Physics 652 Winter 2024

Homework #7

(due Wednesday, March 6, 2024)

1. (10 pts) Show that

$$\left[J_{\pm},T_q^{(k)}\right]=\hbar\sqrt{k(k+1)-q(q\pm1)}T_{q\pm1}^{(k)}$$
$$\left[J_z,T_q^{(k)}\right]=\hbar q T_q^{(k)}$$

Hint: use the definition of the spherical tensor and the case of infinitesimal rotations, i.e. $D_{q'q}^{(k)}=\left\langle k,q'\right|1-\frac{i}{\hbar}d\varphi\mathbf{J}\cdot\mathbf{n}\left|k,q\right\rangle$ as well as properties of the angular momentum operator.

2. (10 pts)

(a) Using appropriate spherical harmonics, calculate $\left\langle 2,0\right|Y_1^0\left|1,0\right\rangle$

(b) Using the result of (a) along with the Wigner-Eckart theorem, calculate the reduced matrix element $\left\langle 2\|Y_1\|1\right\rangle$.

3. (20 pts) Sakurai 3.45.

4. (10 pts) Consider a system of three spin-1/2 particles, whose interaction is described by the following Hamiltonian:

$$H=-A\left(\mathbf{S}_1\cdot\mathbf{S}_3+\mathbf{S}_2\cdot\mathbf{S}_3\right)$$

Find the system's energy levels and their degeneracies.

4. Reading assignment: Sakurai 3.11.