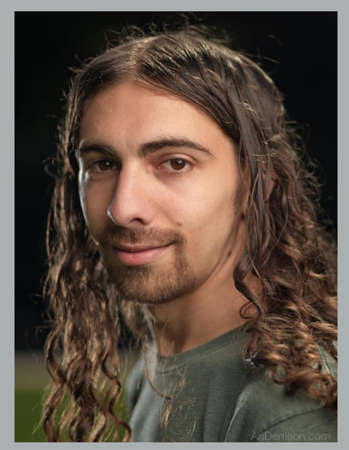
Analogues in Thermodynamics:
Legendre Transformations and the Partial Derivative Machine

Michael Vignal, Corinne A. Manogue, David Roundy, and Elizabeth Gire



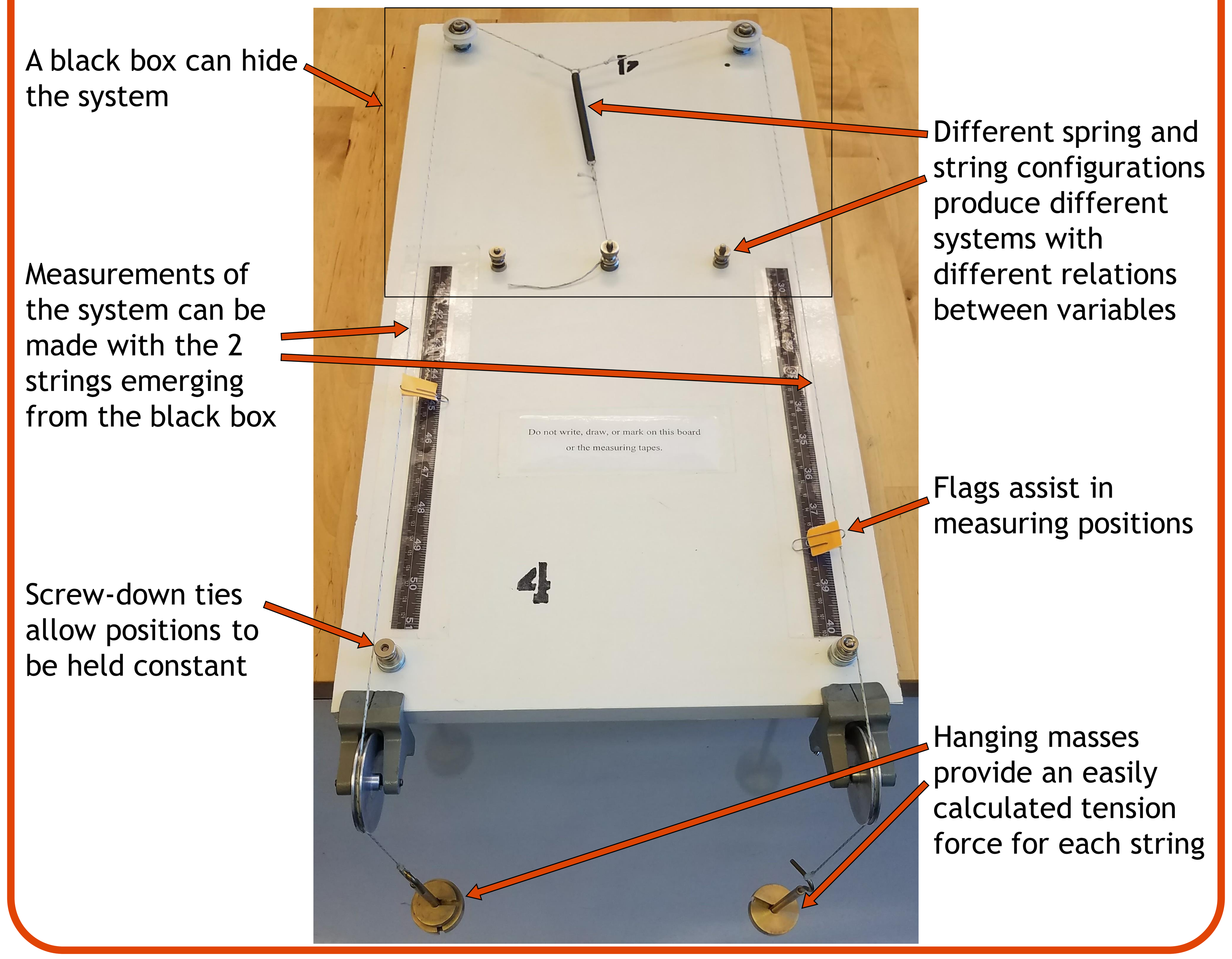
Overview

- The Partial Derivative Machine (PDM) is a mechanical analogue for a thermodynamic system where forces and positions represent thermodynamic variables such as temperature and entropy [1].
- We conducted 12 teaching interviews with the PDM on the topic of Legendre transformations to explore student understanding and use of the PDM.
- We found that students understand and use the PDM in different, though generally productive, ways.

