Lorentz Transformations for Electromagnetism

Working in groups of two or three, do the following problems.

The electric field of an infinite metal sheet with charge density σ points away from the sheet and has the constant magnitude

$$|E| = \frac{\sigma}{2\epsilon_0} \tag{1}$$

The magnetic field of such a sheet with current density $\vec{\boldsymbol{\kappa}}$ has constant magnitude

$$|B| = \frac{\mu_0}{2}|\vec{\boldsymbol{\kappa}}| \tag{2}$$

and direction determined by the right-hand-rule.

1. Consider a capacitor consisting of 2 horizontal ($y =$ constant!) parallel plates, with equal and opposite charge densities. For definiteness, take the charge density on the bottom plate to be σ_0, and suppose that the charges are at rest, that is, that the current density of each plate is zero. Determine the electric field $\vec{\boldsymbol{E}}_0$ between the plates. *(What is the electric field elsewhere?)*

2. Now let the capacitor move to the *left* ($-x$ direction) with speed u, while you remain at rest. Then the *width* of the plate is unchanged, but you observe the *length* to be Lorentz contracted. What charge density do you observe on each plate?

3. What current density do you observe on each plate?

4. What electric and magnetic fields do you observe between the plates? *Express your answers in terms of* $E_0 = |\vec{\boldsymbol{E}}_0|$ *and* α, *where* $u = c\tanh\alpha$.

Before moving on, compare your results with another group!

5. The above discussion gives the electric ($\vec{\boldsymbol{E}}$) and magnetic ($\vec{\boldsymbol{B}}$) fields due to parallel plates moving to the left *as seen by an observer at rest.* What electric ($\vec{\boldsymbol{E}}'$) and magnetic ($\vec{\boldsymbol{B}}'$) fields are seen by an observer *moving to the right* with speed $v = c\tanh\beta$?

   You may wish to recall that

$$\begin{aligned}\sinh(\alpha \pm \beta) &= \sinh\alpha\cosh\beta \pm \cosh\alpha\sinh\beta \\ \cosh(\alpha \pm \beta) &= \cosh\alpha\cosh\beta \pm \sinh\alpha\sinh\beta\end{aligned}$$

6. If you have not already done so, express the components E'_y and B'_z in terms of E_y and B_z. That is, eliminate E_0 and α in favor of E^y and B^z. *Recall that* $\mu_0\epsilon_0 c^2 = 1$.

7. Interpret your results.

by Tevian Dray & Mary Bridget Kustusch
©2012 Tevian Dray