Group Activity 11: The Fishing Net

I Essentials

(a) Main ideas

- Practice doing surface integrals
- The Divergence Theorem

(b) Prerequisites

- Ability to do flux integrals
- Definition of divergence
- Statement of Divergence Theorem
  This lab can be used prior to covering the Divergence Theorem in class with either a minimal introduction or a restatement of the last question based on the assumption that the given vector field doesn't "lose" anything going through the net.

(c) Warmup

- Perhaps a reminder about what the Divergence Theorem is.

(d) Props

- whiteboards and pens
- a model of the fishing net, made from any children's building set

(e) Wrapup

- Reiterate that the Divergence Theorem only applies to closed surfaces.
- Emphasize that the Divergence Theorem is one of several astonishing theorems relating what happens inside to what happens outside.
- Have several students show how they computed $d\vec{\boldsymbol{A}}$, since most likely different choices were made for $d\vec{\boldsymbol{r}}_i$ and hence the limits.

II Details

(a) In the Classroom

- By now the groups should be working well. Sit back and watch!
- The main thing to watch out for is whether students choose the correct signs, both for the normal vectors and the limits of integration. Reiterate that one should *always* write $d\vec{\boldsymbol{r}} = dx\,\hat{\boldsymbol{\imath}} + dy\,\hat{\boldsymbol{\jmath}} + dz\,\hat{\boldsymbol{k}}$; there should *never* be minus signs in this equation. The signs will come out right provided one integrates in the direction of the vectors chosen.
- Most students will realize quickly that there is no flux through the triangular sides.
- Some students will try to do the surface integrals! Point out that this isn't possible — and that the instructions say not to.
- Student may be surprised at first when they calculate $\vec{\nabla} \cdot \vec{\boldsymbol{F}} = 0$, especially since they (correctly) won't think that the surface integrals will add to zero. Use this to motivate the "missing top".
- Some students incorrectly think that $d|z| = |dz|$.

(b) Subsidiary ideas

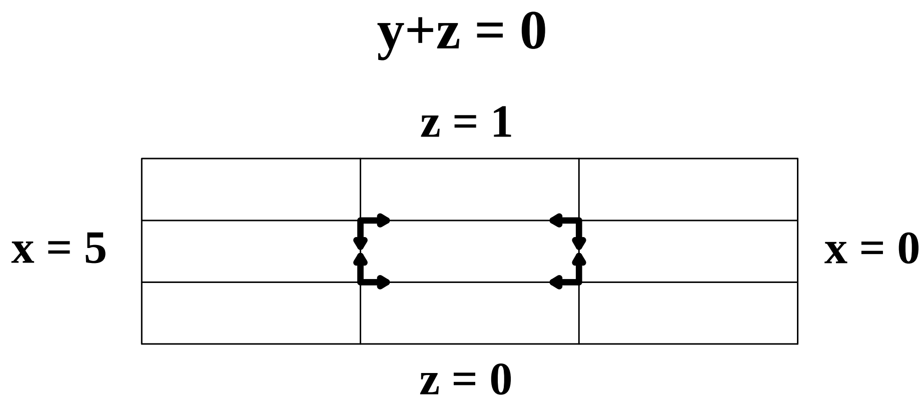
- The geometry of flux.

(c) Homework (none yet)

(d) Essay questions (none yet)

(e) Enrichment

- The surface integrals can in fact be done — provided one adds them up prior to evaluating the integrals.
- This lab provides a good opportunity for students to visualize the flux: It's easy to see that the flux of the horizontal component of this vector field must be zero geometrically. (It's even easier to see that the vertical flux must be zero.)



- During the wrapup (or the following lecture), draw a picture such as the one above of one of the rectangular faces, showing all 4 possible choices for $d\vec{\boldsymbol{r}}_1$ and $d\vec{\boldsymbol{r}}_2$ (and which is which!), and discuss the integration limits in each case.

- An alternative approach to this problem is to determine dA geometrically, compute $\vec{\boldsymbol{F}} \cdot \hat{\boldsymbol{n}}$ explicitly, and then do the integral using "standard" (increasing) limits. There is nothing wrong with this approach, but we would discourage the use of the $d\vec{\boldsymbol{r}}$ notation here for fear of making sign errors.

- One could show students the remarkable trick for integrating e^{-x^2} from 0 to ∞, by squaring and evaluating in polar coordinates.