

Word	Bra-ket	\hat{i} & \hat{j} Vectors	Matrix	Arrows
Vector	$ r\rangle$	$\vec{r} = r_x \hat{i} + r_y \hat{j}$	$\begin{pmatrix} r_x \\ r_y \end{pmatrix}$	
Adjoint Vector	$\langle r $	$\vec{r}^\dagger = r_x^* \hat{i} + r_y^* \hat{j}$	$(r_x^* \quad r_y^*)$	
Ket Basis	$ 1\rangle$	\hat{i}	$\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	
	$ 2\rangle$	\hat{j}	$\begin{pmatrix} 0 \\ 1 \end{pmatrix}$	
Bra Basis	$\langle 1 $	\hat{i}	$(1 \quad 0)$	
	$\langle 2 $	\hat{j}	$(0 \quad 1)$	
Orthogonality	$\langle 1 2\rangle$	$\hat{i} \cdot \hat{j}$	$(1 \quad 0) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 0$	
Normalization	$\langle 1 1\rangle$	$\hat{i} \cdot \hat{i}$	$(1 \quad 0) \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 1$	
	$\langle 2 2\rangle$	$\hat{j} \cdot \hat{j}$	$(0 \quad 1) \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$	
Length (Real)	$\begin{aligned}\langle r r\rangle &= (r_x\langle 1 + r_y\langle 2)(r_x 1\rangle + r_y 2\rangle) \\ &= r_x^2\langle 1 1\rangle + r_x r_y\langle 1 2\rangle \\ &\quad + r_y r_x\langle 2 1\rangle + r_y^2\langle 2 2\rangle \\ &= r_x^2 + r_y^2\end{aligned}$	$\begin{aligned}\vec{r} \cdot \vec{r} &= (r_x \hat{i} + r_y \hat{j}) \cdot (r_x \hat{i} + r_y \hat{j}) \\ &= r_x^2 \hat{i} \cdot \hat{i} + r_x r_y \hat{i} \cdot \hat{j} \\ &\quad + r_y r_x \hat{j} \cdot \hat{i} + r_y^2 \hat{j} \cdot \hat{j} \\ &= r_x^2 + r_y^2\end{aligned}$	$(r_x \quad r_y) \begin{pmatrix} r_x \\ r_y \end{pmatrix} = r_x^2 + r_y^2$	
Components (Real)	$\begin{aligned}\langle 1 r\rangle &= \langle 1 (r_x 1\rangle + r_y 2\rangle) = r_x\langle 1 1\rangle + r_y\langle 1 2\rangle \\ &= r_x\end{aligned}$ $\langle 2 r\rangle =$	$\hat{i} \cdot \vec{r} = \hat{i} \cdot (r_x \hat{i} + r_y \hat{j}) = r_x \hat{i} \cdot \hat{i} + r_y \hat{i} \cdot \hat{j} = r_x$		
Inner Product (Real)	$\langle s r\rangle = (s_x\langle 1 + s_y\langle 2)(r_x 1\rangle + r_y 2\rangle) =$			

Length (Complex)	$\begin{aligned}\langle r r \rangle &= (r_x^* \langle 1 + r_y^* \langle 2)(r_x 1\rangle + r_y 2\rangle) \\ &= r_x^* r_x \langle 1 1\rangle + r_x^* r_y \langle 1 2\rangle \\ &\quad + r_y^* r_x \langle 2 1\rangle + r_y^* r_y \langle 2 2\rangle \\ &= r_x^* r_x + r_y^* r_y = r_x ^2 + r_y ^2\end{aligned}$		$(r_x^* \quad r_y^*) \begin{pmatrix} r_x \\ r_y \end{pmatrix} = r_x ^2 + r_y ^2$	
Components (Complex)	$\begin{aligned}\langle 1 r \rangle &= \langle 1 (r_x 1\rangle + r_y 2\rangle) = r_x \langle 1 1\rangle + r_y \langle 1 2\rangle \\ &= r_x\end{aligned}$ $\begin{aligned}\langle 2 r \rangle &= \langle 2 (r_x 1\rangle + r_y 2\rangle) = r_x \langle 2 1\rangle + r_y \langle 2 2\rangle \\ &= r_y\end{aligned}$		$(1 \quad 0) \begin{pmatrix} r_x \\ r_y \end{pmatrix} = r_x$ $(0 \quad 1) \begin{pmatrix} r_x \\ r_y \end{pmatrix} = r_y$	
Inner Product (Complex)	$\begin{aligned}\langle s r \rangle &= (s_x^* \langle 1 + s_y^* \langle 2)(r_x 1\rangle + r_y 2\rangle) \\ &= s_x^* r_x \langle 1 1\rangle + s_x^* r_y \langle 1 2\rangle \\ &\quad + s_y^* r_x \langle 2 1\rangle + s_y^* r_y \langle 2 2\rangle \\ &= s_x^* r_x + s_y^* r_y\end{aligned}$		$(s_x^* \quad s_y^*) \begin{pmatrix} r_x \\ r_y \end{pmatrix} = s_x^* r_x + s_y^* r_y$	