Prep 9-10

Recommended finish date: Wednesday, November 28

The formats (type, length, scope) of these Prep problems have been purposely created to closely parallel those of a typical exam (indeed, these problems were taken from past exams). To get an idea of how best to approach various problem types (there are three basic types), refer to these Sample Problems.
1. a. Evaluate the following statements (T/F/N). As always, explain your reasoning.

(i) A normal force cannot do work.

(ii) If you hold an object in your hand and slowly lower your hand at constant speed, you are doing work on that object.

(iii) Kinetic energy is conserved when two identical objects, moving on a level, frictionless surface at equal speeds, collide head-on and stick together.

(iv) If a mass is swinging as a pendulum on a wire, the wire is doing no work on that mass.

(v) A friction force always does work on an object.

(vi) If an object is moving in the negative x-direction, you would be doing negative work on it by pushing on it in the positive x-direction.

(vii) One “physics” definition of energy is: It’s the ability to do work.

(viii) If you drop two objects of unequal mass, from rest, from the same height above the ground, the object with more mass will have more kinetic energy at the moment before impact. (Neglect air resistance.)

(ix) In an isolated system, mechanical energy is always conserved.

(x) A parked car could have more mechanical energy than the same car traveling at 30 m/s.

b. A 92 N force acts on an object at an angle of 45° west of north. While the force is acting, the object moves 100 m at an angle of 36° west of south. What work is done on the object?
c. In the diagram, $m_2 > m_1$, and both masses are pulled to the right at the same constant velocity for a distance $d$. $\mu_k$ is the same for both masses. Evaluate the following statements (T/F/N). Explain your reasoning.

(i) The magnitude of the negative work done on mass 2 could be equal to the magnitude of the negative work done on mass 1.
(ii) The work done by the pulling force ($F$) on mass 1 is the same as the work done by mass 1 on mass 2.
(iii) The net work done on each mass is the same.

(i) False. Is it possible that $m_1 g d = [F \cos \theta - \mu_k (m_1 g - F \sin \theta)] d$, or $m_2 g d = [m_2 g + \mu_k (m_1 g - F \sin \theta)] d$? Only if $m_1 g = F \sin \theta$ (that is, $F_{N1} = 0$). We're given no explicit data in the problem, but the picture and the problem statement both imply that there is contact and therefore a non-zero normal force between the surface and mass 1.

(ii) False. The claim is that $F \cos \theta d = F_T d$, or $F \cos \theta = F_T$, or $F \cos \theta = m_2 \mu_k g$. But we know from the above that $F \cos \theta = m_2 \mu_k g + \mu_k (m_1 g - F \sin \theta)$, so the claim would be false, provided that $m_1 g \neq F \sin \theta$. We're given no explicit data in the problem, but the picture and the problem statement both imply that there is contact and therefore a non-zero normal force between the surface and mass 1.

(iii) True. Since there is no change in either height or speed for each object, there is no net work being done on either: $W = 0$ for each.
2. a. In which situation is there at least one force that does non-zero work?
   (i) A block slides down a slope at constant speed.
   (ii) A satellite orbits the earth in a circular path.
   (iii) A block sits at rest on a (vertical) spring.

b. Two horses pull a floating barge along a straight, level canal at constant speed. Each horse walks on a parallel tow road alongside the canal, as shown in this overhead view (not to scale). $\theta = 78.0^\circ$; $F_T = 345 \text{ N}$. What work is done by the water on the barge per km traveled?

c. As shown here (overhead view), two masses ($m_2 > m_1$) begin from rest at position A, side-by-side. Then the same constant eastward force $F$ is exerted on each mass until that mass has moved (along the same uniform, level, frictionless surface) eastward to position B. Evaluate (T/F/N) each statement below, and don’t forget to explain your reasoning and justify each answer.
   (i) Comparing the masses’ kinetic energies as each reaches point B: $K_{2,B} > K_{1,B}$
   (ii) Comparing the masses’ momentum magnitudes as each reaches point B: $P_{2,B} > P_{1,B}$
   (iii) If the above scenario is repeated on a level, uniform surface that is not frictionless (but has the same coefficient of kinetic friction with each mass), it is possible that $v_{2,B} = v_{1,B}$. 


3. a. Shown here is the graph of the $x$-component of the net force acting on a 3-kg mass that moves only along the $x$-axis.

The velocity of the mass is $5.00 \text{ m/s}$ when it is located at the origin. Evaluate (T/F/N) each statement below. Explain your reasoning and justify each answer.

(i) The mass has its greatest speed at $x = 2.0 \text{ m}$.

(ii) As the mass travels from the origin to $x = 10 \text{ m}$, the total (net) work done on it is negative.

(iii) There is some point other than the origin ($0 < x \leq 8 \text{ m}$) where the mass has a velocity of $5.00 \text{ m/s}$.

b. A 10-kg projectile launched over level ground has three times the kinetic energy at the moment of its launch as it does at its peak height. From launch to peak, it gains 15,000 J in gravitational potential energy. Find its impact velocity. Ignore any rotational motion.

c. A 1-kg stone, falling vertically at 30 m/s, strikes and stays on a 7-kg sled that was moving horizontally over level, frictionless ice at 2 m/s.

(i) How much kinetic energy is lost in the collision?

