Group Activity 5: The Pretzel

I Essentials

(a) Main ideas

- Calculating (scalar) line integrals.
- Use what you know!

(b) Prerequisites

- Familiarity with $d\vec{\boldsymbol{r}}$.
- Familiarity with "Use what you know" strategy.

(c) Warmup

It is *not* necessary to explicitly introduce scalar line integrals, before this lab; figuring out that the (scalar) line element must be $|d\vec{\boldsymbol{r}}|$ can be made part of the activity (if time permits).

(d) Props

- whiteboards and pens
- "linear" chocolate covered candy (e.g. Pocky)

(e) Wrapup

- Emphasize that students must express each integrand in terms of a single variable prior to integration.
- Emphasize that each integral must be positive!
- Discuss several different ways of doing this problem (see below).

II Details

(a) In the Classroom

- Make sure the shape of the pretzel is clear! It might be worth drawing it on the board.
- Some students will work geometrically, determining ds on each piece by inspection. This is fine, but encourage such students to try using $d\vec{\boldsymbol{r}}$ afterwards.
- Polar coordinates are natural for all three parts of this problem, not just the circular arc.
- Many students will think that the integral "down" the y-axis should be negative. They will argue that $ds = dy$, but the limits are from 2 to 0. The resolution is that $ds = |dy\,\hat{\boldsymbol{\imath}}| = |dy| = -dy$ when integrating in this direction.
- Unlike work or circulation, the amount of chocolate does not depend on which way one integrates, so there is in fact no need to integrate "down" the y-axis at all.
- Some students may argue that $d\vec{\boldsymbol{r}} = \hat{\boldsymbol{T}}\,ds \Longrightarrow ds = d\vec{\boldsymbol{r}} \cdot \hat{\boldsymbol{T}}$, and use this to get the signs right. This is fine if it comes up, but the unit tangent vector $\hat{\boldsymbol{T}}$ is not a fundamental part of our approach.
- There is of course a symmetry argument which says that the two "legs" along the axes must have the same amount of chocolate — although some students will put a minus sign into this argument!
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(b) Subsidiary ideas

- $ds = |d\vec{\boldsymbol{r}}|$

(c) Homework (none yet)

(d) Essay questions (none yet)

(e) Enrichment (none yet)