HW 1

Due: Tuesday, April 24, 6:00 p.m.

Print your full LAST name: ____________________________________________

Print your full first name: ______________________________________________

Print your full OSU student ID#: ________________________________________

Turn this assignment in to box #15 (located outside of Wngr 234)
OR
email a pdf document of the complete file,
prepared as directed on the next page.

“I affirm and attest that this HW assignment is my own work.
All the reasoning and results here are my own doing.”

Sign your name (full signature): _______________________________________

Print today’s date: ____________________________________________________
HW 1

Directions:

Solve these four problems fully, just as you would if they were exam problems. Then turn in your entire solutions set (all these pages) to Box 15; OR email a .pdf file to coffinc@physics.oregonstate.edu with a subject line as follows:

HW1_LastName_FirstName

One of your four solutions will be chosen and scored for HW1 credit (35 points). Full solutions to all four problems will be posted soon after the due date/time.

Note: You’ll do much better with these problems if you work through the Prep sets 1 and 2-3 and also the SMT1 first, because these problems are all variations on selected Prep and SMT1 problems.

Note: The real MT1 (that’s the actual first midterm exam this term) may also include variations on selected Prep and SMT1 problems.
1. Two protons are fixed in place along the y-axis. One proton is located at \((0, d)\); the other is located at \((0, -d)\). An electron is then released from rest at some point \((x, 0)\) on the x-axis.

a. Derive an expression (a function of position \(x\)) for the force exerted on the electron by the protons.

b. Show that, for small magnitudes of \(x\) (i.e. when \(|x| \ll d\)), the above expression has the form

\[ F(x) = -\{\text{some positive constant}\} \cdot x \]

c. Describe the motion of the electron assuming that its release point was indeed near the origin (so that \(|x| \ll d\)). Denoting the mass of the electron as \(m_e\), find an expression for the frequency (in Hz) of its oscillation.

d. Now suppose the electron is brought to rest at the origin. Find an expression for the approximate electric field (both magnitude and direction) at the point \((-7500d, -10,000d)\) caused by the motionless three-particle system.

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a. From the diagram, we can see that the two forces are equal in magnitude (since each proton has the same magnitude and exerts its pulling force on the electron from the same distance). And the symmetry of the geometry will cause the y-components of the two forces to sum to zero.

Thus:

\[ F_{net,y} = 0 \]

The x-sum:

\[ F_{net,x} = -F_1\cos\theta - F_2\cos\theta \]

Same force magnitudes:

\[ F_{net,x} = -2F_1\cos\theta \]

We know \(F_1 = ke^2/r^2\):

\[ F_{net,x} = -2(ke^2/r^2)\cos\theta \]

And \(\cos\theta = x/r\):

\[ F_{net,x} = -2(ke^2/r^2)x \]

...where \(r = (x^2 + d^2)^{1/2}\)

Therefore:

\[ F_{net} = F_{net,x} = -[2ke^2/(x^2 + d^2)^{3/2}] \cdot x \]

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*(Full solutions and answers for parts b, c and d to be posted soon also, but they will not be scored.)*
Use this page as additional space, if needed, for problem 1
2. a. You have a continuous distribution of charge in the form of a line segment of length \( L \) \((L = 0.243 \text{ m})\). This segment is fixed in position along the positive \( y \)-axis, so that one end is at the origin and the other end is at \((0, L)\). The segment carries a total charge of \(-1.78 \text{ nC}\), and this charge is uniformly distributed along it. Using tools we have already derived, calculate the \( x \)-component of the total electric field (both magnitude and direction) at the point \((0.560, 0)\) m.

b. Now calculate the \( y \) component of the electric field that exists at the point \((0.560, 0)\) m, due to the above charge distribution. To do this, you will need to derive a new integral expression (“build the integral”). Draw the picture carefully—use symbols for all the values—and build the integral. Then go ahead and evaluate it with the known numbers; a \( u \)-substitution should work.  (Note: You would not be asked to do this evaluation step on an exam.)

c. Now calculate the initial acceleration (both magnitude and direction) of a proton that is released from rest at the point \((0.560, 0)\) m.

d. Now suppose the charge along the above line segment is re-arranged so that its charge density can be expressed as \( \lambda = by^{2/3} \), where \( \lambda \) is in C/m, and \( y \) is in m. Calculate the value of \( b \), including its units.  (Note: This kind of simple integration is fair game on the exam.)

a. We already know the field along the bisector of a uniformly charged line segment. Here, we have half of such a line segment. In other words, if we were to double the length of the given segment (and also double its total charge), we would have a ready-made formula for the \( x \)-directed \( E \)-field of that double-segment. After calculating that \( E \) value for the doubled segment, we would then just divide the result by two—because only the top half of that doubled segment is actually present.