(ii) What form does that energy take—and where does it go?
3. d. Two masses \((m_1 = 3.00 \text{ kg}; \ m_2 = 9.70 \text{ kg})\) are suspended independently by massless threads, each 1.64 m long, from the same overhead support. Initially, \(m_1\) is held at rest, its thread straight and horizontal, as shown; and \(m_2\) hangs at rest. Then \(m_1\) is released, and it swings down to strike \(m_2\). After the collision, the two masses have equal speeds in opposite directions. You may disregard air resistance and the diameters of the masses.

(i) Setting \(U_G = 0\) at the initial height of \(m_2\), what percentage of the total initial mechanical energy in this system is lost in the collision?

(ii) Find the angle formed by the two threads when the masses have reached maximum height after the collision.
4. a. (i) Express kinetic energy in fundamental (base) SI units (kg, m, s).
   (ii) Express the units of spring stiffness in fundamental (base) SI units (kg, m, s).

b. Evaluate (T/F/N) each statement below. Explain your reasoning and justify each answer.
   (i) A rubber band at rest must always have less mechanical energy than when it travels as a projectile toward a physics instructor.

   (ii) The force exerted by an ideal spring is always in the negative direction.

c. An ideal spring-mass system ($k = 35$ N/m; $m = 2.0$ kg) is oscillating horizontally on a frictionless surface. At the moment when the spring is compressed by 0.40 m, the system’s total mechanical energy is 1/3 kinetic and 2/3 potential. Find the mass’s maximum speed.

d. An ideal spring with a stiffness of 379 N/m is attached to a wall, and its other end is attached to a block that has a mass of 16.0 kg. The spring/block system is then stretched away from the spring’s relaxed position until 59.0 J of mechanical potential energy is stored in the spring. Then the system is released to oscillate freely, with the block sliding horizontally on the level, frictionless floor. Find the speed of the block when the spring is compressed by 13.4 cm.
4. e. A 46.5 kg block sits at rest on a frictionless, horizontal surface. It is connected horizontally, via a massless, ideal spring to a stationary wall. A force is then used to displace and hold the block in a new position (so that the spring is compressed). Then the block is released to move freely. In that first instant of motion, its acceleration is 9.73 m/s$^2$, and when it passes through its original (first) rest position, its speed is 18.2 m/s. What is the spring constant, $k$?

The work-energy equation is the best tool here:

$$E_{mech.f} = E_{mech.i} + W_{ext}$$

Or, to write it out fully:

$$K_{T.f} + U_{G.f} + U_{S.f} = K_{T.i} + U_{G.i} + U_{S.i} + W_{ext}$$

From the moment the block is released to oscillate freely, the only external force (i.e. force other than gravity and the spring) acting on the block is the normal force by the surface—but since that's always perpendicular to the block's movement, that force does no work. Therefore, $W_{ext} = 0$; the mechanical energy of the system stays constant throughout the block's motions.

At the moment the block is released to oscillate freely (at its maximum displacement, $x = –A$), its $x$-acceleration is, of course given by $a_x = SF_x/m$. And in this case, the only $x$-force is the spring force, $F = kA$. Therefore, $a_x = kA/m$, or $kA = (46.5)(9.73) = 452.445$ N.

Then, when the block passes through the rest position ($x = 0$), at that point all energy in the system is in the form of translational kinetic energy; the spring is unstrained, so there's no potential energy in it (the block's rotational speed, $w$, is always zero; and its height, $h$, never changes—we can call that $h = 0$).

So the total mechanical energy of the system (at any time during oscillation) is the same. So it's equal to the amount of kinetic energy it has at $x = 0$:

$$E_{mech} = K_T = \frac{1}{2}mv^2 = \frac{1}{2}(46.5)(18.2)^2 = 7701.33 \text{ J}$$

That must also be the total energy when the block is at the spring's maximum displacement—the amplitude—when the only form of energy is the spring's potential energy:

$$E_{mech} = U_S = 7701.33 = \frac{1}{2}kA^2$$

So we have two equations in two unknowns, the amplitude ($A$) and the spring stiffness ($k$):

$$kA = 452.445 \quad \frac{1}{2}kA^2 = 7701.33$$

Solving the first equation for $A$, we have:

$$A = \frac{452.445}{k}$$

Substitute this into the second equation: $(1/2)k(452.445/k)^2 = 7701.33$

Simplifying: $(452.445)^2/k = 2(7701.33)$

Solving for $k$: $(452.445)^2/[2(7701.33)] = k = 13.3 \text{ N/m}$

The spring constant is 13.3 N/m.
5. A block of mass 7.00 kg is oscillating up and down on an ideal spring that is suspended vertically from a ceiling, as shown here (not to scale). The relaxed length of the spring is 25.0 cm. Ignore air resistance.

Point A is the highest point in the block’s motion. At that point, the spring’s length is 12.0 cm.

At point B, the block is moving upward at 1.74 m/s, and the spring’s length is 43.0 cm.

a. If the block were hanging at rest (motionless), how long would the spring be?

b. What work would you need to do on the block (which is already oscillating as described above) so that it would hang at rest on the spring?

\[
\begin{align*}
K_B + U_G.B + U_S.B &= K_A + U_G.A + U_S.A \\
\text{(1/2)mv}_B^2 + (1/2)kx_B^2 &= mgh_A + (1/2)kx_A^2
\end{align*}
\]

Solve for \( k \):

\[
k = \frac{(2mgh_A - (1/2)mv_B^2)}{(x_B^2 - x_A^2)}
\]

\[
k = \frac{(2(7.00)(9.80)(0.31) - (7.00)(1.74)^2)}{(-0.18)^2 - (0.13)^2}
\]

\[
k = 1376.70 \text{ N/m}
\]

a. The spring must exert a tension force to match the gravitational force on the block:

\[
|F\text{spring.block}| = k|x| = mg
\]

Thus:

\[
x = mg/k = (7.00)(9.80)/1376.70 = 0.04983 \text{ m}
\]

This is how much the spring stretches to hold the block at rest. So the spring’s full length then is 0.25 + 0.0498 = 0.29983 m = 30.0 cm or 0.300 m

b. Let the block’s hanging rest position (calculated in part a above) be called point C.

