Physics 652 Winter 2024

Homework #2

(due Wednesday, January 24, 2024)

1. (10 pts) Consider a system which is initially in the state

$$\psi(\theta,\varphi)=\frac{1}{\sqrt{5}}Y_1^{-1}(\theta,\varphi)+\frac{\sqrt{3}}{\sqrt{5}}Y_1^{0}(\theta,\varphi)+\frac{1}{\sqrt{5}}Y_1^{1}(\theta,\varphi)$$

(a) Find $\langle\psi|L_+|\psi\rangle$

(b) If L_z were measured what values would one obtain and with what probabilities?

(c) If after measuring L_z we find l_z = -ħ, what are the uncertainties ΔL_x and ΔL_y?

2. (15 pts) Consider a system whose wave function is

$$\psi(x,y,z)=\frac{1}{4\sqrt{\pi}}\frac{2z^2-x^2-y^2}{r^2}+\sqrt{\frac{3}{\pi}}\frac{xz}{r^2}$$

(a) Calculate $L^2\psi(x,y,z)$ and $L_z\psi(x,y,z)$. Find the total angular momentum of this particle.

(b) Calculate $L_+\psi(x,y,z)$ and $\langle\psi|L_+|\psi\rangle$.

(c) If a measurement of L_z is carried out, what values would one obtain and with what probabilities?

3. (30 pts) One of the most important skills for a successful researcher is the ability to read scientific literature. In this homework problem, you are asked to read the attached paper (PRL 2004) entitled “Fullerene Quantum Gyroscope”. This is a paper related to the rotational spectra of quantum rigid rotators (e.g. diatomic molecules) discussed in Lecture 3, and the purpose of this homework is to present you with an opportunity to relate the “textbook stuff” we learn in class and with the “real stuff”, i.e. research frontier. (As you know, *Physical Review Letters* (PRL) is a prestigious physics journal designed for a broad physics

audience, which publishes “hot” results and has very strict criteria for the results that are eligible for publication.)

Try to obtain the following information from the paper:

1) What is the claim of the paper? (1 pt)
2) Did the authors do experiment, calculations or both? (1 pt)
3) What kind of experiment did the authors do? (1 pts)
4) What are the measured quantities? (2 pts)
5) How is the experimentally measured line separation (see Fig.2) related to the difference in energy levels (e.g. E_l-E_{l-1}) ? (5 pts)
6) In the first column of page 2, the authors state that if the motion of the rotator is confined to a plane, then the energies are $E(m)=Bm^2$. Why? (5 pts)
7) What are the selection rules for Raman scattering? What line separation is expected and why? (5 pts)
8) How is the rotational constant B derived from the experimental data? What is the obtained value for B? (3 pts)
9) What are the obtained values for the moment of inertia and a distance between the atoms (i.e. r_e)? (1 pt)
10) Under what conditions the model of an unperturbed rotator is applicable to this “real” system? (2 pts)
11) What do the authors introduce in the Schroedinger equation to better describe the experimental results? (2 pts)
12) Which model provides a better agreement with the experiment (see Table I)? (2 pts)