Prep 5

*Recommended finish date:* Monday, October 29

The formats (type, length, scope) of these Prep problems have been purposely created to closely parallel those of a typical exam (indeed, these problems were taken from past exams). *To get an idea of how best to approach various problem types (there are three basic types), refer to these Sample Problems.*
1. a. Which is a vector quantity? (As always, explain your reasoning.)
   a. \( F_{\text{avg}}(\Delta t) \)
      Yes, this is a vector, because force is a vector quantity; time is a scalar.
   b. \( m|\Delta v| \)
      No, this is a scalar, because the magnitude of velocity is a scalar; mass is also a scalar.
   c. \( |F_{\text{avg}}|(\Delta t) \)
      No, this is a scalar, because the magnitude of force is a scalar; time is also a scalar.
   d. \( J \)
      Yes, this is a vector, because \( J = \int F dt \), and force is a vector quantity; time is a scalar.
   e. What is the common name we use for the rate of change of linear momentum?
      in any object’s change of momentum are force and time. Thus the time derivative of momentum is force.

b. Evaluate the following statements (T/F/N). As always, explain your reasoning.
   (i) The units of force are the same as the units of impulse.
      False. An impulse has the same units as its result—a change of momentum: kg·m/s
      Force has units of kg·m/s², because momentum is the product of force and time.
   (ii) No impulses can be exerted within an isolated system.
      False. Impulses are exerted in any exchange of momentum, which can certainly happen as objects within an
      insolated system collide.
   (iii) If object A strikes object B, the impulse that A exerts on B is always along the direction of object A’s
      velocity right before the impact.
      False. Not always. Only when the initial velocities of A and B are collinear. For any other combination of
      initial velocities, \( J_{AB} \) is not parallel to \( P_A \). \( J_{AB} \) is always parallel to \( \Delta P_B \). As a good example, see the
      solution to problem 3e in this Prep set.
   (iv) The sum of all impulses exerted within an isolated system must be zero.
      True. Since impulses result in momentum changes, this is just a re-statement of the principle of conservation
      of linear momentum in an isolated system. In fact there are several ways to state it:
      \[
      P_{\text{isolated.TOTAL.i}} = P_{\text{isolated.TOTAL.f}} \quad \Delta P_{\text{isolated.TOTAL}} = 0 \quad \Sigma (\Delta P)_{\text{isolated}} = 0 \quad \Sigma J_{\text{isolated}} = 0
      \]
   (v) If skaters A and B are initially together at rest on frictionless ice, and then they push each other apart,
      skater A has a non-zero change in his momentum.
      True. Skater A’s momentum has changed from zero (they were both at rest initially) to some non-zero value;
      he has moved, so his velocity has to have been non-zero. Skater B has exerted an impulse on A (and of
      course, A has exerted the opposite impulse on B.)
   (vi) Units of linear momentum could be correctly written also as N·s.
      True. The units of momentum are kg·m/s, which are equivalent to (kg·m/s²)·s, which are N·s.
   (vii) Impulse is scalar quantity.
      False. Impulse is a vector quantity, the product of force (a vector) and time (a scalar).
      And its result—a change in momentum, \( \Delta P \)—is also a vector.
   (viii) If two skaters (standing on level, frictionless ice) push horizontally on each other, each receives the
      same (identical) impulse.
      False. The magnitudes of their impulses are the same, but their directions are opposite; so the two impulses
      are not identical.
   (ix) The units of momentum are the same as the units of impulse.
      True. Impulse has units of force·time ((kg·m/s²)·s), which are equivalent to units of momentum (kg·m/s).
2. For each item, be sure to show your work and/or explain your reasoning.