The system’s total mechanical energy is the sum \( E_{\text{mech}} = K + U_G + U_S \), at all times and all positions.

So, analyze the external work necessary to transition the block from its prior energy at any point (say, point A) to hanging at rest at point C:

\[
K_C + U_G.C + U_S.C = K_A + U_G.A + U_S.A + W_{\text{ext}}
\]

Now let \( h_C = 0 \) (and note that \( K_A = K_C = 0 \)):

\[
U_S.C = U_G.A + U_S.A + W_{\text{ext}}
\]

Solving for \( W_{\text{ext}} \):

\[
W_{\text{ext}} = U_S.C - U_G.A - U_S.A
\]

In detail:

\[
W_{\text{ext}} = \frac{1}{2}kx_C^2 - mgh_A - \frac{1}{2}kx_A^2
\]

\[
= \frac{1}{2}(1376.70)(-0.0498)^2 - (7.00)(9.80)(0.17983) - \frac{1}{2}(1376.70)(0.13)^2
\]

\[
= -22.3 \text{ J}
\]
5  c.  Evaluate (T/F/N) the final statement in each item below. Fully justify your answers with any valid mix of words, drawings and calculations.

(i)  A mass is attached to a hanging ideal spring and is oscillating vertically between point A (its highest position) and point B (lowest position) without any air resistance or rotational motion. There is at least one position in the mass’s motion where its entire mechanical energy could be in the form of $K_T$.

True. If we designate $h = 0$ at the same point where the spring is at its relaxed length ($x = 0$), then as the mass passes through that point, $U_G = U_S = 0$. So $E_{mech.total} = K_T$ at that point.

(ii) A mass is attached to an ideal spring and is oscillating horizontally on a level, frictionless surface between points A and B, which are 1.00 m apart. When the mass is 0.250 m from point B, its speed is half of its maximum speed.

False. In this system, $K_{T.\text{max}} = U_{S.\text{max}} = E_{mech.total}$. That is, at $x = x_{\text{max}} = A$, all $E_{mech}$ is in the form of $U_S$; but at $x = 0$, all $E_{mech}$ is in the form of $K_T$. Thus:

$E_{mech.total} = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$  

Or:

$v_{\text{max}} = A\sqrt{\frac{k}{m}}$

At $x = A/2$, $E_{mech.total}$ is still the same value (since no external work has been done), but it is now shared between $K_T$ and $U_S$:

$K_T + U_S = E_{mech.total}$.

That is:

$\frac{1}{2}mv^2 + \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{1}{2}kA^2$

That is:

$v^2 = \left(\frac{k}{m}\right)\left(\frac{3A^2}{4}\right)$

Or:

$v = \sqrt{3}\left(\frac{A}{2}\right)\sqrt{\frac{k}{m}}$.  But this is not $v_{\text{max}}/2$, which is $\left(\frac{A}{2}\right)\sqrt{\frac{k}{m}}$.  

6. a. A certain ideal spring \( k = 100 \text{ N/m} \) hangs vertically at rest, and its relaxed length is 45.0 cm. Then a 2-kg mass is attached and the system is set into vertical motion (i.e. the mass and spring oscillate up and down with no energy losses). If the spring is 35.0 cm long when the mass reaches its highest point, how fast is the mass moving when the spring is 60.0 cm long? You may assume there is no rotational motion.

b. Block A \( (m_A = 1 \text{ kg}) \) is at rest on a frictionless surface, its right end just touching the end of a horizontal unstretched spring \( k = 200 \text{ N/m} \). The other end of the spring is fixed to a wall on the far right. Block B \( (m_B = 2 \text{ kg}) \), sliding along the surface at 4 m/s, collides with block A from the left, and the two blocks stick together and move together to the right, compressing the spring. What distance is the spring compressed when the two blocks (momentarily) stop?

c. In the diagram below, a block (mass = 5.10 kg) is sliding across a level surface \( (\mu_s = 0.973; \mu_k = 0.684) \) when it hits the left end of an ideal spring sitting relaxed and at rest at point B (right end anchored as shown). The block’s speed at point A is 12.0 m/s, and it comes to rest against the spring at point C. Then (with the block at rest against the spring at C), you move the spring’s anchor gradually to the left until the block slips and begins to slide again. How far did you need to move the anchor?
6. A block (mass = 7.84 kg) is attached to one end of an ideal, linear spring (k = 56.4 N/m). The spring is also attached to the upper end of a long, flat surface that is sloped at an angle of 20.0° above the horizontal. The block and surface have friction coefficients $\mu_S = 0.569$, and $\mu_K = 0.123$. The block has been moved (by an applied force) up the slope, so that the spring then has a total length of 1.37 m (as shown). At that point, the applied force is removed, and the spring is just barely able to get the block moving, but it does, and it pushes the block down the slope.

a. How fast is the block sliding when the spring is at its relaxed length?