So consider now a line segment that has a length \( L \) of \( 2(0.243 \text{ m}) = 0.486 \text{ m} \) and a net charge \( Q \) (uniformly distributed) of \( 2(–1.78 \text{ nC}) = –3.56 \text{ nC} \). Then from class derivations for a point \((r, 0)\) on its bisector...

We know this: \( E_x(r) = [2kQ/(Lr)][1 + (4r^2/L^2)]^{1/2} \)

\[ = \{2(8.99 \times 10^9)(–3.56 \times 10^{-9})/(0.486)(0.560)\}\{1 + [4(0.560)^2/(0.486)^2]\}^{1/2} \]

\[ = –93.621 \text{ N/C} \]

So for half the segment: \( E_x(r) = –93.621/2 = –46.811 = –46.8 \text{ N/C} \)

b. To calculate the \( y \) component of the electric field that exists at the point \((0.560, 0)\) m, due to the above charge distribution, we will need a fresh integral solution....

c. So the total field at \((0.560, 0)\) is: \( E = [–46.81, yyy] \text{ N/C} \)

As magnitude and angle: \( E = \sqrt{E_x^2 + E_y^2} \text{ N/C} \angle \theta \)

\[ = \sqrt{(46.81^2 + xxxx^2)} \text{ N/C} \angle [\tan^{-1}(yyy/–46.81) + 180] \]

\[ = E \text{ N/C} \angle qqqq^\circ \]

So the force on the proton: \( F = eE = (1.60 \times 10^{-19})(E) \text{ N} \angle qqqq^\circ \)

\[ = x.xx \times 10^{-x} \text{ N} \angle qqqq^\circ \]

(Solution and answer to be posted soon.)
d. Since $by^{2/3}$ must produce units of C/m for $\lambda$, the units of $b$ must be C/m$^{5/3}$.

The total value of $Q$ does not change; it’s still $-1.78$ nC. But now it’s non-uniform; you would sum it like this...

The summation:

$$Q = \int dq = \int \lambda dy = \int by^{2/3}dy$$

Set limits; evaluate:

$$Q = b \int_{0}^{0.243} y^{2/3} dy$$

Thus:

$$-1.78 \times 10^{-9} = (3/5)b(0.243)^{5/3}$$

Solve for $b$:

$$b = (5/3)(-1.78 \times 10^{-9})/(0.243)^{5/3}$$

$$b = -3.14 \times 10^{-8} \text{ C/m}^{5/3}$$
3. In each of the first three situations described here (A, B and C), all charges are fixed in position (nothing is moving), and the total electric field at point P is zero. Drawings are not necessarily to scale. Gravity is not a factor here.

**Situation A:** An infinite plane of uniformly distributed positive charge (+\(\eta\)) and two particles carrying equal but opposite net charges. The two particles \(q_1\) and \(q_2\) are separated by a distance \(s\) and are equidistant from the (empty) point P, and they are aligned along the perpendicular to the plane, as shown.

**Situation B:** Same plane of charge, same point P as above. But the two particles are removed and instead an infinite line of uniformly distributed charge is placed as shown—parallel to the plane and a distance \(s\) toward the plane from point P.

**Situation C:** Same plane of charge, same line of charge, and same point P as above. But now the line is located midway between the plane and point P. And now there is a disk of charge with the same uniform charge density as the plane. The disk’s surface is parallel to the plane, and point P is midway between the disk’s center and the line. The diameter of the disk is equal to the distance from the disk’s center to the plane.

Now in a final experiment (depicted here), there is only the plane of charge (still of value +\(\eta\)) and the single particle, \(q_2\), from above. But this time, \(q_2\) will move: It is released at point P with an initial velocity, \(v_i\), directed down the page—parallel to the plane, as shown. As you would expect, its path intersects the plane of charge. The point of intersection is located a distance \(R\) from point P.

These are the known values:

- \(\eta = 1.20 \mu C/m^2\)
- \(s = 2.50 \text{ cm}\)
- \(v_i = 8.74 \times 10^6 \text{ m/s}\)
- \(R = 13.9 \text{ cm}\)

Calculate \(m_2\), the mass of the \(q_2\) particle.
From situation A: If $q_2$ were negative and $q_3$ were positive, both their fields at point P would be to the right (call that the +x-direction); that would make it impossible for their fields and the plane’s field to sum to zero at point P. So $q_2$ must be negative and $q_3$ must be positive, and both their fields at point P must be to the left (the –x-direction).