a. A 4-kg cat accelerates horizontally from 0 to 2 m/s. Calculate the impulse magnitude.
\[ |\Delta P| = |P_f - P_i| = m(v_f - v_i) = |4(2 - 0)| = 8 \text{ kg}\cdot\text{m/s} \]

b. A book weighing 5.00 N falls freely for 1.50 s. Calculate the impulse magnitude.
\[ |\Delta P| = |\mathbf{F}_{\text{avg}}(\Delta t)| = |5.00(1.50)| = 7.50 \text{ kg}\cdot\text{m/s} \]

c. A basketball (1.00 kg) has its 2 m/s velocity reversed (i.e., same speed but opposite direction).
Calculate the impulse magnitude.
\[ |\Delta P| = |P_f - P_i| = m(v_f - v_i) = |1(-2 - 2)| = 4 \text{ kg}\cdot\text{m/s} \]

d. A snowball (0.300 kg), initially moving at 25.0 m/s, stops. Calculate the impulse magnitude.
\[ |\Delta P| = |P_f - P_i| = m(v_f - v_i) = 0.300(0 - 25.0) = 7.50 \text{ kg}\cdot\text{m/s} \]

e. You shoot a 0.25 kg arrow at an angle of 69.7° above the positive x-axis. It reaches a peak height of 237 m. Calculate the impulse that acted on it while it was a projectile—from the start to its peak height.
\[ J = \Delta P = P_f - P_i \]
At its peak, the projectile’s y-momentum is zero, so its total momentum at that point (i.e., the final momentum of the analysis) is simply its initial x-momentum: \( P_f = P_{ix} \)
Therefore:
\[ J = \Delta P = P_{ix} - P_i \]
But:
\[ P_{ix} - P_i = -(P_i - P_{ix}) = -P_{iy} = m[0, -v_{iy}] \]
Find \( v_{iy} \) (for \( v_{fy} = 0 \)):
\[ v_{f,y}^2 = v_{i,y}^2 + 2a\Delta y \]
So:
\[ 0 = v_{i,y}^2 + 2(-9.80)(237) \]
Therefore:
\[ v_{i,y} = \sqrt{2(9.80)(237)} = +68.16 \text{ m/s} \]
And so:
\[ J = \Delta P = -P_{iy} = m[0, -v_{iy}] = 0.25[0, -68.18] = [0, -17.0] \text{ kg}\cdot\text{m/s} \]

f. An ice hockey puck slides along the ice with a speed \( v \) (m/s). A hockey stick delivers an impulse magnitude of \( J \) (kg·m/s), causing the puck to reverse its velocity (same speed but opposite direction). What is the mass, \( m \) (kg), of the puck, as expressed in terms of \( v \) and \( J \)?
\[ J = |\Delta P| = |P_f - P_i| = m(v_f - v_i) = m(-v - v) = 2mv \quad \therefore \quad m = J/(2v) \]

g. A raindrop and a hailstone, with equal masses, fall from rest from the same height. Both strike the level top of the same stationary car. Each impact takes the same amount of time. The raindrop does not bounce after impact; the hailstone bounces directly upward. Neglecting air resistance, evaluate the following statements (T/F/N).

(i) The hailstone exerts a larger average force on the car than does the raindrop.
True. If their impacts happen over equal intervals of time, then the hailstone’s average force must be bigger, because its’s equal in magnitude to the average force of the car on it—which must be bigger than the average force of the car on the raindrop because the momentum change of the hailstone was greater (motion reversed rather than just stopped).

(ii) The vertical portion of the car’s impulse on the raindrop is upward.
True. Yes, because \( J = \Delta P \) and \( \Delta P \) of the raindrop was upward (positive):
\[ \Delta P = m(v_f - v_i) = m(0 - v_i) = -mv_i \]
This is positive because \( v_i \) was downward (negative).

(iii) The sum of the impulses of each falling object on the car must be zero.
False. No, all the impulses by either rain or hail on the car are in the same direction (downward). There’s no way the sum of vectors can add to zero if all the vectors are in the same direction.
3. A westbound truck \((m_T = 28,000 \text{ kg})\) traveling at 25 m/s collides at a rural crossroads with a northbound car \((m_C = 1500 \text{ kg})\) traveling at 36 m/s. The impact itself lasts for 0.714 s, and the vehicles stick together as a result. Ignore road friction and any other interference with the impact.

a. Draw the momentum vector triangle of the system (the collection of two objects) in this collision.

b. Find the speed of the wreckage immediately after the impact.