b. The spring stretches to a maximum total length of 2.512 m. How far does it then pull the block back up the slope?
7. a. You are pulling a 5-kg block to the right, as shown (but the drawing and angle are not to scale), across a level surface at a steady speed of 3 m/s. The coefficient of kinetic friction between the block and surface is 0.27. The magnitude of your pulling force is twice as much as that of friction force. What work will you do on the block to pull it a distance of 4.00 m?
7. b. In attempting to pass the puck to a teammate, a hockey player gives it an initial speed of 3.0 m/s, but because of the kinetic friction between the puck and ice, the puck travels only half the distance between the players before coming to rest. Assuming that the ice’s friction is the same everywhere, what minimum initial speed did the player need to give the puck in order for it to reach the teammate?

\[ v_{i,A} = \sqrt{\frac{2 m_k g d_A}{2 m_k g d_B}} \]

But \( d_B = 2 d_A \), so substitute:

\[ v_{i,B} = \sqrt{\frac{4 m_k g d_A}{2 m_k g d_A}} \]

Now divide the equations:

\[ \frac{v_{i,B}}{v_{i,A}} = \frac{\sqrt{4}}{\sqrt{2}} = 2^{1/2} \]

So

\[ v_{i,B} = (2^{1/2}) v_{i,A} = (2)^{1/2}(3.0) = 4.24 \text{ m/s} \]

**Analysis of the work being done:**

\[ W_{ext} = F_k d \cdot \cos \theta \]

\[ = m_k F_N d \cdot \cos \theta \]

\[ = -m_k mg d \]

The fundamental Work-Energy equation:

\[ E_{mech.f} = E_{mech.i} + W_{ext} \]

\[ K_{T.f} + U_{G.f} + U_{S.f} = K_{T.i} + U_{G.i} + U_{S.i} + W_{ext} \]

Detailed expansion of the Work-Energy equation for this situation:

\[ \left( \frac{1}{2} \right) m \left( v_{i}^2 - 0^2 \right) + 0 + mg(0) = \left( \frac{1}{2} \right) mv_i^2 + 0 + mg(0) - m_k mg d \]

Simplify:

\[ 0 = \left( \frac{1}{2} \right) v_i^2 - m_k gd \]

Add \( m_k gd \):

\[ m_k gd = \left( \frac{1}{2} \right) v_i^2 \]

Multiply by 2:

\[ 2 m_k gd = v_i^2 \]

Take the square root:

\[ \sqrt{2 m_k gd} = v_i \]
7.  c.  Block A \((m = 1.20 \text{ kg})\), originally at rest, is pushed with a net horizontal force of 7.86 N for 5.34 m as it slides across a horizontal, frictionless surface. Then the force is removed and the Block A slides up a frictionless slope to another horizontal, frictionless surface, gaining 1.60 m in altitude as a result. On the upper surface, block A collides with block B \((m = 2.34 \text{ kg})\), which is sitting at rest, and the two blocks stick together thereafter. The combined block AB continues to move at some constant speed until it comes to the abrupt edge of the surface and falls freely to the floor, 6.57 m below. Ignore any rotational motion or air resistance.

(i) At what horizontal distance does block AB land on the floor, measured from the abrupt edge?
(ii) What is its impact speed?
8. Crate A \((m_A = 150 \text{ kg})\) containing a smaller, heavier crate B \((m_B = 300 \text{ kg})\) is being air-dropped by parachute, as shown. Crate A has an accurate scale built into its floor. When that crate floor is 40.2 m above the level ground, the scale reads 1680 N, and it is moving vertically downward at a speed of 9.70 m/s.

When it reaches the ground, the crate lands on a platform mounted on a spring \((k = 3.96 \times 10^3 \text{ N/m})\) that is embedded in the earth, as shown.

Before the impact, the platform’s top surface is level with the ground. After impact...

(a) when the platform’s top surface is 4.50 m below ground level, what is the speed of the crates?

(b) when the platform’s top surface is 4.50 m below ground level, what is the reading on crate A’s scale?

*Neglect the mass of the platform, spring, parachute and ropes.*

*The tension from the parachute disappears when the crate hits the platform.*
Both crates have this same acceleration, so the acceleration of the entire package (AB) is also \(-4.20 \text{ m/s}^2\).

Use this fact to find the tension in the parachute cord:

Thus:

\[
\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgh_i - FTd
\]

Solving for \(v_f\):

\[
\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgh_i - FTd - \frac{1}{2}kx_f^2
\]

\[
v_f^2 = v_i^2 + 2gh_i - 2FTd/m - kx_f^2/m
\]

\[
= \left[v_i^2 + 2gh_i - 2FTd/m - kx_f^2/m\right]^{1/2}
\]

\[
= \left[9.70^2 + 2(9.80)(44.7) - 2(2520)(40.2)/450 - (3960)(4.50^2)/450\right]^{1/2}
\]

\[
= 18.5 \text{ m/s}
\]
Strategy: Simply do a force analysis (Newton's Laws and FBD's, etc.) at the ending point specified. This will require two steps.

