FROM FEAR TO FUN IN THERMODYNAMICS: Partial Derivatives in Thermodynamics (PERC 2013)

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The Activity

Given the following equations of state for the *total magnetization* M, and the entropy S, in terms of the temperature T and the magnitude B of the magnetic field:

$$M = N\mu \frac{e^{\frac{\mu B}{k_B T}} - e^{-\frac{\mu B}{k_B T}}}{e^{\frac{\mu B}{k_B T}} + e^{-\frac{\mu B}{k_B T}}}$$

$$S = Nk_B \left(\ln\left(e^{\frac{\mu B}{k_B T}} + e^{-\frac{\mu B}{k_B T}} \right) - \frac{\mu B}{k_B T} \frac{e^{\frac{\mu B}{k_B T}} - e^{-\frac{\mu B}{k_B T}}}{e^{\frac{\mu B}{k_B T}} + e^{-\frac{\mu B}{k_B T}}} \right)$$

Compute the following derivatives:

$$\chi_T = \left(\frac{\partial M}{\partial B} \right)_T$$

$$\chi_S = \left(\frac{\partial M}{\partial B} \right)_S$$

(χ_T is the *isothermal magnetic susceptibility*; χ_S is the *adiabatic magnetic susceptibility*).

For further information about this session, scan the QR code at the right, or navigate to:
`http://physics.oregonstate.edu/portfolioswiki/workshops:perc13:start`
or contact David Roundy `<roundyd@physics.oregonstate.edu>`.

Solution

The Substitution Game

Solve the second equation for T in terms of S (!), then substitute into the first equation and differentiate...

The Partial Derivatives Game

Use the chain rule:

$$\left(\frac{\partial M}{\partial B}\right)_S = \left(\frac{\partial M}{\partial B}\right)_T + \left(\frac{\partial M}{\partial T}\right)_B \left(\frac{\partial T}{\partial B}\right)_S$$

and the cyclic identity

$$\left(\frac{\partial T}{\partial B}\right)_S = \left(\frac{\partial S}{\partial B}\right)_T \Big/ \left(\frac{\partial S}{\partial T}\right)_B$$

The Differentials Game

Zap both equations with d:

$$dM = \left(\frac{\partial M}{\partial B}\right)_T dB + \left(\frac{\partial M}{\partial T}\right)_B dT$$
$$dS = \left(\frac{\partial S}{\partial B}\right)_T dB + \left(\frac{\partial S}{\partial T}\right)_B dT$$

Solve the second equation for dT, substitute into the first equation, and read off the coefficient of dB. Alternatively, solve $dS = 0$ for dT, substitute into dM, divide by dB.

Chain Rule Diagrams

The second diagram below encodes the chain rule used in the partial derivatives game, with derivatives taken in the direction of the arrows. The first diagram is similar, but encodes the known information $M = M(B,T)$, $S = S(B,T)$; the derivatives along the given arrows can be computed directly. Comparing these two diagrams can can be used to determine the cyclic identity needed to transform the first diagram into the second. Replacing each variable in these diagrams by its differential (e.g. M by dM) makes them appropriate for the differentials game, where (pairs of) arrows now represent an expansion in terms of a basis. The last diagram represents an alternative strategy for either game, in which the order of operations is reversed, eliminating the need for a cyclic identity in the partial derivatives game.