Use conservation of linear momentum—in the \(x\)- and \(y\)-directions:
\[
P_{\text{total}i} = P_{\text{total}f}
\]
Expand:
\[
(m_T)[v_{T,i} \cos 180^\circ, v_{T,i} \sin 180^\circ] + (m_C)[v_{C,i} \cos 90^\circ, v_{C,i} \sin 90^\circ] = (m_T + m_C)[v_{f.x}, v_{f.y}]
\]
Simplify:
\[
[–m_T v_{T,i}, 0] + [0, m_C v_{C,i}] = (m_T + m_C)[v_{f.x}, v_{f.y}]
\]
Calculate:
\[
-(28,000)(25) = (28,000 + 1,500)\, v_{f.x} \quad \Rightarrow \quad v_{f.x} = -23.73
\]
\[
(1,500)(36) = (28,000 + 1,500)\, v_{f.y} \quad \Rightarrow \quad v_{f.y} = 1.831
\]
Therefore:
\[
v_f = \sqrt{(-23.7288)^2 + 1.8305^2} = 23.8 \text{ m/s}
\]

c. Now draw the momentum vector triangle for only the truck during the collision.

d. Find the magnitude of the average force exerted by the car on the truck during the impact.

\[
F_{\text{avg,CT}} = \frac{\Delta P_T}{\Delta t_{\text{impact}}} = \frac{\Delta P_T}{\Delta t_{\text{impact}}}
\]
So
\[
\Delta P_T = P_{T,f} - P_{T,i}
\]
Therefore:
\[
F_{\text{avg,CT}} = \sqrt{(35,560^2 + 51,268^2)/0.714} = 8.74 \times 10^4 \text{ N}
\]

e. Find the direction of the average force by the truck on the car during the impact.

Because
\[
\Delta P_C = -\Delta P_T
\]
we know:
\[
\Delta P_{C_x} = -\Delta P_{T_x} = -35,560
\]
\[
\Delta P_{C_y} = -\Delta P_{T_y} = -51,268
\]
And because
\[
F_{\text{avg,TC}} = \frac{\Delta P_C}{\Delta t_{\text{impact}}}
\]
the direction of \(F_{\text{avg,TC}}\) is also the direction of \(\Delta P_C\):
\[
\theta_{FTC} = \tan^{-1}[-(51,268)/(-35,560)] = 55.3^\circ
\]
This is 55.3° south of west (or \(\angle 235^\circ\) or \(\angle -125^\circ\))
4. **a. Textbook problem 60 (page 290).**

By definition:
\[
\Delta P = \int F(t) \, dt
\]
\[
= 10 \sin(\pi t/2) dt
\]
\[
= -(20/\pi) \cos(\pi t/2) \bigg|_0^2
\]
\[
= -(20/\pi) \cos(\pi) - \cos(0)
\]
\[
= 40/\pi
\]

But:
\[
\Delta P = m(v_f - v_i)
\]
\[
= mv_f
\]

Therefore:
\[
v_f = \frac{40}{\pi}
\]

Thus:
\[
v_f = \frac{40}{(0.250 \pi)} = 50.9 \text{ m/s}
\]

**b. Two masses** \(m_A = 2.00 \text{ kg}; m_B = 3.00 \text{ kg}\) **collide on a level, frictionless surface.** Just before the collision, mass A was moving directly east at 5.00 m/s; and mass B was moving directly north, also at 5.00 m/s. Just after the collision, mass A is moving directly north at 1.50 m/s. The masses are in contact during the collision for a total of 0.0653 s.

*Evaluate (T/F/N) each statement. As always, you must justify your answers with a valid mix of words drawings and calculations.*

(i) Mass B is moving faster after the collision than before the collision.

**True.** Using conservation of linear momentum...

Before:

\[
\begin{align*}
\text{Before:} & & \text{After:} \\
\vec{P}_{T,i} & & \vec{P}_{T,f} \\
\vec{P}_{A,i} & & \vec{P}_{A,f} \\
\vec{P}_{B,i} & & \vec{P}_{B,f}
\end{align*}
\]

Thus:
\[
m_B v_{Bf} = |P_{Bf}| = \sqrt{12^2 + 10^2}
\]

So:
\[
v_{Bf} = \frac{\sqrt{(244)}}{3} = 5.21 \text{ m/s}
\]

(ii) Denoting east as \(\angle 0^\circ\) (and north as \(\angle 90^\circ\)), the direction of \(J_{AB}\) is approximately \(\angle 50.2^\circ\).

