Midterm Exam 2

Print your full LAST name: ____________________________________________

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Adjustment: __________
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Exam total: __________ / 250
1. **(60 points total)** Three blocks are on a level, frictionless surface. Their undersides always maintain contact with that surface. The blocks interact as follows:

Block A \((m_A = 2.00 \text{ kg})\) is initially sliding to the right at 5.00 m/s; block B \((m_B = 4.00 \text{ kg})\) and mass C \((m_C = 1.00 \text{ kg})\) are sitting together, at rest (see diagram above).

Block A then collides with the left side of block B. After the collision, the three blocks are separated, and these facts are known: Block A is at rest, and block B is sliding to the right at \(+1.00 \text{ m/s}\).

Some time later, block C strikes the wall; it is in contact with the wall for 40.0 ms. Block C then collides with block B. As a result of this second collision, blocks B and C both come to rest. **Find the average force (magnitude and direction) that block C exerts on the wall.**

(You may specify the direction of this force in any way you wish, so long as it is clearly stated.)

The initial x-momentum of the entire system (the three blocks) is simply \(P_{A,x,i}\).

That is: \[ P_{\text{total},x,i} = P_{A,x,i} \]

\[ = m_A v_{A,x,i} = (2.00)(5.00) = (+)10.0 \text{ kg·m/s} \]

**This must still be the x-momentum of the system right after the first collision.**

That is: \[ P_{\text{total},x,f} = P_{\text{total},x,i} = (+)10.0 \text{ kg·m/s} \]

Itemized: \[ m_A v_{A,x} + m_B v_{B,x} + m_C v_{C,x} = (+)10.0 \text{ kg·m/s} \]

Simplified: \[ (5)(0) + (4)(1) + (1)v_{C,x,f} = (+)10.0 \text{ kg·m/s} \]

Clearly: \[ v_{C,x} = (+)6.00 \text{ m/s} \]

And at that velocity, block C hits the wall, then rebounds back toward the oncoming block B.

When those two collide, they come to rest—the momentum of the pair (B & C) is zero afterward. So it was zero before they collided, too.

That is: \[ P_B + P_{C,\text{rebound}} = 0 \]

Itemized: \[ m_B v_{B,x} + m_C v_{C,\text{rebound},x} = 0 \]

Simplified: \[ (4)(1) + (1)v_{C,\text{rebound},x} = 0 \]

Clearly: \[ v_{C,\text{rebound},x} = -4.00 \text{ m/s} \]

So, what did the wall do to block C?? It exerted an impulse on C that changed C’s momentum.

In other words: \[ J_{\text{WC},x} = \Delta P_{C,x} \]

Itemized: \[ F_{\text{WC},x,\text{avg}} \Delta t = P_{C,x,f} - P_{C,x,i} \]

Solve for \(F_{\text{WC},x}:\)

\[ F_{\text{WC},x,\text{avg}} = (P_{C,x,f} - P_{C,x,i})/\Delta t \]

N’s 3rd Law: \[ F_{\text{CW},x,\text{avg}} = -F_{\text{WC},x,\text{avg}} = -(P_{C,x,f} - P_{C,x,i})/\Delta t \]

\[ = -m_C(v_{C,\text{rebound},x} - v_{C,x})/\Delta t = -(1)(-4.00 - 6.00)/0.040 \]

\[ = (+250 \text{ N}) \]

or \(250 \text{ N directed to the right}\)

Any variation equivalent to this is OK. 6 pts. total: value = 2 pts.; dir. = 2 pts.; units = 1 pt.; sig. figs = 1 pt.
Alternate reasoning:

The final momentum of the entire system is **zero** — all three blocks are at rest.
The initial momentum of the entire system was +10.0 kg·m/s.
So the wall’s impulse on C must have contributed a total of –10.0 kg·m/s of momentum to the system.
That is: \( J_{w,C,x} = -10.0 \text{ kg·m/s} = \Delta P_{C,x} \)
2. (60 points total) Refer to the diagram (a side view—not to scale).
The level slab is suspended above level ground. The pulleys are ideal, and
the ideal wires supporting the slab are vertical. Everything is at rest.

Calculate the magnitude of the friction force acting on \( m_1 \).

The data: \( m_1 = m_2 = m_3 = 15.4 \text{ kg} \quad \mu_{S,1} = \mu_{S,2} = 0.623 \quad \theta = 17.0^\circ \quad g = 9.80 \text{ m/s}^2 \)

First, this observation: Everything about \( m_1 \) is also applicable to \( m_2 \) — with the single exception of the directions of the tension forces acting on them, but even those are symmetrically opposite. Therefore, every aspect of an analysis of \( m_1 \) will apply to \( m_2 \) accordingly; we can analyze \( m_1 \) and simply extend the results to \( m_2 \).

Now recognize: \( F_{N,13} = F_{N,31} = F_{N,23} = F_N \)
And of course: \( m_1 = m_3 = m \)

Simplified y-info: 
\[
F_y + F_T \cos \theta - mg = 0 \quad \text{and} \quad 2F_T - 2F_N - mg = 0
\]
Thus: 
\[
F_T = mg - F_T \cos \theta
\]
Substitute: 
\[
2F_T - 2(mg - F_T \cos \theta) - mg = 0
\]
Expand: 
\[
2F_T - 2mg + 2F_T \cos \theta - mg = 0
\]
Collect terms: 
\[
2F_T + 2F_T \cos \theta = mg + 2mg
\]
Solve for \( F_T \): 
\[
F_T = \frac{3mg}{2 + 2\cos \theta}
\]

Now the x-info: 
\[
F_{S,31} = F_T \sin \theta
\]
\[
= \frac{3mg \sin \theta}{2 + 2\cos \theta}
\]
\[
= 3(15.4)(9.80)(\sin 17)/(2 + 2\cos 17)
\]
\[
= 33.8 \text{ N}
\]

6 pts. total: value = 3 pts.; units = 2 pt.; sig. figs = 1 pt.
Use this page as additional space, if needed, for problem 2
3. (60 points total) Refer to the diagram here (a side view).

A block is sliding to the right over a level surface.

At a certain time \( t_1 \), the block is sliding over a frictionless section of the surface.

At a later time \( t_2 \), the block is sliding over a section of the surface that does offer friction.

At no time during the interval \( t_1 \leq t \leq t_2 \) are any other new forces applied to the block (just the surface friction).

The block’s weight magnitude is 10% greater at time \( t_2 \) than at time \( t_1 \).

Find the coefficient of kinetic friction between the block and the rough (friction) portion surface.

By definition:

\[
||W_m|| = ||\Sigma F_{Contact.m}|| = ||F_{net.m} - F_{G,Em}||
\]

At \( t_1 \), when the block is sliding without friction, it has zero acceleration.

So \( F_{net.m} = 0 \).

That means:

\[
||W_1|| = ||0 - F_{G,Em}|| = mg
\]

Or, this way:

\[
||W_1|| = ||\Sigma F_{Contact.m}|| = ||F_{N,Sm}|| = ||F_{G,Em}|| = mg
\]

At \( t_2 \), when the block is sliding with friction, there is \(-\) acceleration.

So now:

\[
||W_2|| = ||\Sigma F_{Contact.m}|| = ||F_{N,Sm} + F_{K,Sm}||
\]

The vector sum:

But we know:

\[
F_{N,Sm} = mg
\]

And:

\[
F_{K,Sm} = \mu_k(F_{N,Sm}) = \mu_k(mg)
\]

And (given):

\[
||W_2|| = 1.1||W_1|| = 1.1(mg)
\]

Pythagoras:

\[
[1.1(mg)]^2 = (mg)^2 + [\mu_k(mg)]^2
\]

Simplified:

\[
1.1^2 = 1 + \mu_k^2
\]

Solve for \( \mu_k \):

\[
\mu_k = \sqrt{0.21} = 0.458
\]

6 pts. total: value = 5 pts.; sig. figs = 1 pt.
Use this page as additional space, if needed, for problem 3
4. **(70 points total)** Refer to the diagram shown here for both items a and b.

A horizontal force $F$ is applied as shown to a block of known mass $m$, which is at rest on a slope of angle $\theta$. The slope offers known friction to the block.

a. **(_____/35 points)** For this part a only, you may assume the following data: $m = 2.34$ kg, $\mu_s = 0.613$, $\theta = 25.4^\circ$, $g = 9.80$ m/s$^2$.

Find the maximum magnitude of $F$ for which the block will not slip.

(Let the block be $m_1$ here.)

\[
\begin{align*}
\Sigma F_{,x} &= m_1a_{,x} \\
F_x - F_{,S1}^{\text{max}} - F_{,G1,x} &= m_1a_{,x} \\
F\cos\theta - \mu_s F_{,N1} - m_1g\sin\theta &= 0 \\
\end{align*}
\]

Thus:

\[
F_{,N1} = F\sin\theta + m_1g\cos\theta
\]

Therefore:

\[
F\cos\theta - \mu_s(F\sin\theta + m_1g\cos\theta) - m_1g\sin\theta = 0
\]

Solve for $F$:

\[
F\cos\theta - \mu_s F\sin\theta = m_1g\sin\theta + \mu_s m_1g\cos\theta
\]

Result:

\[
F = \frac{(m_1g\sin\theta + \mu_s m_1g\cos\theta)}{(\cos\theta - \mu_s \sin\theta)}
\]

This expression gives the maximum $F$ can be but still not move the block — it was derived assuming static conditions (but with $F_{,S1}^{\text{max}}$ in effect).

Calculate $F$:

\[
F = \frac{[(2.34)(9.80)(\sin25.4^\circ) + (0.613)(2.34)(9.80)(\cos25.4^\circ)]}{[\cos25.4^\circ - (0.613)(\sin25.4^\circ)]} = 35.2 \text{ N}
\]

**4 pts. total: value = 2 pts.; units = 1 pt.; sig. figs = 1 pt.**
b. (_______ / 35 points) Refer to the same drawing (see previous page), but now assume that only the following symbols represent known values (they are not the numbers given in part a): \( m, \mu_s, g \)

Find an algebraic expression (containing only known values) for the slope angle \( \theta \) for which there is no possible magnitude of the force \( F \) (applied horizontally, in the direction shown) that will cause the block to slip.

Same analysis as before:

\[
\begin{align*}
\Sigma F_{1x} &= m_1a_{1x} \\
F_x - F_{SSI}^{max} - F_{G,E1,x} &= m_1a_{1x} \\
F \cos \theta - \mu_s F_{NSI} - m_1g \sin \theta &= 0 \\
\end{align*}
\]

\[
\begin{align*}
\Sigma F_{1y} &= m_1a_{1y} \\
F_{NSI} - F_y - F_{G,E1,y} &= m_1a_{1y} \\
F_{NSI} - F \sin \theta - m_1g \cos \theta &= 0 \\
\end{align*}
\]

Thus: \( F_{NSI} = F \sin \theta + m_1g \cos \theta \)

Therefore: \( F \cos \theta - \mu_s (F \sin \theta + m_1g \cos \theta) - m_1g \sin \theta = 0 \)

Solve for \( F \): \( F \cos \theta - \mu_s F \sin \theta = m_1g \sin \theta + \mu_s m_1g \cos \theta \)

Result: \( F = (m_1g \sin \theta + \mu_s m_1g \cos \theta)/(\cos \theta - \mu_s \sin \theta) \)

This expression gives the maximum \( F \) can be but still not move the block—it was derived assuming static conditions (but with \( F_{SSI}^{max} \) in effect).

Now notice: When \( \cos \theta - \mu_s \sin \theta \) (that’s the denominator of the above solution) becomes zero, \( F \) becomes infinite; the maximum force that will still not move the block is infinite. There is no way to exceed infinity; no matter how large the force, the block won’t budge—once \( \theta \) satisfies this equation: \( \cos \theta - \mu_s \sin \theta = 0 \)

Solve for \( \theta \): \( \cos \theta = \mu_s \sin \theta \)

\( \theta = \tan^{-1}(1/\mu_s) \)

or \( \theta = \cot^{-1}(\mu_s) \)

3 pts. (either expression OK)
GENERAL DIRECTIONS

Fill out the cover sheet completely, as indicated. Then follow the general guidelines below and the specific directions on each page for each item.

For ALL items (unless directed otherwise):
- In all expressions and symbolic solutions, reduce them to simplest form.
- In all final numerical answers, use standard SI units and three significant digits.
- No item will be given full credit if it does not include valid reasoning/work to justify the solution/answer. Correct answers alone are generally worth about 10% of the points.

For T/F/N items: Evaluate each statement as being either… demonstrably True (T), or demonstrably False (F), or with Not enough information (N) to declare it either True or False.
- You must fully explain your reasoning, using any valid mix of words, diagrams and/or equations.
- Little credit will be given for a correct T/F/N answer without a valid explanation to accompany it.

For items that give numeric values for the “knowns:”
- Some credit (10%) will be offered for a numeric answer, with a little additional (about 15-20%) for showing the math to arrive at that answer. The rest of the credit (70-75%) is offered for your valid physics reasoning and work—the “setup” (equations and process) before the math starts.
- Very little credit will be given for a correct answer without any work shown.

For items asking for symbolic solutions (algebraic expressions) and/or solution descriptions:
- These items will state which symbols may be considered “known values” (and which may therefore appear in your final expression or be used in the first step of your solution description).
- For an algebraic expression, again, about 75% of the credit is showing how the physics leads to the correct equations and use of the known values; the rest of the credit will be in solving/simplifying.
- For a solution description problem, full credit (100%) is offered for a valid description. Remember that the test of a valid solution description is this: Could another person who knows math (but not physics) use your solution description—-together with the known values given—to arrive at a correct answer?

Physical constants and other possibly useful information:

\[ g_{\text{free fall}} = 9.80 \text{ m/s}^2 \]