Therefore: \[ E_{\text{plane,P}} - E_{q_2,P} - E_{q_3,P} = 0 \] 3 pts.
In detail: \[ \eta/(2\varepsilon_o) - 2k|q_2|/(s/2)^2 - k|q_3|/(s/2)^2 = 0 \] 1 pt.
Simplify: \[ \eta/(2\varepsilon_o) - 8k|q_2|/s^2 = 0 \] (noting that $|q_2| = |q_3|$) 1 pt.
Solve for $|q_2|$: \[ |q_2| = \frac{\eta s^2/(16\varepsilon_o)}{\pi\eta s^2/4} = \pi(1.20 \times 10^{-6})(0.025)^2/4 = 5.89 \times 10^{-10} \text{ C/m} \] 1 pt.

From situation B: The line must have negative charge, because if it were to have positive charge, that would make it impossible for its field and the plane’s field to sum to zero at point P. So the line’s field at point P must be to the left (the –x-direction).

Therefore: \[ E_{\text{plane,P}} - E_{\text{line,P}} = 0 \] 2 pts.
In detail: \[ \eta/(2\varepsilon_o) - 2k|\lambda|/s = 0 \] 1 pt.
Solve for $|\lambda|$: \[ |\lambda| = \frac{\eta s/(4k\varepsilon_o)}{\pi\eta s} = \pi(1.20 \times 10^{-6})(0.025) = 9.42 \times 10^{-8} \text{ C/m} \] 1 pt.

From situation C: Let the distance from the plane to the line be called $d$. Then the distance from the line to point P is also $d$; and the distance from point P to the disk is also $d$. And therefore the radius of the disk is $3d/2$.

Therefore: \[ E_{\text{plane,P}} - E_{\text{line,P}} - E_{\text{disk,P}} = 0 \] 3 pts.
In detail: \[ \eta/(2\varepsilon_o) - 2k|\lambda|/d - [\eta/(2\varepsilon_o)]\{1 - d/(d^2 + (d/2)^2)^{1/2}\} = 0 \] \[ \eta/(2\varepsilon_o) - 2k|\lambda|/d - \eta/(2\varepsilon_o) + [\eta d/(2\varepsilon_o)]/[d^2 + (3d/2)^2]^{1/2} = 0 \]
Expand: \[ \frac{\eta}{\varepsilon_0}\sqrt{13} = 2k|\lambda|/d \]
Solve for $d$: \[ d = 2\sqrt{(13)}|k|\varepsilon_0/\eta = \sqrt{(13)}|\lambda|/(2\pi\eta) \]
Substitute for $|\lambda|$: \[ d = \sqrt{(13)}(\pi\eta s)/(2\pi\eta) = \sqrt{(13)}s/2 \]

Situation D: Constant-acceleration kinematics apply here. There is zero acceleration of $q_2$ in the y-direction, and in the x-direction, the plane’s constant force acts on it. Let $\Delta t$ be the time interval from the moment when $q_2$ is given its initial velocity at point P to the moment it encounters the plane of charge. Here’s what we now know:

\[ \begin{align*} x\text{-direction} & & y\text{-direction} \\
\Delta x &= -2d = -\sqrt{(13)}s & \Delta y &= -\sqrt{(R^2 - \Delta x^2)} = -\sqrt{(R^2 - 13s^2)} \\
v_{x,i} &= 0 & v_{y,i} &= -v_i \\
v_{x,f} &= ?? & v_{y,f} &= -v_i \\
a_x &= F_i/m_2 = -|q_2|E_{\text{plane}}/m_2 = -\{\pi\eta s^2/4\}[(\eta/(2\varepsilon_o))]m_2 = -\pi\eta s^2/(8m_2\varepsilon_o) & a_y &= 0 \\
\Delta t &= ?? \\
\text{Use } \Delta y: & & \Delta y = v_{y,f}\Delta t \\
\text{Solve for } \Delta t: & & \Delta t = \Delta y/v_{y,i} = -\sqrt{(R^2 - 13s^2)}/(-v_i) = \sqrt{(R^2 - 13s^2)}/v_i \\
\text{Use } \Delta x: & & \Delta x = (1/2)a_x(\Delta t)^2 \\
\text{Substitute: } & & -\sqrt{(13)}s = (1/2)[-\pi\eta s^2/(8m_2\varepsilon_o)](R^2 - 13s^2)/v_i^2 \\
\text{Solve for } m_2: & & m_2 = \pi\eta s(R^2 - 13s^2)/(16\varepsilon_o v_i^2\sqrt{(13)}) \\
& & = \pi(1.20 \times 10^{-6})(0.025)(0.139^2 - 13(0.025)^2)/(16(8.74 \times 10^{-12})(8.74 \times 10^9)\sqrt{(13)}) \\
& & = 3.25 \times 10^{-20} \text{ kg} \]
4. Refer to the diagram here—an axial or “end-on” cutaway view (not to scale): An infinitely long solid cylinder of insulative material (glass) is located at the center of an infinitely long, hollow cylinder of metal, as shown. The glass cylinder has a radius \( R_G \). The metal cylinder has an outer radius \( R_M \). The radius of the hollow (empty) cavity is \( R_c \). Both cylinders are motionless.

