

Quantum Calculations on a Ring I

In this activity, your group will carry out calculations on each of the following normalized abstract quantum states on a ring:

$$|\Phi_a\rangle = \sqrt{\frac{2}{12}} |3\rangle + \sqrt{\frac{1}{12}} |2\rangle + \sqrt{\frac{3}{12}} |0\rangle + \sqrt{\frac{2}{12}} |-1\rangle + \sqrt{\frac{1}{12}} |-3\rangle + \sqrt{\frac{3}{12}} |-4\rangle$$

$$|\Phi_b\rangle \doteq \begin{pmatrix} 0 \\ \sqrt{\frac{2}{12}} \\ \sqrt{\frac{1}{12}} \\ 0 \\ \sqrt{\frac{3}{12}} \\ \sqrt{\frac{2}{12}} \\ 0 \\ \sqrt{\frac{1}{12}} \\ \sqrt{\frac{3}{12}} \end{pmatrix}$$

$$\Phi_c(\phi) \doteq \sqrt{\frac{1}{24\pi r_0}} \left(\sqrt{2}e^{i3\phi} + e^{i2\phi} + \sqrt{3} + \sqrt{2}e^{-i1\phi} + e^{-i3\phi} + \sqrt{3}e^{-i4\phi} \right)$$

For each of the following questions, state the postulate of quantum mechanics you use to complete the calculation and show explicitly how you use that postulate to answer the question.

1. For each state above, what is the probability that you would measure the z -component of angular momentum to be $-4\hbar$? $0\hbar$? $-2\hbar$? $3\hbar$?

2. What other possible values for the z -component of angular momentum could you have obtained with non-zero probability?

3. For each state, what is the probability that you would measure the energy to be $\frac{16\hbar^2}{2I}$? 0? $\frac{4\hbar^2}{2I}$? $\frac{9\hbar^2}{2I}$?
4. If you measured the energy, what possible values could you have obtained with non-zero probability?
5. How are the calculations you made for the different state representations similar and different from each other? Be prepared to compare and contrast the calculations you made for each of the different representations (ket, matrix, wavefunction).