Operators & Functions

Your group will discuss one of the sets: if you finish early, move on to another, swapping roles of taskmaster, cynic, & recorder. Everyone participates in the intellectual process, but the taskmaster manages and keeps people on-task, the cynic looks for holes in arguments, and the recorder makes sure that no information is lost.

You are given an operator and the mathematical instruction that represents it. You are also given a number of wave functions.

- Test each function to see if it is an eigenfunction of the operator.

- If it is, what is the eigenvalue?

- If it is not, can you write it as a superposition of functions that are eigenfunctions of that operator?

1. $\hat{p} \rightarrow i\hbar \frac{d}{dx}$

$\psi_1(x) = Ae^{-ikx}$ $\psi_2(x) = Ae^{+ikx}$ $\psi_3(x) = A\sin(kx)$

2. $\hat{H} \rightarrow -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

$\psi_1(x) = Ae^{-i\frac{p}{\hbar}x}$ $\psi_2(x) = Ae^{+i\frac{p}{\hbar}x}$ $\psi_3(x) = A\sin\left(\frac{p}{\hbar}x\right)$

3. $\hat{H} \rightarrow -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

$\psi_1(x) = A\sin(kx)$ $\psi_2(x) = A\cos(kx)$ $\psi_3(x) = Ae^{ikx}$

4. $\hat{S}_z \rightarrow \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$|\psi_1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $|\psi_2\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $|\psi_3\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

In what context have you encountered eigenfunctions and eigenvalues before?

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