The glass has a net positive charge distributed throughout its volume according to this density function: \( \rho_G = a(r^{3\alpha}) \), for \( 0 \leq r \leq R_G \) (\( r \) in m; \( \rho_G \) in C/m³)

An electron orbits in a circular path at constant speed around the metal cylinder.

The hollow metal cylinder is electrically neutral (has no net charge) and is in electrostatic equilibrium. The magnitude of the net electric field very near to (but in the empty space next to) its inner cavity surface is \( E_c \).

Known values: \( R_G = 1.23 \) m \( R_c = 4.56 \) m \( R_M = 7.89 \) m \( E_c = 432 \) N/C \( m_{\text{electron}} = 9.11 \times 10^{-31} \) kg

\[ k = 8.99 \times 10^9 \text{ N·m}^2/\text{C}^2 \quad \varepsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/(\text{N·m}^2) \quad e = 1.60 \times 10^{-19} \text{ C} \]

\[ L = 432 \text{ cm} \]

a. Calculate the charge density of the metal’s outer surface.
b. Calculate the speed of the electron.
c. Calculate the electric field (magnitude and direction) at a distance of 0.678 m from the center of the glass cylinder.

First, since this charge distribution is of infinite length, for this entire analysis, we will consider a certain segment—a segment of length \( L \).

a. First define a cylindrical Gaussian surface of radius \( R_c \) (and length \( L \)).

Gauss’s Law:
\[ \Phi_{E.C} = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

In detail:
\[ E_c A_c = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

That is:
\[ E_c (2\pi R_c L) = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

Solve for \( Q_{\text{encl}} \):
\[ \frac{Q_{\text{encl}}}{\varepsilon_0} = E_c (2\pi R_c L) \]

This value \( Q_{\text{encl}} \) is an important result, because it’s the total charge contained in a length \( L \) of the inner glass cylinder—and that’s the total net charge of the entire sample of length \( L \) (because the metal is electrically neutral). Therefore, the value of \( Q_{\text{encl}} \) is also the value of \( Q_{\text{encl}} \) for any other cylindrical Gaussian surface we wish to define that has a radius of at least that of the glass cylinder (\( R_G \)).

For example, now we define a cylindrical Gaussian surface of radius \( R_M \) (and length \( L \)).

Gauss’s Law:
\[ \Phi_{E.M} = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

In detail:
\[ E_M A_M = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

That is:
\[ E_M (2\pi R_M L) = \frac{Q_{\text{encl}}}{\varepsilon_0} \]

Solve for \( E_M \):
\[ E_M = \frac{Q_{\text{encl}}}{(2\pi R_M L \varepsilon_0)} \]

But \( Q_{\text{encl}} = Q_{\text{encl}} \):
\[ E_M = \frac{Q_{\text{encl}}}{(2\pi R_M L \varepsilon_0)} \]

Substitute:
\[ E_M = E_c (2\pi R_c L \varepsilon_0) / (2\pi R_M L \varepsilon_0) \]

Simplify:
\[ E_M = E_c R_c / R_M \quad \text{This is the field at the metal’s outer surface.} \]

Now we use what we know about the E-field at the surface of any conductor in electrostatic equilibrium....

