Phys 651 Fall 2023

Homework #7

(due Wednesday, November 22, 2023)

1. (20 pts) Consider a particle of mass m in the attractive 1D delta potential given by

$V(x) = -V_0\,\delta(x)$, where $V_0 > 0$.

(a) In the case of negative energies, show that this particle has only one bound state; find the binding energy and the wave function.

(b) What is the probability that the particle remains bound when V_0 suddenly changes to V_0' ?

(c) Study the scattering case (i.e. $E > 0$) and calculate the transmission and reflection coefficients as a function of the wave number k (or energy E).

2. (20 pts) Consider a particle of mass m in the 1D potential well given by $V(x) = -V_0$ if $|x| < a$ and $V(x) = 0$ if $|x| > a$, where V_0 is a positive number.

(a) Write down the Schroedinger equation for the wave functions in three regions ($x < -a$, $-a < x < a$, $x > a$)

(b) Write down a general form of the physically admissible solution

(c) Find the energy spectrum of the bound states (you will encounter some transcendental equations – solve them graphically)

(d) How does the number of the bound states depend on the parameters of the well (i.e. V_0 and a) ?

3. (20 pts) A particle of mass m is subject to an attractive double-delta potential $V(x) = -V_0\,\delta(x-a) - V_0\,\delta(x+a)$, where $V_0 > 0$. Consider only the case of negative energies.

(a) Obtain the wave functions of the bound states. **Hint**: do not forget to use symmetry arguments !!

(b) Derive the eigenvalue equations (you should get two transcendental equations, one for the odd wavefunctions and one for the even wavefunctions)

(c) Specify the number of bound states and the limit of their energies. Is the ground state an even state or an odd state?

(d) Estimate the ground state energy for the limits $a \to 0$ and $a \to \propto$.

4. Reading assignment: Sakurai 2.4-2.5; also look at modern research utilizing scanning tunneling microscopy - Nature Physics 12, 92 (2016).