**False.** The direction of the impulse by A on B \((J_{AB})\) is not positive.

(iii) \(|F_{BA|} \approx 160 \text{ N.}\)
5. A student \( m = 75 \text{ kg} \) falls freely from rest and strikes the ground, coming to rest in a time interval of \( 0.0158 \text{ s} \). The average force magnitude exerted on him by the ground is \( 1.96 \times 10^4 \text{ N} \). From what height did he fall?

Impulse produces change of momentum:

\[ \Delta P = (m)(v_{\text{impact}} - v_{\text{impact}}) = (F_{\text{avg}})(\Delta t) \]

Simplify (student comes to rest after impact):

\[ (m)(-v_{\text{impact}}) = (F_{\text{avg}})(\Delta t) \]

\[ v_{\text{impact}} = -(F_{\text{avg}})(\Delta t)/m \]

Kinematics of the fall:

Solve for \( v_{\text{fall}} \) of the fall:

But \( v_{\text{fall}} = v_{\text{impact}} \). So:

Solve for \( h \):

\[ h = [(F_{\text{avg}})(\Delta t)/m]^2/(2g) \]

\[ = [(19600)(0.0158)/75]^2]/(2)(9.80) \]

\[ = 0.870 \text{ m} \]
6. In this problem, use at least the E-S steps of the ODAVEST problem-solving protocol.

a. A projectile (mass = m) is launched vertically upward with initial velocity \( v \). When it has reached a height \( h \) above its launch position, it is still moving upward, but at that moment it explodes into two pieces. Immediately after the explosion, piece A (mass = \( m_A \)) is moving at a speed \( v_A \) at an angle of \( \theta_A \) below the horizontal. A certain time \( \Delta t \) after the explosion, piece B strikes a nearby cliff with an average force magnitude \( F_{\text{avg,Bcliff}} \) exerted on it by the cliff during its impact (and the piece stops completely as a result). For what time interval was this impact force exerted?

You may consider these values as known: \( m, v, h, m_A, v_A, \theta_A, \Delta t, F_{\text{avg,Bcliff}}, g \).

Since there are no external forces interfering with or affecting the initial motions of the pieces (in either the \( x \)- or \( y \)-direction) immediately after the explosion, we can predict those initial motions via conservation of momentum.

Note: The initial momentum was only in the \( y \)-direction:

\[
P_{T,y} = m(v_{y,f})
\]
\[
P_{T,x} = m(v_{x,0})
\]

Find the \( y \)-velocity of the projectile right before it explodes:

Use this equation:

\[
v_{y,f} = v_{y,i} + 2a_y(A\Delta y)
\]

Use these knowns:

\( a_y = -g \) (given)

\( \Delta y = h \) (given)

Solve for: \( v_{y,f} \)

(choose the positive root)

III. Find the \( x \)-momentum, \( P_{B,\text{expl,x}} \), of piece B right after the explosion (and let that direction be positive \( x \)):

Use this equation:

\[
P_{T,x} = P_{T,f,x} = P_{A,\text{expl,x}} + P_{B,\text{expl,x}}
\]

Use these knowns:

\( P_{T,x} = 0 \) (from step I)

\( P_{A,\text{expl,x}} = -m_A v_A \cos \theta_A \) (all values given)

Solve for: \( P_{B,\text{expl,x}} \)

IV. Find the \( y \)-momentum, \( P_{B,\text{expl,y}} \), of piece B right after the explosion:

Use this equation:

\[
P_{T,y} = P_{T,f,y} = P_{A,\text{expl,y}} + P_{B,\text{expl,y}}
\]

Use these knowns:

\( P_{T,y} = m(v_{y,f}) \) (from step I; \( v_{y,f} \) from step II)

\( P_{A,\text{expl,y}} = -m_A v_A \sin \theta_A \) (all values given)