First, analyze the acceleration of the entire package (both crates together):

\[ F_{N.AB} = \frac{(3960)(4.50) - (450)(9.80)}{450} = 29.8 \text{ m/s}^2 \]

Now use this known acceleration value to analyze the forces on Crate B. The force \( F_{y} \) is the answer:

\[ F_{y} - F_G = m_{AB} a_y \]
\[ F_{N.AB} - m_B g = m_B (29.8) \]
\[ F_{N.AB} = m_B (9.80 + 29.8) = 300(9.80 + 29.8) = 1.19 \times 10^4 \text{ N} \]
9. a. Which is a vector quantity and which is a scalar quantity? (And state how you know.)
   (i) \( k|x| \)
   (ii) Power
   (iii) Work
   (iv) Displacement

b. (i) \( T/F/N? \) Units of power could be correctly expressed as \( \text{N/(m·s)} \)
   (ii) \( 1 \text{ kW·hr} = \_\_\_\_\_\_\_ \text{ J} \).

c. A 710-kg car drives at a constant speed of 23.0 m/s. It is subject to a total resistive (wind drag + rolling friction) force of 500 N. What power is required from the car’s engine to drive the car...
   (i) …on level ground?
   (ii) …up a hill with a slope of 2.0°.

d. In the diagram (standard \( g \); neglect air resistance), the block is sliding horizontally \( (\mu_k = 0.320) \). Evaluate \( T/F/N? \) the following statement. Fully justify your answer with any valid mix of words, drawings and calculations. The mechanical power being delivered to the block by the level surface is greater in magnitude than the mechanical power being delivered to the block by the 10 N force.
9. e. Shown here is the net $s$-force acting on an object ($m = 2.00$ kg) as it moves along the $s$-axis. The object is released from rest at $s = 0$. Evaluate (T/F/N) each statement. Fully justify your answers with any valid mix of words, drawings and calculations.

(i) The object is moving faster at $s = 2.00$ m than at $s = 7.00$ m.

False. Comparing the total amount of net area under the curve for those two points, it's clear that there is more such net area ("area above the axis minus area below the axis") for $0 \leq s \leq 7.00$ than for $0 \leq s \leq 2.00$. That means more net work is done for $0 \leq s \leq 7.00$, so the object's speed is greater there.

(ii) The mechanical power being delivered to the mass at $s = 3.00$ m is about 81.0 W.

False. First, find the speed at $s = 3$, by calculating the work done on the object for $0 \leq s \leq 3.00$. One easy way is to subtract the work done for $3.00 \leq s \leq 6.00$ from the work done for $0 \leq s \leq 6.00$:

$$\frac{1}{2}(36\cdot6) - \frac{1}{2}(18\cdot3) = 81 \text{ J.}$$

The object started from rest, so all the work shows up as $K_T$:

$$\frac{1}{2}mv^2 = 81$$

Thus:

$$v_s(3) = 9 \text{ m/s}$$

And (reading the graph):

$$F_s(3) = 18 \text{ N}$$

Therefore, at $s = 3.00$ m:

$$P_{mech} = F_s \cdot v_s = 18 \cdot 9 = 162 \text{ W}$$

(iii) This graph could be an accurate representation of the force exerted on a mass by an ideal spring.

True. The force is a linear dependency on position, and it's symmetric about a zero point ($F = 0$ where $s = 6.00$). So the graph could be accurately depicting $F_s = -kx$, where $k = 6$ N/m, and $x = s - 6$; that is, the "relaxed position" of the spring (where $x = 0$) is at $s = 6$. 

![Graph of net force vs. displacement](image)
10. A runaway locomotive (mass = \( m_1 \)) is switched onto a railroad siding (a separate section of track), so that it will crash safely at a “dead-end.” That’s where another “junk” rail car (mass = \( m_2 \)) sits at rest on a level track against a relaxed ideal spring (stiffness = \( k \)), as shown in Fig. A.

The locomotive’s engine has been shut off, but its brakes have failed completely, and it’s still going at speed \( v_i \) as it rolls onto the siding from the main line. Its travel along the siding is shown here in two different views. As shown in Fig. B (a side view) here, it climbs a slope (with an altitude gain of \( h \)), then crosses the level width of a plateau to where the junk car and spring are located.

Fig. C shows an overhead view of this. It also shows the wind blowing horizontally (across that plateau only) at an angle \( \theta^\circ \) north of east. The average force of that wind on the locomotive (exerted in the direction of that wind) is \( F_{\text{air}} \).

The total distance traveled along the siding from the main line to the junk car is \( d \). The plateau width is \( w \). The average rolling resistance of the rails (and still air) opposing the locomotive’s motion all along the siding is \( F_r \). Find the maximum compression of the spring after impact.

**A summary of all known values:** \( m_1, m_2, k, v_i, h, \theta, F_{\text{air}}, d, w, F_r, g \) (use ODAVEST here)
Data:
- \( m_1 \), \( m_2 \) the masses of the engine and junk car, respectively.
- \( k \) the stiffness of the spring.
- \( v_i \) the speed of the engine as it turns onto the siding from the main line.
- \( h \) the net elevation gain of the siding from the switch to the plateau level.
- \( d, F \) the total length of the siding—all the way from the switch to the point of impact—and the average rolling resistance force (due to rolling friction and still air) acting all along this length.
- \( w \) the width of the plateau.
- \( F_{\text{air}}, q \) the extra force exerted on the engine by the wind on the plateau only; and the angle of that wind (and its force), measured from "head on."
- \( g \) the local free-fall acceleration magnitude.

Assumptions:
- Masses: We treat the masses as particles, with no rotation/tipping.
- Alignment: We assume that spring and junk car (\( m_2 \)) are positioned along straight and level track.
- \( F_{\text{air}} \): We assume that \( F_{\text{air}} \) goes to zero (the wind calms) at the point of impact.
- \( F_r \): We assume that \( F_r \) goes to zero at impact.
- Collision: We assume that \( m_1 \) and \( m_2 \) stay together, moving as one mass after impact. We also assume the collision does not result in any disintegration along the axis of motion.
- Wind: We assume the engine travels through still air except on the plateau.
- \( g \): We assume the \( g \) value is constant for all relevant heights here.

