface). Therefore:} \]

\[ h = E_{D\text{net.enclosed}} = 3 \]

\[ \frac{Q}{2} = 2 \]

\[ \text{is the area of each circular end of the cylindrical Gaussian surface we have chosen).} \]

\[ \text{Thus:} \]

\[ \int \frac{Q}{h} = \int \frac{e}{h} + \int \frac{b}{c} = 3 \]

\[ \frac{a}{3} \]

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\[ \frac{a}{3} \]

\[ \text{Thus:} \]

\[ \int \frac{Q}{h} = \int \frac{e}{h} + \int \frac{b}{c} = 3 \]

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\[ \int \frac{Q}{h} = \int \frac{e}{h} + \int \frac{b}{c} = 3 \]

\[ \frac{a}{3} \]

\[ \text{Thus:} \]

\[ \int \frac{Q}{h} = \int \frac{e}{h} + \int \frac{b}{c} = 3 \]

\[ \frac{a}{3} \]
12. h. Refer to the diagram below. Three positively charged objects are fixed along the x-axis.

Object A has a known total net charge $Q_A$. It is a “halo” of charge—a thin disk of uniformly distributed charge (of known outer radius $R_A$) but with a central hole (of known radius $r_A$) where there is no charge. The center of the halo is located at $(-2d, 0, 0)$, and the x-axis forms the halo’s central axis. ($d$ is known and is positive.)

Object B carries a known net charge $Q_B$. It is a thin ring of uniformly distributed charge of radius $R_B$, centered at the origin $(0, 0, 0)$, with the x-axis forming its central axis (i.e. it’s co-axial with the halo).

Object C is a line segment of charge, of known length $L$ and unknown charge density that has the form $\lambda = cvx$ (so $c$ is unknown), with the least-dense end on the left (i.e. positioned at the origin, as shown) and the line segment lying along the positive x-axis.

At the point $(0, 0, d \times 10^6)$, the E-field magnitude is known—call this $E_z$.

Find an expression (using any/all the known values given here, as needed) for the total E-field (magnitude and direction) at point P, which is located at $(-d, 0, 0)$.

*Note: There are two integrals necessary here. One is very straightforward, and you should evaluate it. The other is messy and you can leave it simply expressed as an integral. So your final expression may include one integral that you have not evaluated.*
That is, distribution., which means we'll need to find the total charge, the charge line segment's field magnitude at P requires an integration of all charge elements dq.

Therefore, dE = \( q \, dz \) and \( \sum dE \) is the total charge we will get by integrating \( dz \):

\[
\sum dE = \frac{Q}{C} \cdot 10^3 \, Q \cdot \frac{1}{2} \, dz.
\]

The halo is simply a disk.R, is given by

\[
\frac{dE}{Q} = \frac{C}{R} = \left( \frac{3}{2} \right) \, \frac{C}{R} + \frac{Q}{2}.
\]

The vector sum of all three objects' fields therefore looks like this: