FROM FEAR TO FUN IN THERMODYNAMICS: Thermodynamics of a Rubber Band (PERC 2013)

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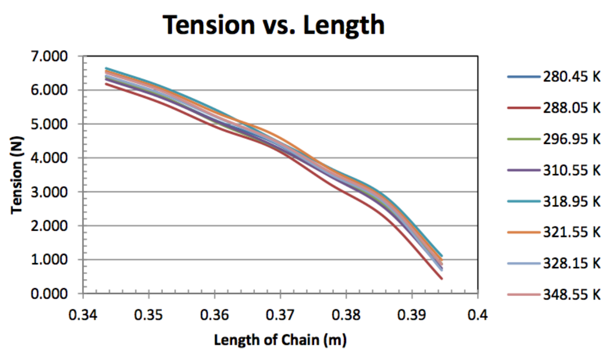
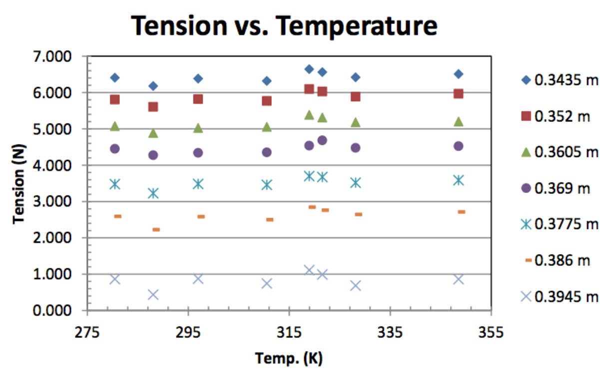
The Activity

Using the apparatus on display (or see link at the bottom of the page) a student measured the tension as a function of length and temperature. Below is the data from her lab report:





Using the plots above, answer the following questions

1. What is the change in free energy for an isothermal stretch at $T = 296.95\text{K}$ as the chain length is decreased from 0.39m to 0.35m? The Helmholtz free energy is defined as

$$F = U - TS$$

and its differential is

$$dF = -SdT - \tau dL$$

where τ is the tension in the rubber band and L is the length of the rubber band.

2. What is the change in entropy for the same isothermal stretch? To solve this, you will need the Maxwell relation that is derived from the Helmholtz free energy:

$$-\left(\frac{\partial S}{\partial L}\right)_T = \left(\frac{\partial \tau}{\partial T}\right)_L .$$

3. What is the change in the internal energy for the same isothermal stretch?

For further information about this session, scan the QR code at the right, or navigate to:
`http://physics.oregonstate.edu/portfolioswiki/workshops:perc13:start`
or contact David Roundy `<roundyd@physics.oregonstate.edu>`.

Solution

Free energy

To solve for the change in free energy, we use the total differential $dF = \tau\, dL - S\, dT$. Since this was an isothermal process, we know $dT = 0$, therefore $dF = \tau\, dL$. Integrating this expression gives us

$$\Delta F = \int \tau\, dL.$$

Entropy

To find the change in entropy of the process, we can utilize the Maxwell relation provided. From the second plot, we can find an approximate value for $\left(\frac{\partial \tau}{\partial T}\right)_L$ since the data looks like it might be linear. The Maxwell relation tells us that we can switch from $\left(\frac{\partial \tau}{\partial T}\right)_L$ to $-\left(\frac{\partial S}{\partial L}\right)_T$. Finally, we just need to integrate:

$$\Delta S = \int -\left(\frac{\partial S}{\partial L}\right)_T dL.$$

Internal energy

The definition of Helmholtz free energy tells us $F = U - TS$, and solving this expression for U gives us a second equation, $U = F + TS$. Since we already know ΔF and ΔS, and T is a constant, the change in total energy is given by

$$\Delta U = \Delta F + T\,\Delta S.$$