Visual Representation:

Equations:

I. \( W_r = F_r d \cos(180°) = -F_r d \)

II. \( W_{\text{air}} = F_{\text{air}} w \cos(180° - q) \)

III. \( W_{\text{ext}} = W_r + W_{\text{air}} \)

(Energy analysis, with the initial point at the start of the siding and the final point just before impact; with \( h_f = h \) and \( h_i = 0 \).) See above.
Diagram of $\mathbf{p}$-vectors: $\mathbf{p}$-momentum cons. during collision.

$$(m_1 v_1) = (m_1 + m_2) v_{12}$$

Diagram of two cars moving as one, starting with the spring at its relaxed length, ending with the spring at maximum compression.

$$\frac{1}{2} (m_1 + m_2) v_{12}^2 = \frac{1}{2} k x^2$$

Solving: Solve I for $W_r$. Substitute that result into III. Solve II for $W_{air}$. Substitute that result into III. Solve III for $W_{ext}$. Substitute that result into IV. Solve IV for $v_f$. Substitute that result ($v_1$) into V. Solve V for $v_{12}$. Substitute that result into VI. Solve VI for $x$.

Testing: Dimensions: The solution for the compression distance should have dimensions of length. Dependencies: A greater locomotive mass, $m_1$, would be more mechanical energy in the system initially; hence a greater final spring compression. A greater junk car mass, $m_2$, would result in more mechanical energy lost to thermal energy in the collision; hence a smaller final spring compression. A stiffer spring (greater $k$) would result in a smaller compression length. A greater initial speed, $v_i$, would be more mechanical energy in the system initially; hence a greater final spring compression. A greater plateau altitude, $h$, or gravitational value, $g$, would result in a smaller spring compression length, as more mechanical energy would be in the form of $U_G$, leaving less $K$ to transform into $U_S$. (And an increased $g$ would likely increase $F_r$, too, since rolling friction depends on the $F_N$ acting between the two surfaces. This would also decrease the spring compression—see next item.) A greater siding length, $d$, or a greater rolling resistive force, $F_r$, would result in a smaller spring compression length, because more negative work would be done by the force while the engine travels the length of the siding. A greater plateau width, $w$, or a greater wind resistance force, $F_{air}$, or a smaller angle, $q$, would result in a smaller spring compression length, because more negative work would be done by that wind force while the engine travels the width of the plateau.
11. a. In the testing of a real spring being stretched near its elastic limit, the force it exerts is described by the equation $F = -kx + \beta(x)^3$, where $x$ is the displacement stretched. If $k = 10$ N/m and $\beta = 100$ N/m$^3$, calculate the work done against this force when the spring is stretched 0.10 m.

b. The spring shown is compressed 50 cm and used to launch a 100-kg physics student. The track is frictionless until it starts up the incline. The student’s coefficient of kinetic friction on the 30º incline is 0.15.

(i) How far up the incline does the student go?
(ii) The spring is replaced with a “sproing,” which exerts a force described by $F = k(\Delta s)^3$. If this Sproing ($k = 80,000$ N/m$^3$) is compressed 50 cm, what is the student’s speed just as it loses contact with it?

c. Shown here is the graph of the total potential energy of an 8-kg mass as it moves only along one axis (call it the $r$-axis). Its velocity along this axis, $v_r$, is $2.00$ m/s when it is located at $r = 8.00$ m. No external work is done on the mass. Evaluate (T/F/N) each statement below, and don’t forget to explain your reasoning and justify each answer.

(i) The mass does not achieve the position of $r = 14.0$ m.

(ii) At $r = 9.00$ m, $F_{net,r}$ is exerted in the negative $r$-direction.

(iii) At $r = 3.00$ m, $a_r = 5.00$ m/s$^2$. 

Because that would require more total energy than it has.
11. d. The graph shows the potential energy that a certain object ($m = 3.00$ kg) would have along the $r$-axis. The object is actually released from rest at $r = 2.00$ m. Evaluate (T/F/N) each statement. As always, you must fully justify your answers with a valid mix of words, drawings and calculations.

(i) The object's initial $r$-acceleration ($a_r$) will be $-6.67$ m/s$^2$.

(ii) Assuming no external work is done on the object (and all its motion is translational—no rotational motion), the object’s speed at $r = 6.00$ m will be 6.32 m/s.

(iii) In order for the object to reach the position $r = 13.0$ m, at least 20 J of external work must be done on it.
12. a. Some kids have built a pellet launcher consisting of an ideal, massless spring ($k$) placed inside a frictionless tube attached to a ramp (see diagram). Both the tube and the ramp are inclined at an angle of $\theta$ above the horizontal, but unlike the tube, the ramp’s surface does have friction ($\mu_k$). The tube’s bottom end is plugged. The relaxed spring length fits exactly into the remaining space inside the tube.

The kids compress the spring by a distance $x$, place the pellet in the tube and release the spring. The pellet is ejected out of the tube, then it slides (without rolling) up the ramp an additional distance of $d$, and then it becomes a projectile.

The pellet mass = $m_1$. The target is a ball ($m_2$), hanging vertically (at rest) from a mass-less thread (length $L$), on a nearby tree. At the peak of its motion as a projectile, the pellet hits the ball and bounces off at a speed of $v$, in the direction opposite to its impact velocity. The ball swings upward in a circular arc.

What angle does the thread make (measured from the vertical) at the highest point in the ball’s swing?

The known values: $k$, $\theta$, $\mu_k$, $x$, $d$, $m_1$, $m_2$, $L$, $v$, $g$.

Use the full, seven-step ODAVEST procedure. Numeric values are given as a study aid.

Assume that there is no air resistance and no rotational motion of the pellet.

(DRAWING IS NOT TO SCALE)
Assumptions:

Surfaces
We assume there is no seam, lip, bump or other disruption of motion of the pellet as it emerges from the tube onto the ramp.
We also assume that the pellet and ramp surfaces are uniform.

Alignment
We assume that the spring's motion is only longitudinal up the tube—no sideways shimmy or vibration.
We also assume that the end of the spring that pushes on the pellet is perpendicular to the axis of the spring's motion; that the pellet is propelled directly along that axis.
We also assume that the pellet strikes the ball "head-on"—so that the motions of the pellet and ball are at all times in the same plane.

Pellet
We will model the pellet as a particle that won't tip or rotate at any time.

Ball
We assume the ball does not spin or twist after being struck by the pellet.
And for the purposes of the collision and the ball's subsequent motion, we model the ball as a particle.

Air and g
We assume the value is constant for all relevant heights here.
We disregard any effects of wind or air drag.

Visual Rep(s):

Equations:

I. \( h_f = (x + d) \cdot \sin q \)

II. \( F_{N.S1} - m_1 g \cdot \cos q = 0 \)

III. \( F_{K.S1} = m_K F_{N.S1} \)

IV. \( W = F_{K.S1} \cos(180°) \cdot d \)

V. \( \frac{1}{2} m_1 v_f^2 + m_1 gh_f = \frac{1}{2} k x_i^2 + W \)

VI. \( v_{\text{impact}} = v_i \cdot \cos q \) (initial velocity vector triangle)

VII. \( m_1 v_{\text{impact}} = -m_1 v_1 + m_2 v_2 \) (sketch of collision of pellet with ball: initial and final momentum vectors)

VIII. \( m_2 gh_2.f = \frac{1}{2} m_2 v_2^2 \)

IX. \( h_2.f = L - L \cdot \cos q \)

Solving: Solve I for \( h_f \).  Substitute that result into V.
Solve II for \( F_{N.S1} \).  Substitute that result into III.
Solve III for \( F_{K.S1} \).  Substitute that result into IV.
Solve IV for \( W \).  Substitute that result into V.
Solve V for \( v_f \).  Substitute that result into VI (as \( v_i \)).
Solve VI for \( v_{\text{impact}} \).  Substitute that result into VII.
Solve VII for \( v_2 \).  Substitute that result into VIII.
Solve VIII for \( h_2.f \).  Substitute that result into IX.
Solve IX for \( q_2 \).
Testing:

Dimensions: The ball's maximum upswing angle, \( q^2 \), should be dimensionless.

Dependencies: If the spring were stiffer (greater \( k \)), \( q^2 \) would be greater. If the angle of incline, \( q \), of the tube and ramp were greater, this would imply a higher arc for the pellet as a projectile, and thus a lesser impact speed and therefore a smaller \( q^2 \). If \( m_K \) were greater, the pellet would have less energy at the time of impact, so \( q^2 \) would be smaller. If the spring were compressed farther (greater \( x \)), \( q^2 \) would be greater. If \( d \) were greater, the pellet would have less energy at the time of impact, so \( q^2 \) would be smaller. If the pellet were more massive (\( m_1 \) greater), it would have less kinetic energy at impact (because the investment in \( UG \) would be greater), but \( m_1 \) would also have more effect in the collision. So an explicit solution is needed to reveal the dependencies clearly.

If the ball were more massive (\( m_2 \) greater), it would have less kinetic energy after impact (because its momentum would be more invested in \( m \) than \( v \)), so \( q^2 \) would be smaller. If the thread were longer (\( L \) greater), the same maximum height of the ball would be achieved with a smaller \( q^2 \). If the rebound speed, \( v \), of the pellet were greater, momentum conservation would require a greater speed for the ball after impact; \( q^2 \) would be greater. Increasing the value of local \( g \) would decrease \( q^2 \).
12. b. Refer to the diagram shown below. A block of mass \( m \) is sitting at rest at point A on the level portion of a surface when it is suddenly acted upon (at an angle \( \theta \), as shown) by a steady applied force, \( F \). The force continues steadily for a time interval \( \Delta t \), then ceases (disappears completely). As a result, the block slides across the level portion of the surface and up the incline. After passing point C, the block is not in contact with anything solid until the moment it is again moving horizontally, when it strikes a vertical wall at point D (which is a vertical distance \( H \) higher than point B). There the block comes to rest after embedding itself into that wall’s material by a short distance \( x \).

The block and the surface have a coefficient of kinetic friction \( \mu_k \) between points A and B, which are a distance \( L \) from each other. However, the block is at some unknown location between A and B when \( F \) ceases. The surface is frictionless from point B to point C. The wall’s material exerts on the block a compression-resistance force \( F(c) = \beta c^4 \), where \( \beta \) is a known coefficient and \( c \) is the distance by which the wall material has compressed as the block embeds.

Estimate the block’s total loss of mechanical energy due to air resistance from point A to point D.

Here is a summary of all known values: \( m, \theta, F, \Delta t, H, x, \mu_k, L, \beta, c, g \)
Data:
- $m$: The mass of the block.
- $F$: The magnitude of the force applied to the block.
- $q$: The angle below the positive horizontal at which the force $F$ is applied to the block.
- $t$: The time interval during which the force $F$ is applied.
- $K$: The coefficient of kinetic friction between the block and the rough portion of the surface.
- $L$: The length of the block's travel over the rough portion of the surface.
- $H$: The height above the block's initial position where it impacts the vertical wall.
- $b$, $c$: The wall's force compression function, which has the form $F_{\text{compress}} = bc^3$, where $b$ is a known coefficient and $c$ is the known distance by which the block has compressed (by penetrating into) the wall's material.
- $g$: The local free-fall acceleration magnitude.

Assumptions:
- Surfaces: We assume the sliding surfaces are uniform and planar.
- Alignment: We assume that the force shown in the diagram is acting completely in the plane of the paper. We also assume that the block's impact velocity is normal to the wall (that it doesn't strike it obliquely).
- Block: We also model the block as a particle for the purposes of crossing from the rough portion to the frictionless portion of the surface.
- Motion: We assume that the block has no rotational motion at any time.
- Impact: We assume that the block's impact into the wall is with exactly the profile modeled by the force compression equation. We also ignore any other thermal losses at impact—assume all mechanical energy loss is due to the negative work done by the compressing wall.

We assume the value of $g$ is constant for all relevant heights here.
Visual Representations:

Energy diagram from start to impact (can be similar to diagram given in problem, but for full credit, must have initial and final points labeled and also all known values labeled):

FBD of block sliding with friction:

Work analysis of applied force:

Work analysis of sliding friction:

Work analysis of wall’s compression:
Equations:

I. \( \frac{1}{2}mv_f^2 + mgh_f + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + mgh_i + \frac{1}{2}kx_i^2 + W_{\text{ext}} \)

II. \( W_{\text{ext}} = W_{\text{F}} + W_{\text{K}} + W_{\text{wall}} + W_{\text{air}} \)

III. \( v_i = v_f = 0 \)

IV. \( h_i = x_i = x_f = 0 \)

V. \( h_f = H \)

VI. \( W_{\text{F}} = (F \cos \theta) d \)

VII. \( d = v_i (Dt) + \frac{1}{2}a (Dt)^2 \)

VIII. \( F \cos \theta - mK_F N = ma \)

IX. \( F_N - F \sin \theta - mg = 0 \)

X. \( W_{\text{K}} = -mK_F N d - mK mg (L - d) \)

XI. \( W_{\text{wall}} = -\int_0^x bc^4 \, dc \)

Solving:

- Substitute III into I and VII.
- Substitute IV into I.
- Substitute V into I.
- Solve IX for \( F_N \).
- Substitute that result into VIII and X.
- Solve XIII for \( a \).
- Substitute that result into VII.
- Solve VII for \( d \).
- Substitute that result into VI and X.
- Solve VII for \( W_{\text{K}} \).
- Substitute that result into II.
- Solve VI for \( W_{\text{F}} \).
- Substitute that result into II.
- Solve XI for \( W_{\text{wall}} \).
- Substitute that result into II.
- Solve I for \( W_{\text{ext}} \).
- Substitute that result into II.
Testing:

Dimensions: The solution for the mechanical energy loss should have dimensions of energy (which is mass·length$^2$/time$^2$).

Dependencies: The effect on the calculation of $W_{\text{air}}$ of increasing the mass, $m$, is not clear without explicitly solving the above equation set as prescribed; the mass affects several parts of those calculations. If $q$ were greater, this would imply a lesser investment of work in the mass, so if its height and embedding distance is still the same, this implies less loss due to air drag. $W_{\text{air}}$ would have a lesser magnitude. If $F$ were greater, this would imply a greater investment of work in the mass, so if its height and embedding distance is still the same, this implies a greater loss due to air drag. $W_{\text{air}}$ would have a greater magnitude. If $D_t$ were greater, this would imply a greater investment of work in the mass, so if its height and embedding distance is still the same, this implies a greater loss due to air drag. $W_{\text{air}}$ would have a greater magnitude. If $H$ were greater (all other given values remaining the same), since the block achieved the same impact height (with more energy invested there due to greater $H$) and it embedded the same distance, there must have been less energy loss to air drag. $W_{\text{air}}$ would have a lesser magnitude. If $x$ were greater (all other given values remaining the same), since the block achieved the same impact height but embedded a greater distance into the wall, there must have been less energy loss to air drag. $W_{\text{air}}$ would have a lesser magnitude. If $m_K$ were greater (all other given values remaining the same), since the block achieved the same impact height and embedded the same distance into the wall, there must have been less energy loss to air drag, in order to account for the greater loss due to surface friction. $W_{\text{air}}$ would have a lesser magnitude. If $L$ were greater (all other given values remaining the same), since the block achieved the same impact height and embedded the same distance into the wall, there must have been less energy loss to air drag, in order to account for the greater loss due to surface friction. $W_{\text{air}}$ would have a lesser magnitude. If $b$ were greater (all other given values remaining the same), since the block achieved the same impact height and embedded the same distance into the wall, there must have been less energy loss to air drag, in order to account for the greater energy to be absorbed at impact. $W_{\text{air}}$ would have a lesser magnitude. If $g$ were greater (all other given values remaining the same), since the block achieved the same impact height (with more energy invested there due to greater $g$) and it embedded the same distance, there must have been less energy loss to air drag; $W_{\text{air}}$ would have a lesser magnitude.