SPINS Lab 2

1.   Start the SPINS program and choose `Unknown #1` under the `Initialize` menu. This causes the atoms to leave the oven in a definite quantum state, which we call $|\psi_1\rangle$. Now measure the six probabilities $|\langle\phi|\psi_1\rangle|^2$, where $|\phi\rangle$ corresponds to spin up and spin down along the three axes. Fill in the table for $|\psi_1\rangle$ on the worksheet. Use your results to figure out what $|\psi_1\rangle$ is, using the following procedure (even though it may be obvious what the unknown state is for the first few cases, follow the procedure as practice for the harder cases to follow):
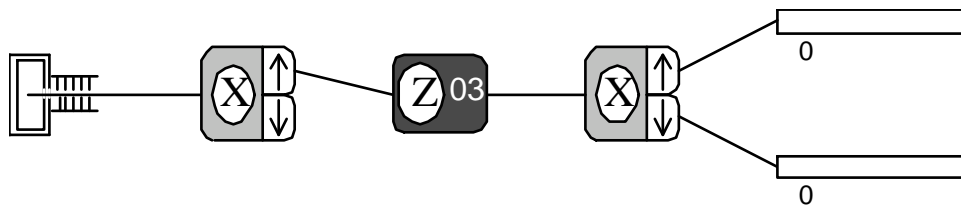
   i)   Assume that we want to write the unknown state vector in terms of the $|\pm\rangle$ basis, *i.e.* $|\psi_1\rangle = a|+\rangle + b|-\rangle$, where a and b are complex coefficients. We thus must use the data to find the values of a and b.

   ii) Use the measured probabilities of spin up and spin down along the z-axis first. This will allow you to determine the magnitudes of a and b. Since an overall phase of the state vector has no physical meaning, we follow the convention that the coefficient of $|+\rangle$ (i.e. a) is chosen to be real and positive. If we write $b = |b|e^{i\phi}$, then you have determined everything except the phase ϕ.

   iii) Use the measured probabilities of spin up and spin down along the x- axis to provide information about the phase of b. In some cases, this will be unambiguous, in other cases not.

   iv) If needed, use the measured probabilities of spin up and spin down along the y- axis.

   v)   Confirm your results by entering your calculated coefficients into the table for `User State` and running the experiment.

Repeat this exercise for `Unknown #2` ($|\psi_2\rangle$), `Unknown #3` ($|\psi_3\rangle$), and `Unknown #4` ($|\psi_4\rangle$). Design an experiment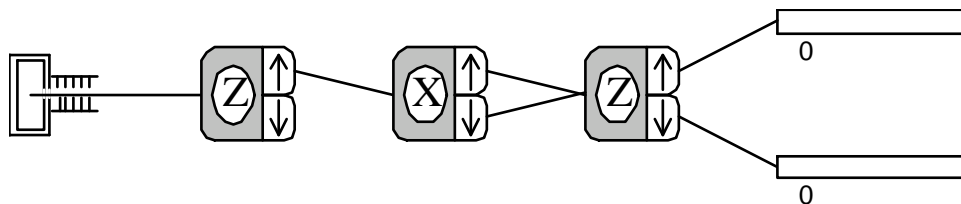 to verify your results (Hint: the general spin 1/2 state vector can be written as $|+\rangle_n = \cos\dfrac{\theta}{2}|+\rangle + \sin\dfrac{\theta}{2}e^{i\phi}|-\rangle$).

2.   Make another unknown state using the following setup with a magnet in between two
Stern-Gerlach analyzers:

Consider the magnet for now as a black box that transforms the input $|+\rangle_x$ state into a
new state $|\psi\rangle$. Use Random as the initial state and set the strength of the magnet to a number
from 1-20 corresponding to the position of your computer in the lab. Use the last analyzer to
measure the probabilities for the state $|\psi\rangle$ to have the six possible spin projections along the
three axes. Keep the first Stern-Gerlach analyzer and the middle magnet oriented as shown in
the figure. Fill in the table on the worksheet and deduce the state $|\psi\rangle$, in terms of the $|\pm\rangle$ basis.
Design an experiment to verify your results. Can you figure out what the magnet does?

3.   Make an interferometer as shown:

Use Random as the initial state. Measure the relative probabilities of spin up and spin
down after the final SG device. Do this for the three cases where (1) the spin up beam from the
middle SG device is used, (2) the spin down beam from the middle SG device is used, and (3)
both beams from the middle SG device are used (as the figure shows). Put your experimental
results in the table on the worksheet. Repeat for the case where the spin down state is used from
the first SG device.

Now calculate the theoretically expected probabilities and fill in the theory part of the
worksheet. When both beams from the middle SG device are used, the input state to the last SG
device (call it $|\psi\rangle$) is a combination (superposition) of the spin up and spin down states with

respect to the axis of the middle SG device. To calculate $|\psi\rangle$ properly, use the projection postulate (5th postulate), which says that the ket after a measurement is the normalized projection onto the subspace corresponding to the measured results. For the case where both spin up and spin down results are obtained (because both beams are sent to the next device), the projector is $P = |+\rangle_{xx}\langle+| \; + \; |-\rangle_{xx}\langle-|$. For the case where the input to the middle SG device is $|+\rangle$, the projection is

$$P|+\rangle = |+\rangle_{xx}\langle+|+\rangle \; + \; |-\rangle_{xx}\langle-|+\rangle \; = \; {}_x\langle+|+\rangle|+\rangle_x \; + \; {}_x\langle-|+\rangle|-\rangle_x.$$

To find $|\psi\rangle$, just normalize this new ket.

4.   Using the interferometer shown above, select `Watch` under the `Control` menu. This feature causes a light to flash at the port where an atom exits. The program now only allows the `Go` mode. Run the experiment and notice how your results compare to the results obtained above. Explain the new results.

SPINS Lab 2 Worksheet

Unknown $|\psi_1\rangle$

Probabilities	Axis		
Result	x	y	z
spin up ↑			
spin down ↓			

Unknown $|\psi_2\rangle$

Probabilities	Axis		
Result	x	y	z
spin up ↑			
spin down ↓			

Unknown $|\psi_3\rangle$

Probabilities	Axis		
Result	x	y	z
spin up ↑			
spin down ↓			

Unknown $|\psi_4\rangle$

Probabilities	Axis		
Result	x	y	z
spin up ↑			
spin down ↓			

State $|\psi\rangle$ made with magnet. B = ____

Probabilities	Axis		
Result	x	y	z
spin up ↑			
spin down ↓			

Spin 1/2 Interferometer

Beams used		Experiment		Theory	
SG1	SG2	P↑	P↓	P↑	P↓
$|+\rangle$	$|+\rangle_x$				
	$|-\rangle_x$				
	$|+\rangle_x , |-\rangle_x$				
$|-\rangle$	$|+\rangle_x$				
	$|-\rangle_x$				
	$|+\rangle_x , |-\rangle_x$				