Physics 651 Fall 2023

Homework #4

(due Wednesday, October 25, 2023)

1. (10 pts) Consider a physical system whose Hamiltonian and initial state are given by $H = \varepsilon_0 \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, $|\psi\rangle = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$, where ε_0 has the dimensions of energy.

(a) What values will we obtain when measuring the energy and with what probabilities?

(b) Calculate the expectation value of the Hamiltonian in both ways: (i) using the results of (a), i.e. eigenvalues and probabilities, and (ii) using the definition of an expectation value and given matrix H and the initial state.

2. (15 pts) Consider a system whose Hamiltonian and an operator A are given by the matrices $H = \varepsilon_0 \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$, $A = a_0 \begin{pmatrix} 0 & 4 & 0 \\ 4 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

(a) If we measure the energy, what values will we obtain?

(b) Suppose that when we measure the energy, we obtain a value of $-\varepsilon_0$. Immediately afterwards, we measure A. What values will we obtain for A and with what probabilities ?

(c) What is the expectation value of A?

3. (15 pts) Consider a physical system whose state and two observables A and B are represented by

$$|\psi\rangle = \frac{1}{6}\begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}, A = \frac{1}{\sqrt{2}}\begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & i \\ 0 & -i & 1 \end{pmatrix}, \ B = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

(a) We first measure A and then B. Find the probability of obtaining a value of 0 for A and a value of 1 for B.

(b) We first measure B and then A. Find the probability of obtaining a value of 1 for B and a value of 0 for A.

(c) Compare the results of (a) and (b) and explain.

4. (15 pts) Consider a physical system which has a number of observables that are represented by the following matrices:

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \ B = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 0 & i \\ -1 & -i & 4 \end{pmatrix}, \ C = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{pmatrix}$$

a. Which among these observables are compatible? Find the results of the measurements of the compatible observables.
b. Give a basis of eigenvectors common to these observables.
c. Do the following constitute a C.S.C.O.: {A}, {B}, {C}, {A,B}, {B,C}, {A,C}?

5. Reading assignment: Sakurai, 1.1-1.5.