Teaching Interview Protocol

- Legendre Transformation Recall Questions
  - i.e. What is a Legendre Transformation?
  - i.e. What are thermodynamic potentials?
- PDM Recall Questions
  - i.e. What do you remember about the PDM from class?
- Teaching Legendre Transformations on the PDM (~25 minutes)
  - See Legendre Transformations
- Transfer Problem
  - See Transfer Problem
- Reflection
  - i.e. Was the PDM useful for solving the transfer problem?

The Partial Derivative Machine (PDM)

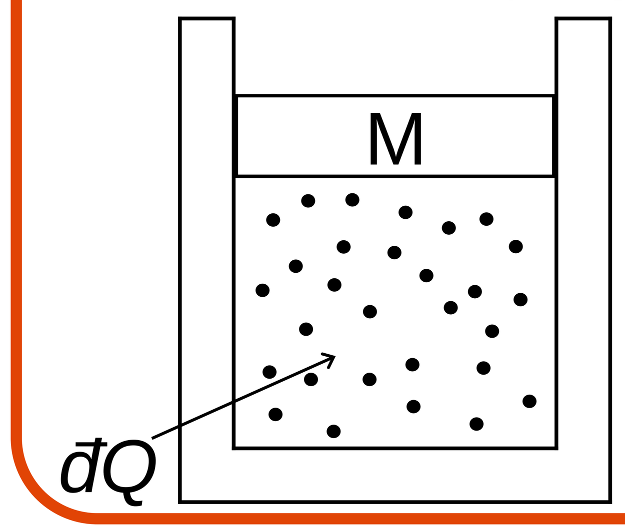


Legendre Transformations

Legendre transformations yield thermodynamic potentials, such as A and H below, with unique independent variables (the dX's). Considering these potentials provides new perspectives of a given physical system.

PDM	Thermodynamics	
$dU = F_1\,dx_1 + F_2\,dx_2$	$dU = T\,dS - p\,dV$	First Law (given)
$dA = \_\_\,dx_1 + \_\_\,dF_2$	$dH = \_\_\,dS + \_\_\,dp$	Target Equation
$A = U - F_2\,x_2$	$H = U + p\,V$	Legendre Transformation
$dA = dU - F_2\,dx_2 - x_2\,dF_2$	$dH = dU + p\,dV + V\,dp$	'Zap with *d*' [2]
$dA = F_1\,dx_1 + F_2\,dx_2 - F_2\,dx_2 - x_2\,dF_2$	$dH = T\,dS - p\,dV + p\,dV + V\,dp$	Substitute in the First Law
$dA = F_1\,dx_1 - x_2\,dF_2$	$dH = T\,dS + V\,dp$	Simplify

Transfer Problem



Consider a gas in a chamber in equilibrium with a massive piston (free to slide up and down) on top. Suppose we add an amount of heat $đQ = TdS$ to the gas (the system is otherwise thermally isolated). A change in which thermodynamic potential would be the easiest for us to measure?

Do students understand the PDM as a mechanical device?

Yes

- All 12 participants demonstrated understanding of the variables (2 forces and 2 positions) that can be measured on the PDM.
- 10 of 12 participants discussed measuring the relations between these 4 variables (such as through partial derivatives).

Sam: *We looked at like every variable that you could control, like the mass* [gestures at right mass], *where your starting distance was* [gestures at right position marker], *whether or not you are holding* [the right position] *constant so it would not be able to move. . . . And then how changing one of those variables affects the other variables in the system.*

Do students understand the PDM as a thermodynamic analogy?

Yes, and in different ways

- 4 participants discussed how the PDM can model a state system.
- 6 participants discussed parallels in the inaccessibility of some quantities.
- 9 participants discussed how the PDM can be used to find relations between different variables in a way that relates to thermodynamics.

Gabriel: *We also used* [the PDM] *to demonstrate that you can describe a certain state of a system using a minimum number of variables. Like, in here* [gestures at black box] *there was the strings coming off the 2 different sides, and you could describe* [the system] *based on I think just 2 variables.*

Do students transfer understanding from the PDM to thermodynamics?

Yes, and in different ways

- 10 participants referred back to some aspect of the PDM during the transfer problem.
- 9 participants referenced equations for the PDM (which have clear parallels to thermodynamic equations) during the transfer problem.
- 4 participants referenced the physical PDM during the transfer problem.

Kai: *If pressure's gonna be constant. . . before we were considering the x's, but we wanted to talk about* F_2*, so I think we want to get to the point where we can talk about a dp. . . . So we could do a Legendre transformation where we're gonna. . . add a Vp to, I want to say, U. Right?* [writes $dA = U + V\,p$].

Oregon State University

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References

[1] G. Sherer, M. B. Kustusch, C. A. Manogue, and D. Roundy, in PERC Proceedings (2013) pp. 341–344.
[2] T. Dray and C. A. Manogue, The College Mathematics Journal 41, 90 (2010).