Solve for: \( P_{B,\text{expl,y}} \)

V. Find the \( y \)-velocity, \( v_{B,\text{expl,y}} \), of piece B right after the explosion:

Use this equation:

\[
P_{B,\text{expl,y}} = m_B v_{B,\text{expl,y}}
\]

Use these knowns:

\( m_B = m - m_A \) (both values given)

Solve for: \( v_{B,\text{expl,y}} \)

VI. Find the \( y \)-velocity of piece B right before its impact against the cliff:

Use this equation:

\[
v_{y,f} = v_{y,i} + a_y(A\Delta t)
\]

Use these knowns:

\( v_{y,i} = v_{B,\text{expl,y}} \) (from step V)

\( a_y = -g \) (given)

\( \Delta t \) (given)

Solve for: \( v_{y,f} \)

VII. Find the \( y \)-momentum, \( P_{B,\text{impact,y}} \), of piece B right before impact:

Use this equation:

\[
P_{B,\text{impact,y}} = m_B v_{B,\text{impact,y}}
\]

Use these knowns:

\( m_B = m - m_A \) (both values given)

\( v_{B,\text{impact,y}} = v_{y,f} \) (from step VI)

Solve for: \( P_{B,\text{impact,y}} \)

VIII. Note the nature of the \( x \)-motion of piece B as a projectile:

Since only gravity affects a projectile—and only in the \( y \)-direction—the \( x \)-velocity (and therefore the \( x \)-momentum) of piece B remains unchanged throughout its flight.

So: \( P_{B,\text{impact,x}} = P_{B,\text{expl,x}} \)
IX. Find the total momentum vector magnitude, $|P_{B,\text{impact}}|$, of piece B right before impact:

Use this equation: $|P_{B,\text{impact}}| = \sqrt{(P_{B,\text{impact},x})^2 + (P_{B,\text{impact},y})^2}$

Use these knowns: $P_{B,\text{impact},x}$ (from step VIII)  
$P_{B,\text{impact},y}$ (from step VII)  

Solve for: $|P_{B,\text{impact}}|$

X. Since the momentum of piece B after impact is zero (the piece comes to rest), the momentum magnitude before impact is also the magnitude of the change:

Use this equation: $\Delta P_B = P_{B,f} - P_{B,\text{impact}} = 0 - P_{B,\text{impact}}$  

So: $|\Delta P_B| = |P_{B,\text{impact}}|$

XI. Find the duration of the impact necessary to cause that change in momentum:

Use this equation: $|\Delta P_B| = |F_{\text{impact}}|(\Delta t_{\text{impact}})$

Use these knowns: $|\Delta P_B|$ (from steps IX and X)  
$|F_{\text{impact}}| = F_{\text{avg.}}$ (given)  

Solve for: $\Delta t_{\text{impact}}$
Two identical projectiles (each with mass \( m \); each moving with speed \( v \)) collide in the air at a height \( h \) above a level patch of frictionless ice. At the moment before the collision: Projectile A is moving at an angle of \( \theta_A \) above the positive horizontal; and projectile B is moving at an angle of \( \theta_B \) above the negative horizontal. The two masses stick together as a result of the collision, and they fall, as one combined mass, to the ice below. When the combined mass hits the ice, it does not bounce; rather, it slides horizontally across that surface. If the ice exerted an average force magnitude of \( F_{\text{avg,icemass}} \) to change the motion of the combined mass during impact, how long was \( F_{\text{avg,icemass}} \) exerted?

You may consider these values as known: \( m, v, h, \theta_A, \theta_B, F_{\text{avg,icemass}}, g \).

I. Note the nature of the collision of the pieces: Since there are no external forces interfering with or affecting the initial motion of the combined mass (in either the \( x \)- or \( y \)-direction) immediately after the explosion, we can predict that initial motion via conservation of momentum:

\[
P_{\text{T,i,x}} = P_{\text{T,f,x}}
\]

\[
P_{\text{T,i,y}} = P_{\text{T,f,y}}
\]

II. Note the nature of the impact of the combined mass with the ice: Since the ice is frictionless, it can exert a force (and therefore a change of momentum—an impulse) only in the \( y \)-direction. So at issue is only the \( y \)-momentum of the falling mass; we need only concern ourselves with the \( y \)-motion in this problem.

