Group Activity 8: Potential Functions

I Essentials

(a) Main ideas

- Finding potential functions.

Students love this activity. Some groups will finish in 10 minutes or less; few will require as much as 30 minutes. [1]

(b) Prerequisites

- Fundamental Theorem for line integrals
- The Murder Mystery Method

(c) Warmup

none

(d) Props

- whiteboards and pens

(e) Wrapup

- Revisit integrating conservative vector fields along various paths, including reversing the orientation and integrating around closed paths.

[1] More accurately, students love the Murder Mystery Method! We often incorporate this activity into an exam review, rather than devoting an entire period to it.

II Details

(a) In the Classroom

- We recommend having the students work in groups of 2 on this activity, and not having them turn anything in.
- Most students will treat the last example as 2-dimensional, giving the answer xyz. Ask these students to check their work by taking the gradient; most will include a $\hat{\boldsymbol{k}}$ term. Let them think this through. The correct answer of course depends on whether one assumes that z is constant; we have deliberately left this ambiguous.

(b) Subsidiary ideas

- 3-d vector fields do not necessarily have a $\hat{\boldsymbol{k}}$-component!

(c) Homework (none yet)

(d) Essay questions (none yet)

(e) Enrichment

- The derivative check for conservative vector fields can be described using the same type of diagrams as used in the Murder Mystery Method; this is just moving down the diagram (via differentiation) from the row containing the components of the vector field, rather than moving up (via integration). We believe this should not be mentioned until after this lab.
  When done in 3-d, this makes a nice introduction to curl — which however is not needed until one is ready to do Stokes' Theorem. We would therefore recommend delaying this entire discussion, including the 2-d case, until then.
- Work out the Murder Mystery Method using polar basis vectors, by reversing the process of taking the gradient in this basis.
- Revisit the example in the *Ampère's Law* lab, using the Fundamental Theorem to explain the results. This can be done without reference to a basis, but it is worth computing $\vec{\nabla}\phi$ in a polar basis.