Outer Product of a Vector on Itself

$|v_1\rangle = |+\rangle \doteq \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ $\qquad$ $|v_2\rangle = |-\rangle \doteq \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$|v_3\rangle = |+\rangle_x \doteq \sqrt{\frac{1}{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\qquad$ $|v_4\rangle = |-\rangle_x \doteq \sqrt{\frac{1}{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$

$|v_5\rangle = |+\rangle_y \doteq \sqrt{\frac{1}{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$ $\qquad$ $|v_6\rangle = |-\rangle_y \doteq \sqrt{\frac{1}{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$

$|v_7\rangle \doteq \frac{1}{5}\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\qquad$ $|v_8\rangle \doteq \frac{1}{5}\begin{pmatrix} 4 \\ -3 \end{pmatrix}$

$|v_9\rangle \doteq \begin{pmatrix} a \\ be^{i\gamma} \end{pmatrix}$ (a and b are real and $a^2+b^2 = 1$ because it's a quantum state.)

$|v_{10}\rangle = |1\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1/\sqrt{2} \\ 1 \\ 1/\sqrt{2} \end{pmatrix}$ $\qquad$ $|v_{11}\rangle = |0\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ $\qquad$ $|v_{12}\rangle = |-1\rangle_x \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1/\sqrt{2} \\ -1 \\ 1/\sqrt{2} \end{pmatrix}$

1. For one of the vectors above, determine the outer product of the vector on itself (i.e., $|v_1\rangle\langle v_1|$).

2. Determine the transformation caused by your outer-product matrix.

3. Find the determinant of your outer-product matrix.

4. Find the square of your outer-product matrix.

Bonus: What happens when you add the outer products for a complete orthonormal basis?