III. Find the total \( y \)-momentum, \( P_{\text{T,i,y}} \), of the pieces right before the collision:

\[
P_{\text{T,i,y}} = P_{A,y} + P_{B,y}
\]

Use these knowns:

\[
P_{A,y} = mvsin\theta_A \quad \text{(all values given)}
\]

\[
P_{B,y} = mvsin\theta_B \quad \text{(all values given)}
\]

Solve for: \( P_{\text{T,i,y}} \)

IV. Find the \( y \)-velocity, \( v_{f,y} \), of the combined mass right after the collision:

\[
P_{\text{T,f,y}} = \frac{2m(v_{f,y})}{m}
\]

Use these knowns:

\[
P_{\text{T,f,y}} = P_{\text{T,i,y}} \quad \text{(from step III)}
\]

Solve for: \( v_{f,y} \)

V. The \( y \)-velocity of the combined mass just after the collision becomes the initial \( y \)-velocity for that mass’s journey to the ice as a projectile. Find the \( y \)-velocity of the mass at impact on the ice:

\[
v_{y,f}^2 = v_{y,i}^2 + 2a_y(\Delta y)
\]

Use these knowns:

\[
v_{y,i} = v_{f,y} \quad \text{(from step IV)}
\]

\[
a_y = -g \quad \text{(given)}
\]

\[
\Delta y = -h \quad \text{(given)}
\]

Solve for: \( v_{y,f} \)

Choose the negative root.

VI. Find the \( y \)-momentum, \( P_{y,f} \), of the combined mass right before impact. (This will be the initial momentum value of the impact event.)

\[
P_{y,f} = 2mv_{y,f}
\]

Use these knowns:

\[
m \quad \text{(given)}
\]

\[
v_{y,f} \quad \text{(from step V)}
\]

Solve for: \( P_{y,f} \)

VII. Find the impulse that brings the combined mass to rest in the \( y \)-direction.

\[
\Delta P_y = P_{y,f} - P_{y,i}
\]

Use these knowns:

\[
P_{y,f} = 0 \quad \text{(given; “no bounce” means mass is at rest in y-direction)}
\]

\[
P_{y,i} = P_{y,f} \quad \text{(from step VI)}
\]

Solve for: \( \Delta P_y \)

VIII. Find the time duration of this impact:

\[
\Delta P_y = F_{\text{avg}}(\Delta t_{\text{impact}})
\]

Use these knowns:

\[
\Delta P_y \quad \text{(from step VII)}
\]

\[
F_{\text{avg}} \quad \text{(given)}
\]

Solve for: \( \Delta t_{\text{impact}} \)
6. c. An object, initially at rest, is pushed across a level, frictionless table by a horizontal force $F_{\text{appObj}}$ for a time interval $\Delta t$. Sometime after the force has ceased its pushing, the object reaches the edge of the table and falls freely toward the floor below. After it has fallen a vertical distance $d$ (but before it reaches the floor, the object explodes into two pieces, $A$ (mass = $m_A$) and $B$ (mass = $m_B$). Piece A’s initial velocity after the explosion is directed vertically upward. When piece B hits the frictionless floor, it slides without bouncing until it collides with, and sticks to, another object, $C$ (mass = $m_C$, which has been at rest up to then). What is the speed of that combined mass (B&C)?

You may consider these values as **known**: $F_{\text{appObj}}, \Delta t, d, m_A, m_B, m_C, g$.

The only quantity of interest is the **horizontal** momentum of the original object, which is conserved through the explosion, the impact on the floor, and the collision with mass $C$.

**Equations:**

I. $P_{fx} - P_{ix} = F_{\text{appObj}} \Delta t$  
   (where $P_{ix} = 0$; the object on the table was originally at rest).

II. $P_{Bx} = (m_B + m_C)(v_{fBC})$

**Solving:**

Solve I for $P_{fx}$. Substitute that result as $P_{Bx}$ into II (piece B carries all the original $x$-momentum)

Solve II for $v_{fBC}$. 