Namely, this:
\[ E_{\text{surface}} = \eta_{\text{surface}} / \varepsilon_0 \]

For this situation:
\[ E_M = \eta_E / \varepsilon_0 \]

Substitute from above:
\[ E_c R_c / R_M = \eta_E / \varepsilon_0 \]

Solve for \( \eta_E \):
\[ \eta_E = \varepsilon_0 E_c R_c / R_M \]

Alternate solution—totally equivalent and valid:
Use the \( E_{\text{surface}} = \eta_{\text{surface}} / \varepsilon_0 \) relation for \( E_c \) and \( \eta_E \), then find the charge \( Q_e \) on the inner metal surface (knowing that surface area. Then \( Q_e \) must be the same (but positive); find the density that implies for the outer surface area.)

\[ \eta_E = (8.85 \times 10^{-12})(432)(4.56)/7.89 = 2.21 \times 10^{-9} \text{ C/m}^2 \quad \text{(or 2.21 nC/m}^2) \]
b. At whatever radius \( r \) the electron is orbiting, it is being acted upon by an electrical force due to the electric field present at that radius—the field caused by the cylindrical charge distribution.

In other words: \( eE_r = \frac{m v_e^2}{r} \)

We need to find the value of \( E_r \). So we do exactly the same sort of Gaussian analysis as we did above—but this time we define our Gaussian cylinder with radius \( r_e \). That is, we now define a cylindrical Gaussian surface of radius \( r_e \) (and length \( L \)).

Gauss’s Law: \( \Phi_{E-r} = Q_{encl}/\varepsilon_0 \)

In detail: \( E_{A-r} = Q_{encl}/\varepsilon_0 \)

That is: \( E_r(2\pi rL) = Q_{encl}/\varepsilon_0 \)

Solve for \( E_r^2 \): \( E_r = Q_{encl}/(2\pi rL\varepsilon_0) \)

But \( Q_{encl} = Q_{encl C} \): \( E_r = Q_{encl C}/(2\pi rL\varepsilon_0) \)

Substitute: \( E_r = E_r(2\pi rL)\varepsilon_0/(2\pi rL\varepsilon_0) \)

Simplify: \( E_r = E_r(2\pi rL)\varepsilon_0 \) \( \text{This is the field at the electron’s orbital radius.} \)

Substitute from above: \( eE_r R_r/r = m v_e^2/r \)

Solve for \( v_e \): \( v_e = [eE_r R_r/m]^{1/2} \)

\[
= [(1.60 \times 10^{-19})(432)(4.56)/(9.11 \times 10^{-31})]^{1/2} \\
= 1.86 \times 10^7 \text{ m/s}
\]

c. First we must find the value of \( a \), by summing all the charge in a length \( L \) of the glass cylinder. The resulting total, \( Q_{encl G} \) will be equal to \( Q_{encl C} \), which we already know.

The summation: \( Q_{encl G} = \int \rho dV = \int a(r^{209})dV = \int a(r^{209})(2\pi rLdr) \)

Simplify; set limits: \( Q_{encl G} = 2\pi a L \int_{0}^{R_G} r^{119} dr \)

Evaluate: \( Q_{encl G} = 2\pi a L (9/20) R_G^{209} \)

But \( Q_{encl G} = Q_{encl C} \): \( Q_{encl C} = 2\pi a L (9/20) R_G^{209} \)

Substitute: \( E_r(2\pi rL)\varepsilon_0 = 2\pi a L (9/20) R_G^{209} \)

Solve for \( a \): \( a = (20/9)E_r C_e \varepsilon_o R_G^{209} \)

Now define a cylindrical Gaussian surface of radius 0.678 m (and length \( L \)).

Gauss’s Law: \( \Phi_{E-678} = Q_{encl 678}/\varepsilon_0 \)

In detail: \( E_{678} A_{678} = Q_{encl 678}/\varepsilon_0 \)

That is: \( E_{678}[2\pi(0.678)L] = Q_{encl 678}/\varepsilon_0 \)

Solve for \( E_{678} \): \( E_{678} = Q_{encl 678}/[2\pi \varepsilon_0 (0.678)L] \)

Now find \( Q_{encl 678} \): \( Q_{encl 678} = 2\pi a L \int_{0}^{678} r^{119} dr \)

Evaluate: \( Q_{encl 678} = 2\pi a L (9/20)(0.678)^{209} \)

Substitute for \( a \): \( Q_{encl 678} = 2\pi [E_r C_e \varepsilon_o L] (0.678/R_G)^{209} \)

Substitute into \( E_{678} \): \( E_{678} = 2\pi [E_r C_e \varepsilon_o L] (0.678/R_G)^{209}/[2\pi \varepsilon_0 (0.678)L] \)

Simplify: \( E_{678} = (E_r C_e /0.678)(0.678/R_G)^{209} \)

\[
= (432)(4.56)/0.678(0.678/1.23)^{209} \\
= 773 \text{ N/C (radially outward)}
\]