Phys 651 Fall 2023

Homework #6

(due Wednesday, November 15, 2023)

1. (10 pts) Show that $[\mathbf{R}\cdot\mathbf{P}, H] = 2\,i\hbar\, T - i\hbar\, \mathbf{R}\cdot\nabla V$, where **R** is the position operator in 3D space, **P** is the momentum operator, H is the Hamiltonian ($H = \mathbf{P}^2/2m + V(\mathbf{R})$), and T is the kinetic energy operator ($T = \mathbf{P}^2/2m$).

2. (20 pts) Sakurai 2.10.

3. (30 pts) Consider a wave packet freely moving in 1D so that the wave function at t = 0 is given by

$$\psi(x,0) = A\exp\left[-\frac{x^2}{2a^2} + i\frac{p_0}{\hbar}x\right],$$

where p_0 is a momentum of the particle, and A is the normalization constant.

(a) What is the probability to find the particle in the region $[-\Delta,\ \Delta]$, where Δ is a very small parameter ?

(b) What is the uncertainty of the measurement of x in this state ?

(c) Now consider the state of this system at some later time t and find $\psi(x,t)$ and the probability density $|\psi(x,t)|^2$.

Hint: expand $\psi(x,0)$ in terms of the momentum eigenstates and then propagate them in time.

Make sure to check your function $\psi(x,t)$ (that at t = 0 you get the initially given $\psi(x,0)$).

Don't be afraid of a very long expression you obtained in (c) – just rearrange the terms in a way that you can actually analyze the function in order to answer the following questions:

(d) Did the probability to find the particle in the region $[-\Delta,\ \Delta]$ change ? If yes, how (a qualitative answer is fine) ?

(e) Did the uncertainty of the measurement of x change ? If yes, how (a qualitative answer is fine)?

4.Reading assignment: Sakurai 2.1-2.2, 2.4.