Group Activity 9: The Grid

I Essentials

(a) Main ideas

- Understanding different ways of expressing area using integration.
- Concrete example of Area Corollary to Green's/Stokes' Theorem.

We originally used this activity after covering Green's Theorem; we now skip Green's Theorem and do this activity shortly before Stokes' Theorem.

(b) Prerequisites

- Familiarity with line integrals.
- *Green's Theorem is not a prerequisite!*

(c) Warmup

- The first problem is a good warmup.

(d) Props

- whiteboards and pens
- a planimeter if available

(e) Wrapup

- Emphasize the magic – finding area by walking around the boundary!
- Point out that this works for any closed curve, not just the rectangular regions considered here.
- Demonstrate or describe a planimeter, used for instance to measure the area of a region on a map by tracing the boundary.

II Details

(a) In the Classroom

- Make sure students use a consistent orientation on their path.
- Make sure students explicitly include all segments of their path, including those which obviously yield zero.
- Students in a given group should all use the same curve.
- Students should be discouraged from drawing a curve whose longest side is along a coordinate axis.
- Students may need to be reminded that $\oint$ implies the counterclockwise orientation. But it doesn't matter what orientation students use so long as they are consistent!
- A geometric argument that the orientation should be reversed when interchanging x and y is to rotate the xy-plane about the line $y = x$. (This explains the minus sign in Green's Theorem.)
- Students may not have seen line integrals of this form (see below).

(b) Subsidiary ideas

- Orientation of closed paths.
- Line integrals of the form $\int P\,dx + Q\,dy$.
  We do not discuss such integrals in class! Integrals of this form almost always arise in applications as $\int \vec{\boldsymbol{F}} \cdot d\vec{\boldsymbol{r}}$.

(c) Homework (none yet)

(d) Essay questions (none yet)

(e) Enrichment

- Write down Green's Theorem.
- Go to 3 dimensions — bend the curve out of the plane and stretch the region like a butterfly net or rubber sheet. This is the setting for Stokes' Theorem!